

# Economic Cycles Theory

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## *Measuring and Monitoring Cycles*

Many large-scale econometric models are now being criticized as representing measurement without theory. The techniques and methods in this lecture provide

***a handbook of cycle tools.***

*Exploratory Data Analysis (EDA)* is a way of summarizing and displaying data to gain the greatest insights. This way first reviews basic growth rate concepts and then discusses methods of calculating trends, cycles, and seasonal patterns. **EDA** serves an important role in helping the analyst judge how best to understand and interpret a particular economic time series .

## **Moving Averages( MA), Moving Differences ( MD), and Moving Medians( MM)**

The most popular way to bring out these underlying patterns while ignoring the noise in the movement is to use certain forms of moving averages or moving medians, or in some cases to eliminate a trend, a moving difference. Two desirable features in the selection of a smoothing routine are

*(1) the smoothing process should not distort the original pattern, and*

*(2) the smoothed pattern should not be adversely affected by an outlier, that is, the method should be robust.*

There are many elaborate smoothing routines.

- (1) the Spencer moving average,*
- (2) the Alexander moving average,*
- (3) moving differences,*
- (4) moving medians, and*
- (5) tapered (moving trimmed) means or medians*

They all share a common purpose -  
to bring out the *underlying pattern* (sometimes referred to  
as a *signal*) while *reducing the random fluctuation*  
(sometimes referred to as *noise*).

## *Spencer moving average*

The Spencer curve is calculated by using a 5\*5\*4\*4 MA, that is a 4-term MA of a 4-term MA of a 5-term MA of a 5-term MA with the final MA being assigned the weights  $-3/4, +3/4, 1, +3/4, -3/4$ .

### *1 Calculate a Four-Term Moving Average.*

$$MA4_1 = (x_1 + x_2 + x_3 + x_4) / 4 \quad (1)$$

$$MA4_2 = (x_2 + x_3 + x_4 + x_5) / 4 \quad (2)$$

$$MA4_3 = (x_3 + x_4 + x_5 + x_6) / 4 \quad (3)$$

$$MA4_4 = (x_4 + x_5 + x_6 + x_7) / 4 \quad (4)$$

where,  $MA4_i$  equals a moving four-term average and  $x_i$  is the time series value.

## 2 *Calculate a Four-Term Moving Average of MA4.*

This takes the same form as formulas 1 to 4.

$$MA4 \times 4 = (MA4_1 + MA4_2 + MA4_3 + MA4_4) / 4 \quad (5)$$

Substitute (1) to (4) into (5) and collect terms:

$$\begin{aligned} MA4 \times 4 &= ((1/4)x_1 + (2/4)x_2 + (3/4)x_3 + (4/4)x_4 + (3/4)x_5 + (2/4)x_6 + (1/4)x_7) / 4 \\ &= (x_1 + 2x_2 + 3x_3 + 4x_4 + 3x_5 + 2x_6 + x_7) / 4 \end{aligned}$$

Then perform this calculation on all the observations of the time series.

## 3 *Calculate a Five-Term Moving Average of MA4 × 4.*

The five-term moving average is:

$$\begin{aligned} MA5 \times 4 \times 4 &= (MA4 \times 4_1 + MA4 \times 4_2 + MA4 \times 4_3 \\ &+ MA4 \times 4_4 + MA4 \times 4_5) / 5 \end{aligned} \quad (6)$$

Substituting (5) into (6), the result for the first term is:

$$MA5 \times 4 \times 4_1 = (1/80)(x_1 + 3x_2 + 6x_3 + 10x_4 + 13x_5 + 14x_6 + 13x_7 + 10x_8 + 6x_9 + 3x_{10} + x_{11}) \quad (7)$$

This same process is calculated for the other expressions,

$$MA5 \times 4 \times 4_2 \quad MA5 \times 4 \times 4_3 \quad MA5 \times 4 \times 4_4 \quad MA5 \times 4 \times 4_5$$

## Homework 4

Formulate the expressions

$$MA5 \times 4 \times 4_2 \quad MA5 \times 4 \times 4_3 \quad MA5 \times 4 \times 4_4 \quad MA5 \times 4 \times 4_5$$

in the way as the expression (7).



#### 4 *Take a Weighted 5-Term MA of MA5 × 4 × 4* .

The final step is to calculate a five term moving average of (6), which have assigned weight. This expression is written as:

$$MA_{\text{Spencer}} = (-3/4)MA5 \times 4 \times 4_1 + (3/4)MA5 \times 4 \times 4_2 + MA5 \times 4 \times 4_3 + (3/4)MA5 \times 4 \times 4_4 - (3/4)MA5 \times 4 \times 4_5 \quad (8)$$

Substituting (6) into (8) yield the sequence, for the first term:

$$MA_{\text{Spencer}} = (-3/320)x_1 + (-6/320)x_2 + (-5/320)x_3 + (3/320)x_4 + (21/320)x_5 + (46/320)x_6 + (67/320)x_7 + (74/320)x_8 + (67/320)x_9 + (46/320)x_{10} + (21/320)x_{11} + (3/320)x_{12} + (-5/320)x_{13} + (-6/320)x_{14} + (-3/320)x_{15}$$

## *Alexander moving average*

Another elaborate smoothing formula is a 7-term moving average by **Alexander**. His formula can be written as:

$$(5x_t + 4x_{t-1} + 3x_{t-2} + 2x_{t-3} + x_{t-4} - x_{t-6}) / 14$$

and corresponds to fitting a second-order polynomial to seven consecutive observations. This moving average has been applied to diffusion indexes that tend to be volatile.

## *Moving Differences and Moving Medians*

An advantage of moving median over a moving average is that a median is less affected by outliers than is the average.

Moving differences can also be useful at times, especially when there is a secular trend. Moving differences provide one way of eliminating the effect of that trend.

## *Tapered (moving trimmed) Means or Medians*

Another robust estimate of central tendency is the trimmed mean or median. To calculate the trimmed average, the values of the series are rank ordered from low to high and then a preset proportion of the series is trimmed or dropped. This method is useful for eliminating the effect of extreme values (resulting from strikes, weather, political events, and other disruptions) on measures of central tendency.

# Homework 5

Employ

- 1) the Spencer moving average,*
- (2) the Alexander moving average,*
- (3) moving differences,*
- (4) moving medians, and*
- (5) tapered (moving trimmed) means or medians*

on the following data sets



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and discuss results.

## The Variable-Length Moving Average

A variant on the standard moving average concept requires adjusting the span of the moving average over time. This smoothing process is known as *the variable-length moving average (VLMA)*. The logic for the VLMA is that if the latest changes are significantly different from its historical average change, this may be a sign that the trend has changed and consequently, it is necessary to adjust the span of the moving average. The calculation of the VLMA requires more thought and effort but the effort may pay off in yielding a more realistic trend.

To construct a VLMA requires the following steps:

- 1 Determine the bounds over which an adjustment will be made to the span of the moving average. A useful application of this methodology is in determining the trend for the NBER's growth cycle.
- 2 Calculate the mean and standard deviation for the data. The sample size should at least equal the longest moving average span.
- 3 Determine the threshold or trigger points when the span of the moving average would change. One such rule to determine the trigger points is to use the mean plus or minus one standard deviation as the criterion for adjust the variable span up or down.

4 Set the framework for determining the partition boundaries. For example

Partition Bound A = mean - ( 1.5 \* standard deviation)

Partition Bound B = mean - ( 0.5 \* standard deviation)

Partition Bound C = mean + ( 0.5 \* standard deviation)

Partition Bound D = mean + ( 1.5 \* standard deviation)

The selection of the number of partition boundaries is somewhat arbitrary, but it is possible to statistically test whether the boundary points maximise the deviation between segments.



5 Preset the amount of observations that the moving average will change when each partition is exceeded.

6 For each period, calculate the new length of the moving average, based on the updated moving mean and standard deviation.

7 Then recalculate the partition boundaries and use that information to calculate the next period's span of the moving average. Loop around Steps 6 and 7 until all the data are used.

# Homework 6

Use the method VLMA for determining trends in the following series:



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## Calculating Growth Rates

The simple growth rate is calculated by:

$$G1 = 100\left(\left(X_t / X_{t-1}\right) - 1\right)$$

where  $G1$  is the simple growth rate and  $X_t$  is the value of the series at time  $t$ .

Growth rates expressed as annual rates, allow direct comparison of quarterly, monthly, and multiyear growth.

This formula is:

$$G2 = 100\left(\left(X_t / X_{t-n}\right)^{(m/n)} - 1\right)$$

$G2$  is the annualized growth rate,  $m$  is the number of periods within the year and  $n$  is the number of periods spanned. For example  $m=4$  for quarterly data and  $n=12$  for 12 periods of quarters.

Another variant on the growth rate calculation is to average the growth rate over a span of time. The formula for the geometric mean is:

$$G3 = \sqrt[n]{R_1 \times R_2 \times \dots \times R_n}$$

where  $R_i$  's are the growth rates that are multiplied together and the  $n$ -th root is taken.

Another variant on calculating growth rates is the smoothed growth rate Geoffrey H. Moore and Victor Zarnowitz have devised this method as a way of smoothing out some of the volatility inherent in percentage changes while attempting to preserve the cycle.

The formulas for these smoothed rates are:

For monthly data

$$G_4 = 100 * \left( \left( \frac{X_t}{\sum_{i=t-1}^{t-13} X_i} \right)^{\frac{12}{6.5}} - 1 \right)$$

For quarterly data

$$G_5 = 100 * \left( \left( \frac{X_t}{\sum_{i=t-1}^{t-5} X_i} \right)^{\frac{4}{2.5}} - 1 \right)$$

**The normal bounds** around the average growth rate are calculated by adding and subtracting one-half of the average absolute deviation around the mean growth rate.

The formulas for these calculations are:

$$B_{upper} = G3 + \left| \sum_{i=1}^n (R_i - G3) \right| / 2n$$

$$B_{lower} = G3 - \left| \sum_{i=1}^n (R_i - G3) \right| / 2n$$

These normal high-low bands provide a monitoring range for quick evaluation of an economic indicator.

# Homework 7

Calculate growth rate and normal bounds for series in the following file:



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## Persons Method of Time Series Decomposition

One of the earliest methods of analysing time series data was to decompose the series into various types of movement. By W.M.Persons, a time series could be divided into four components: *the seasonal, the trend, the cyclical movement, and the residual or irregular component*. This time series decomposition approach also has become the basic for the Census Bureau's X-12 seasonal adjustment method. Conceptually, the series can be defined as either the sum of  $T + C + S + I$  or the product  $T \times C \times S \times I$ , where  $T$  is the trend,  $C$  is the cycle,  $S$  is the seasonal, and  $I$  is the irregular component.

## Next lecture

- *measuring trends and cycles*
- *business cycle summary statistics*