Frequency and Amplitude Estimation Based on the Teager Operator

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Presentation Overview

- Teager (-Kaiser) Operator: ‘energy’ of a signal (Kaiser, 1990)
  - Definition, derivation, behavior, limitations
- Subsequent work
  - Enhancements, extensions, applications
  - “energy separation” and tone resolution
- My project

Kaiser, 1990

- “On a simple algorithm to calculate the ‘energy’ of a signal,” James F. Kaiser, 1990
- Q: What is a signal’s energy?
  - Time domain: average of sum of squares of amplitude?
  - Frequency domain: magnitude of frequency components?

Kaiser 1990…

Kaiser says, "No!"

- Talk about the energy required to generate a signal
- Physics: spring energy

$$E = \frac{1}{2} k x^2 + \frac{1}{2} m \dot{x}^2 .$$

$$E = \frac{1}{2} m \omega^2 A^2$$

$$E \propto A^2 \omega^2 .$$

Kaiser 1990…

Derivation

- Discrete time, single component: $x_n = A \cos(\Omega n + \varphi)$
- Combine x_n, x_{n+1}, x_{n-1} to see $A^2 \sin^2(\Omega) = x_n^2 - x_{n+1} x_{n-1}$
- For small Ω, $\sin(\Omega) \approx \Omega$, so… $A^2\Omega^2 \approx x_n^2 - x_{n+1} x_{n-1}$

Kaiser 1990…

$$E_n = A^2\boldsymbol{\Omega}^2 \approx x_n^2 - x_{n+1}\, x_{n-1}$$

- Instantaneous measurement of E_n
- Reasonable for $f/f_s < 1/8$
- Non-linear
- Easy to calculate! Complexity O(n), minimal delay in real-time
  - Compare to STFT…

Kaiser 1990…

Basic Behavior: Ex. 1

(Steady sine wave: E is constant)

Amplitude tracking:

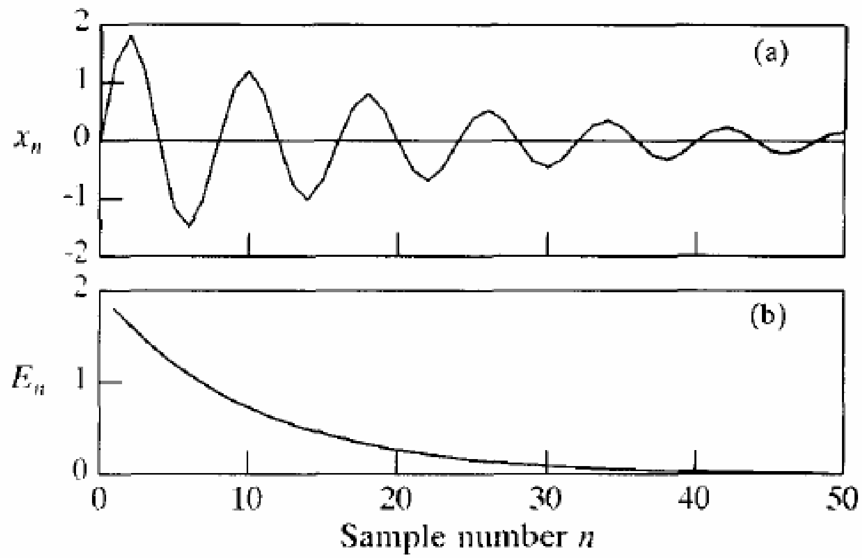


Fig. 1 The algorithm applied to a damped sinusoidal signal. (a) The damped sinusoidal signal $2e^{-0.05n}\sin(n\pi/4)$. (b) The output of the algorithm which varies approximately as $4e^{-0.1n}(\pi/4)^2$.

Kaiser 1990…

Basic Behavior: Ex. 2

Frequency tracking

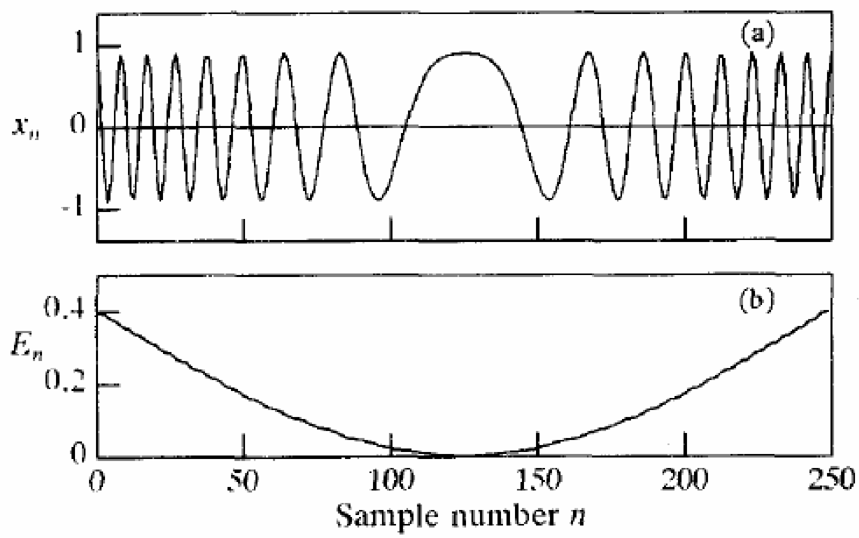


Fig. 2 The application of the algorithm to a chirp signal. (a) The chirp signal of amplitude 0.9 with frequency decreasing linearly from $\pi/4$ to zero and then increasing linearly back to $\pi/4$ in 251 samples. (b) The output of the algorithm which decreases as $\sin^2[\Omega(n)]$ from 0.398 to 0 and then increases similarly back to 0.398.

Kaiser 1990…

Basic Behavior: Ex. 3

Multiple Components

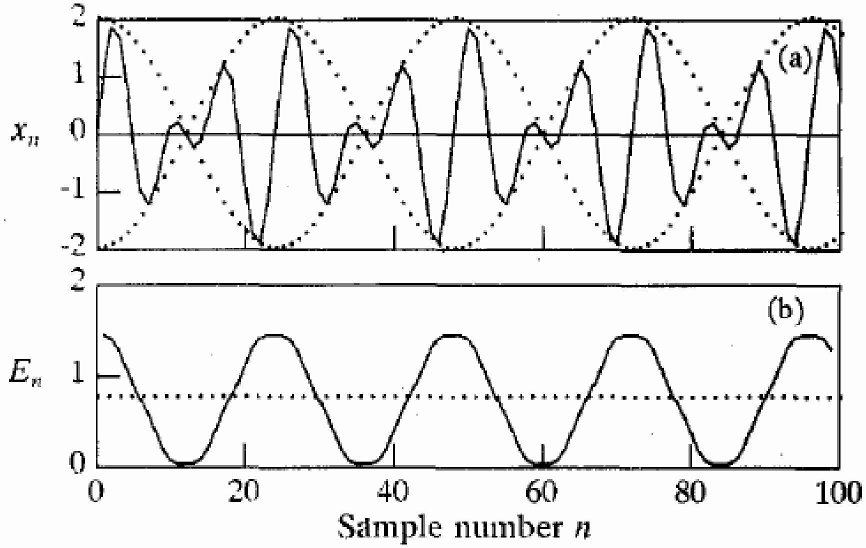


Fig. 4 The algorithm applied to a composite of sinusoids. (a) Sum of two sinusoids of frequency $\pi/4$ and $\pi/6$, each of unit amplitude. (b) The output of the algorithm when applied to (a) oscillates at the difference frequency, $\pi/12$, about the sum of the energies in the two sinusoids as per (31).

Kaiser 1990…

Drawbacks

- $x_c = x_a + x_b \not\rightarrow E_c = E_a + E_b$?
  (it's non-linear)
- Use a filterbank to separate components before applying TKEO
- Very sensitive to noise, but lowpass filter improves performance

Subsequent work

- Continuous domain: $\Psi[x(t)] = \dot{x}^2 - x\ddot{x}$
- Mathematical properties of the operator
- How to separate amplitude & frequency contributions
- How to separate multiple sinusoidal components
- Resolution of two close tones

Maragos, Kaiser, & Quatieri, 1992

- “On Separating Amplitude from Frequency Modulations Using Energy Operators”
- Estimate the amplitude envelope and track the instantaneous frequency (separately!)
- Apply to speech analysis

Maragos et al 1992…

Apply Teager to AM-FM

Discrete AM-FM signal: $x(n) = a(n)\cos[\phi(n)] = a(n)\cos(\Omega_c n + \Omega_m \int_0^n q(k)dk + \theta)$

and
under the assumptions: *(DA1):* a has bandwidth $\Omega_a \ll \Omega_c$.
(DA2): $8\sin^2[(\Omega_a + \Omega_f)/2] \ll [\sin^2(\Omega_i)]_{max}$.
(DA3): (a) $\Omega_f \ll 1$ for AM–FM/Cosine; or
(b) $(\Omega_m/N) \ll [\sin^2(\Omega_i)]_{max}$ for AM–FM/Linear.

Teager estimation: $\Psi[a(n)\cos(\int_0^n \Omega_i(m)dm + \theta)] \approx a^2(n)\sin^2[\Omega_i(n)]$

Maragos et al 1992…

DESA-2

- Find the time-varying frequency and envelope, for f ≤ .25f_s

$$\arcsin\left(\sqrt{\frac{\Psi[x(n+1)-x(n-1)]}{4\Psi[x(n)]}}\right) \approx \Omega_i(n)$$

$$\frac{2\Psi[x(n)]}{\sqrt{\Psi[x(n+1)-x(n-1)]}} \approx |a(n)|$$

Maragos et al 1992…

DESA-2 and DESA-1

- Same idea
- DESA-1 works for $f \leq .5f_s$
- Errors 1% or less!
- Will work with constant frequency and amplitude signals

Kumaresan, et al 1992

- "Instantaneous Non-Linear Operators for Tracking Multicomponent Signal Parameters"
- Extract amplitude and frequency information from a signal having multiple (unrelated) sinusoidal components, without filtering first
- Aim for better resolution than STFT methods

Kumaresan et al 1992…

Enter the Matrix

- Toeplitz matrices: constant diagonals

$$\Psi(x_n) = \mathrm{Det}\begin{bmatrix} x_n & x_{n+1} \\ x_{n-1} & x_n \end{bmatrix}$$

- Determinant is time-invariant

Kumaresan et al 1992…

More matrices

- Extend time-invariance property to arbitrarily large matrices:
  - $X_m(n)$ (values taken from original signal from x_{n-m+1} to x_{n+m-1})
  - $Y_m(n)$ (values taken from the output of a linear time-invariant filter)

Kumaresan et al 1992…

1- and 2-components

- 1-component: DESA-1, DESA-2 are special cases
- 2-component: solve for ω_1 and ω_2 from:

$$\frac{1}{2}\sqrt{\frac{\Delta y_1}{\Delta x}} = \cos\frac{\omega_1(n)-\omega_2(n)}{2} + \cos\frac{\omega_1(n)+\omega_2(n)}{2}$$

$$\frac{1}{2}\sqrt{\frac{\Delta y_2}{\Delta x}} = \cos\frac{\omega_1(n)-\omega_2(n)}{2} - \cos\frac{\omega_1(n)+\omega_2(n)}{2}$$

Kumaresan et al 1992…

Drawbacks

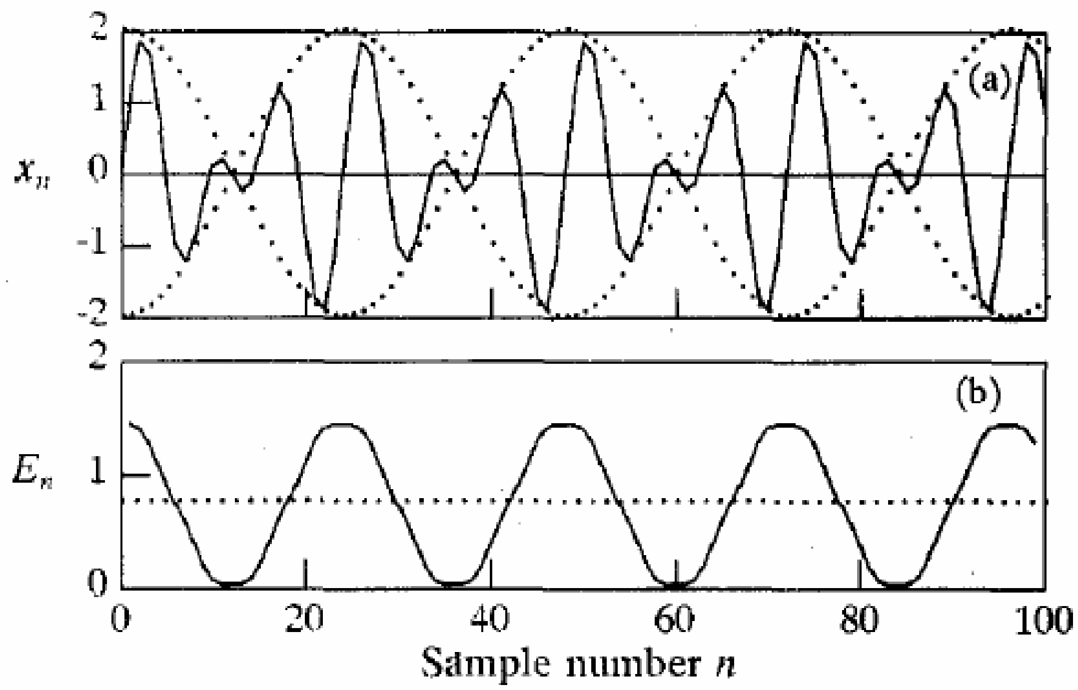
- N > 4 components: Involves rooting a polynomial: similar to Prony’s method
- Efficiency is no longer a benefit
- Noise is a big problem: necessary to use higher-order models

Lin et al, 1995

- "A Generalization to the Teager-Kaiser Energy Function & Application to Resolving Two Closely-Spaced Tones"
- Show a generalized form of the TKEO
- Show how energy measure can be used in conjunction with traditional analysis

Lin et al 1995…

TKEO accentuates difference frequency



Lin et al 1995…

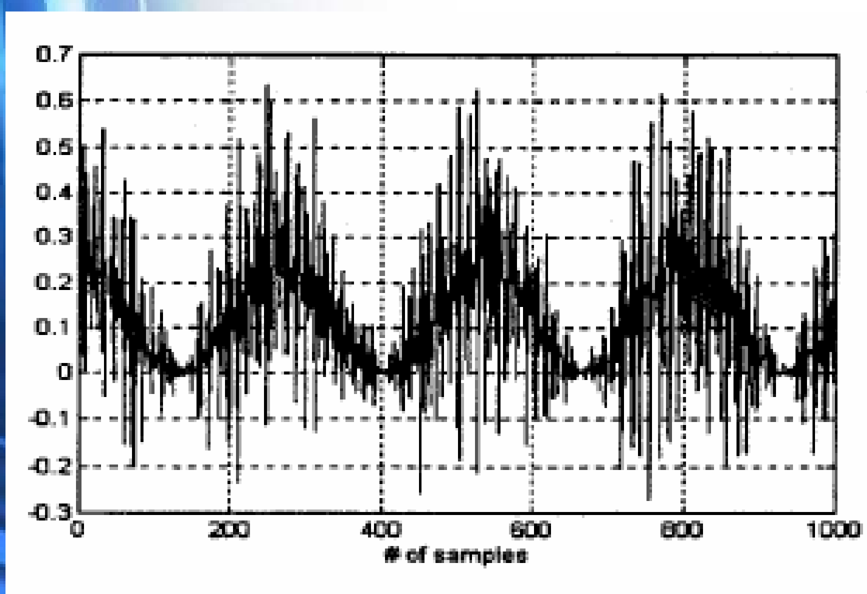
Generalize TKEO

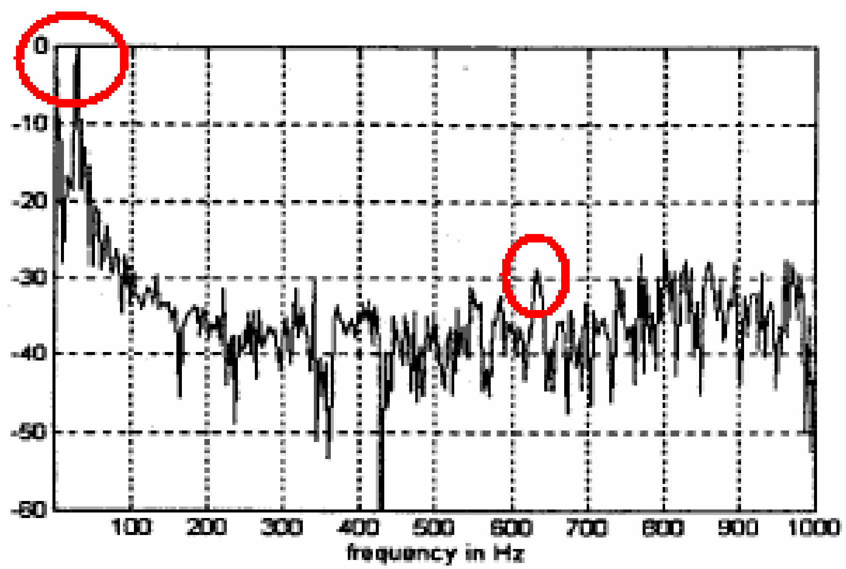
- $\Psi_n = x_n^2 - x_{n+1}\, x_{n-1}$
- Let's examine $x_n^2 - x_{n+M}\, x_{n-M}$
- Findings:
  - M=1 (as in TKEO) emphasizes f_1-f_2 when no noise is present
  - M can be chosen carefully to emphasize either f_1-f_2 or f_1+f_2 in the presence of noise

Lin et al 1995…

Results

- Original TKEO applied to noisy 300Hz + 330Hz signal, time and spectrum behavior:

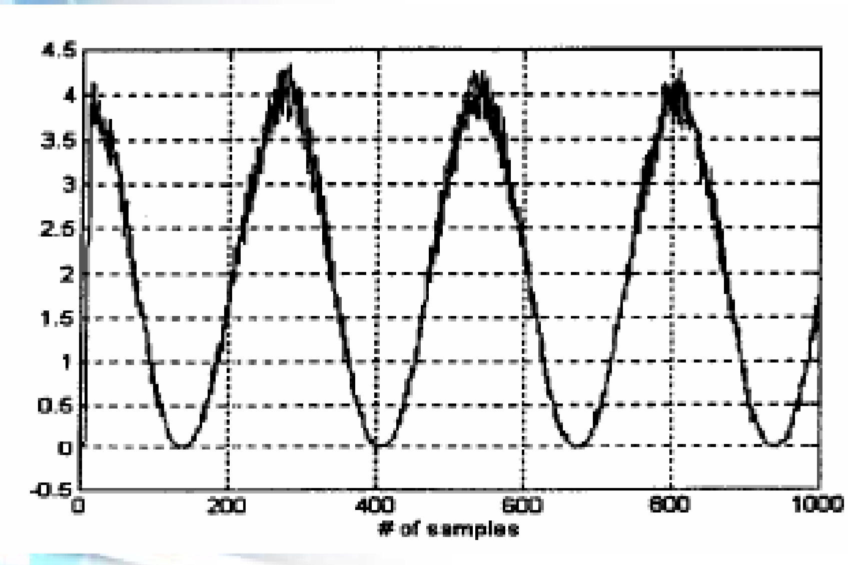


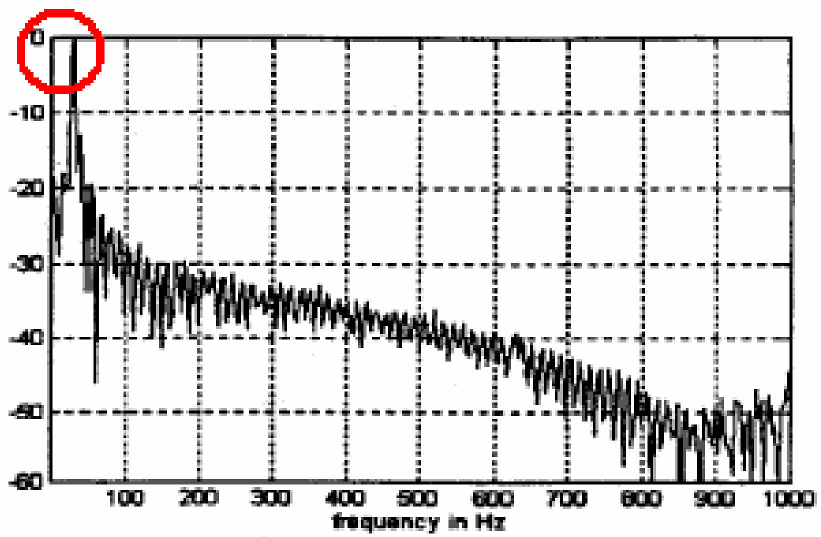


Lin et al 1995…

Results

- Accentuate difference with M=6, time and spectrum behavior:

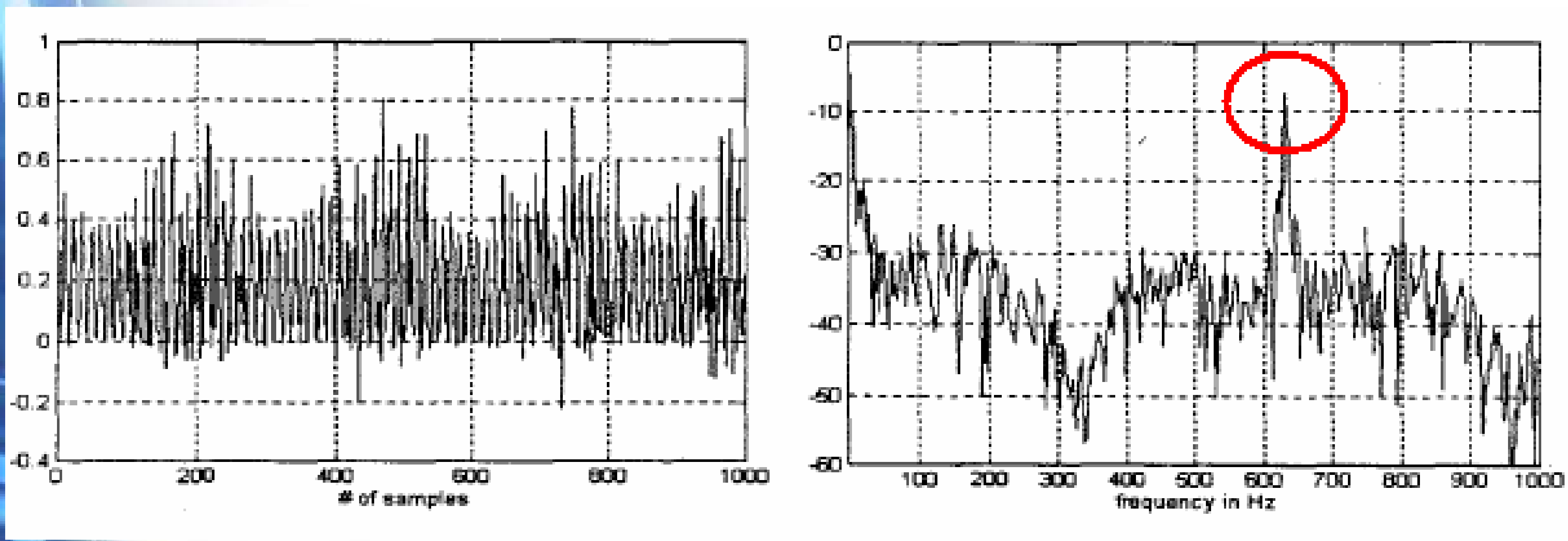




Lin et al 1995…

Results

- Accentuate summation with M=25, time and spectrum behavior:



Lin et al 1995…

Preprocessing application

- Select M_1 and M_2 and apply generalized energy operator:

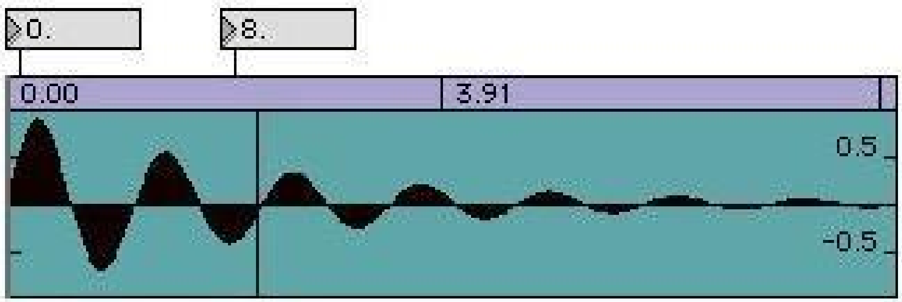
  Transform a signal containing f_1 and f_2 into one containing (f_1+f_2) and (f_1-f_2)
- Then use any spectral analysis method on the output!

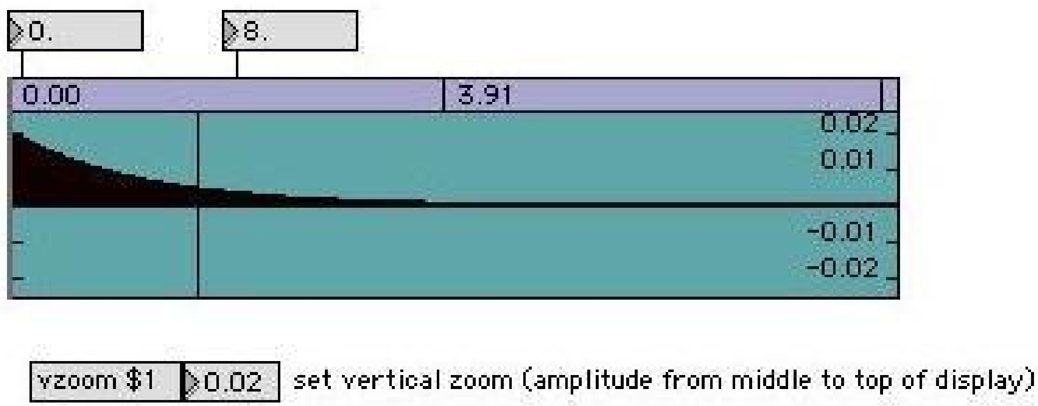
My Project

- Implement a “teager~” object ☑
- Implement examples in Max/MSP showing basic operator behavior on 1 and 2 signals ☑
- Implement DESA-2?
- …?

My Project: Exponential decay

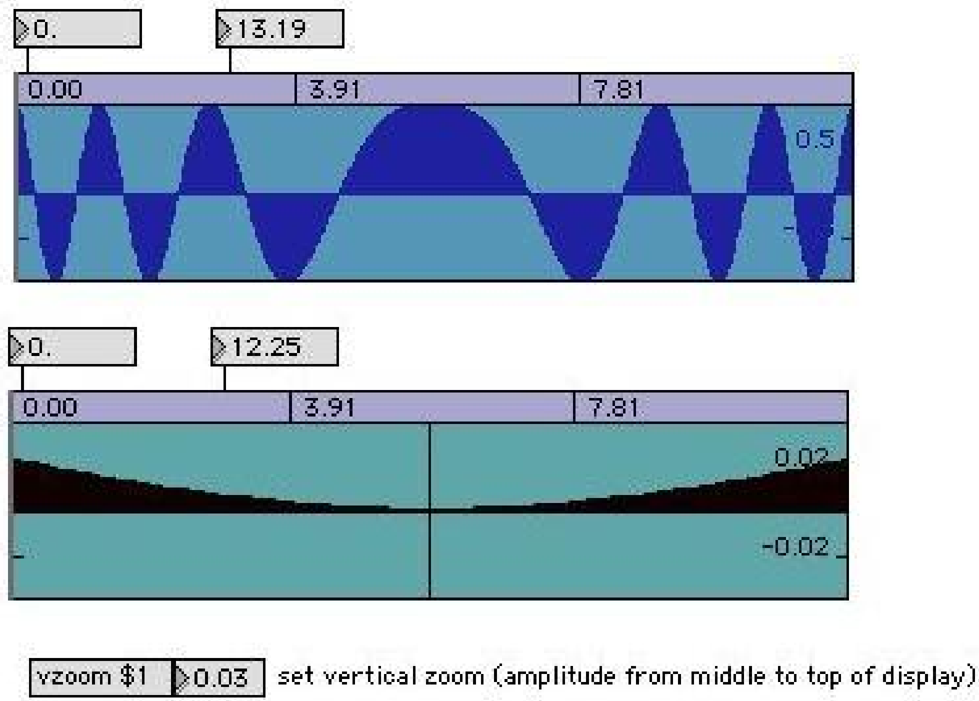
- Signal and Teager function (time domain)





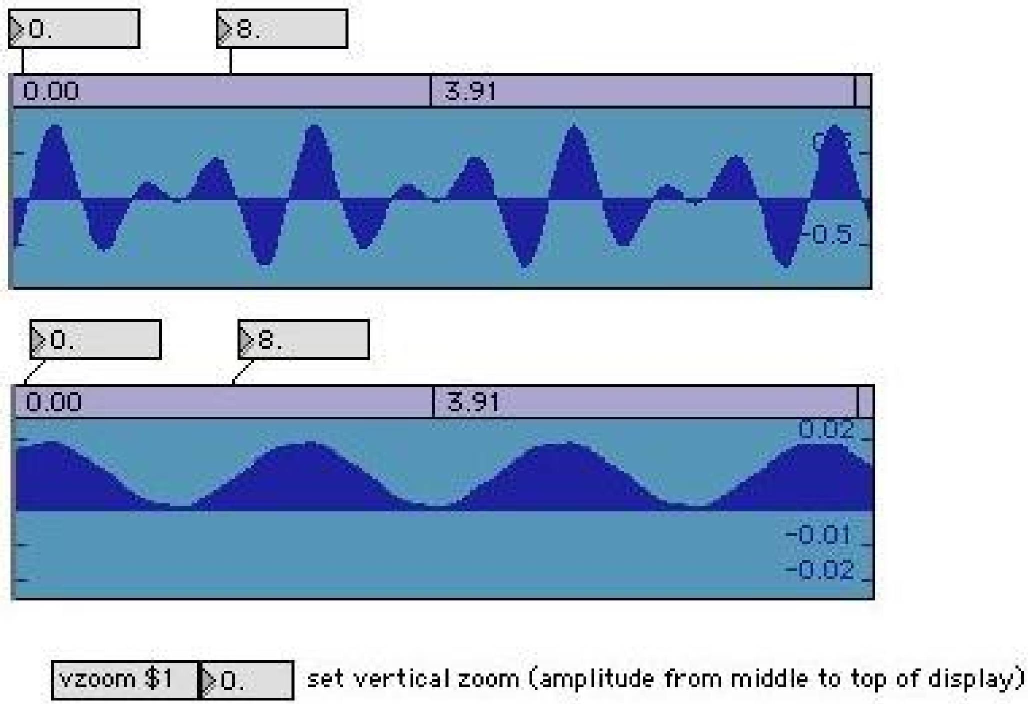
My Project: Chirp

- Signal and Teager function (time domain)



My Project: Two components

- Signal and Teager function (time domain)



Conclusions

- Teager operator is efficient, fast, derived from physics
- Building block for more complex (more useful, and less efficient?) analysis
- Broad range of applications: acoustics, medicine, speech recognition, speech pathology, …
- Still room for more work

References (1 of 2...)

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