# The US Dollar Index Futures Contract

## I. Introduction

Redfield (1986) and Eytan, Harpaz, and Krull (1988) present descriptions and pricing models for the US dollar index (USDX) futures contract. This article updates the original Redfield (1986) and Eytan, Harpaz, and Krull (1988) studies to incorporate changes in the contract specifications.

The USDX futures contract has two features that influence its pricing and its use. First, the USDX index is a geometric average, rather than an arithmetic average, of the constituent currencies. Second, the foreign exchange (FX) rates in the USDX index (in US dollars per foreign exchange rate) are in the denominator of the index, implying that a dollar appreciation leads to a higher index level. Both the geometric averaging and the use of quoting convention have implication for the use of the USDX futures contract in hedging a foreign exchange exposure.

This article begins by providing updated coverage of the USDX futures contract, which has changed contract specifications since these earlier publications. Next, this article presents an intuitive explanation of why the standard cost-of-carry futures pricing formula is inappropriate for the USDX futures contract, which must be adjusted by an adjustment factor (sometimes referred to as a "volatility premium"). This article then notes the factors that would play a role in determining the magnitude of the adjustment factor.

## II. History and Current Status of the USDX Futures Contract

The USDX futures contract began trading on November 20, 1985 on the Financial Instruments Exchange, a division of the New York Cotton Exchange, which is now part of the New York Board of Trade (NYBOT). The USDX index was originally a geometrically weighted average of ten different currencies, with each currency representing a country that was a major trading partner with the United States. With the introduction of the Euro, the USDX index became a geometrically weighted average of six currencies, which represent five major U.S. trading partners and the Euro. Appendix A describes the current contract specifications for the USDX futures contract.

*Index Formula*

The formula for the index level on date $t$ is the product of the six currencies spot rates, each raised a power related to a currency-specific weight. The general formula for the index can be written as

$$USDX_t = K\prod_{i=1}^{N}(FX_{i,t})^{-w_i}$$

where $USDX_t$ is the calculated level of the USDX index on date $t$,
$FX_{i,t}$ is the foreign exchange rate (U.S. dollars per foreign currency unit) for currency $i$ on date $t$,
$w_i$ is the weight associated with currency $i$ (the weights are determined by the contract specs and sum to one, i.e., $\sum_{i=1}^{N} w_i = 1$);
$N$ is the number of currencies in the index (for the USDX index, $N$ is currently six and was formerly ten); and
$K$ is a constant.[1]

Under the current USDX futures contract specs, the USDX index is equal to

$$USDX_t = 50.14348112 \times (Euro_t)^{-0.576} \times (Yen_t)^{-0.136} \times (Sterling_t)^{-0.119} \times (CanadianDollar_t)^{-0.091} \times (SwedishKroner_t)^{-0.042} \times (SwissFranc_t)^{-0.036}$$

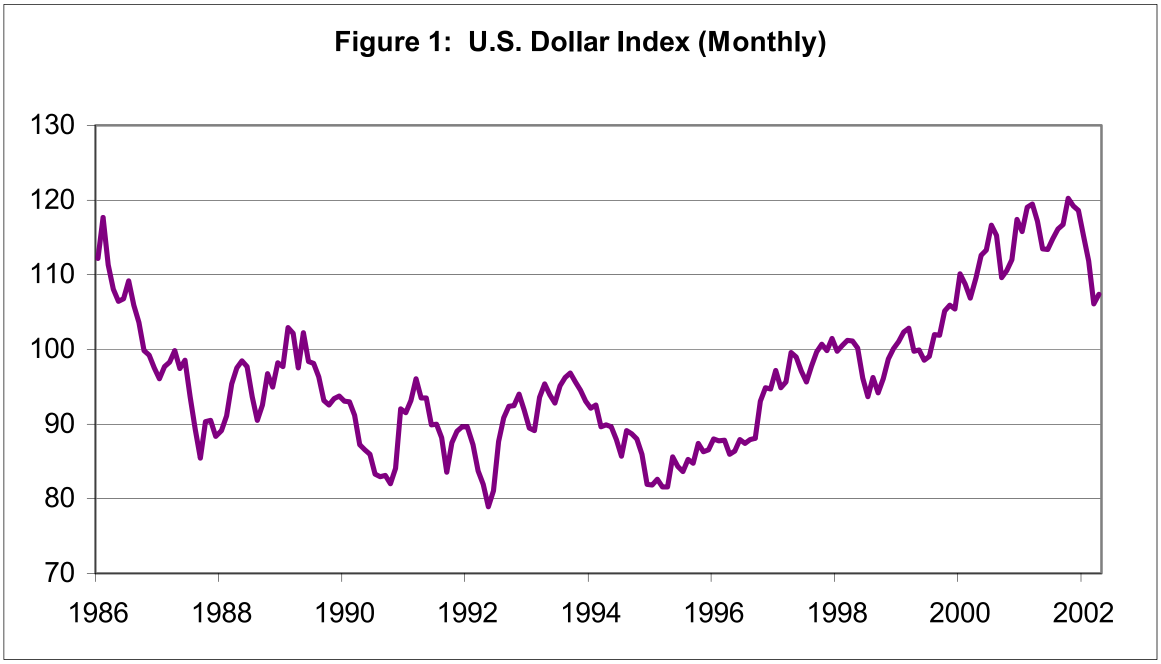
Table 1 shows the FX rates of the constituents currencies on July 31, 2002 (from Bloomberg), the weights applied to the FX rates, the FX rate raised to (the negative of) that weight, and the product of the FX rates raised to their weight. This product, when multiplied by the index constant of 50.14348112, yields the index level of 107.306.

| **TABLE 1 as of July 31, 2002** | | | |
|---|---|---|---|
| **Foreign currency (FX)** | ***FX rate*** | ***Weight*** | ***(FX rate)*** $^{-Weight}$ |
| Euro | 0.978 | 0.576 | 1.013 |
| Yen | 0.008 | 0.136 | 1.917 |
| Sterling | 1.563 | 0.119 | 0.948 |
| Canadian Dollar | 0.631 | 0.091 | 1.043 |
| Swedish Kroner | 0.106 | 0.042 | 1.099 |
| Swiss Franc | 0.673 | 0.036 | 1.014 |
| SUM: | - | 1.000 | - |
| PRODUCT: | - | - | 2.140 |
| INDEX LEVEL* | | | 107.31 |

* Index level is PRODUCT times the index constant of 50.14348112.

Figure 1 shows the monthly levels of the USDX index from the USDX futures contract from April 1986, at a level of 112.140, through July 31, 2002, when it officially closed at 107.41.

[1] The constant, currently equal to 50.14348112, was set back at the initiation of the index in March 1973, in order to make the index equal to 100.00 at that time.




Source: Bloomberg

## III. Determining the Fair Futures Price of the USDX Futures Contract

Two features of the USDX index formula are important in determining the fair futures price for a USDX index futures contract. First, the USDX index is constructed such that, as the U.S. dollar appreciates, the index level rises. Therefore, a position that is long the foreign currencies (and will therefore lose value if the U.S. dollar appreciates) will need a long position in the USDX futures contract to hedge this exposure. This situation is sometimes referred to as a "long-long" hedge. Second, the USDX index level is constructed as a *geometric* mean (where the FX rates are multiplied together), rather than as an *arithmetic* mean, of the constituent FX rates. The use of the geometric mean implies that the standard cost-of-carry model to arrive at the futures price is not applicable for this contract. As this section demonstrates, each of these conventions within the USDX formula play a part in determining how to fairly value a futures contract based on this index.

### *A. Pricing the USDX futures contract when there is no uncertainty in foreign exchange rates*

In order to understand how to price the USDX futures contract, first consider a situation where there is no uncertainty about the level of foreign exchange rates in the future. In this situation, we could calculate the forward rates in each of the six currencies that constitute the USDX index. Table 1 shows the foreign exchange rates, the local 'risk-free' rates (as measured by an estimate of the local interest rates, from Bloomberg), and the resulting forward rates for each of the six currencies, as of July 31, 2002.

| TABLE 2 | | | | |
|---|---|---|---|---|
| **Foreign currency** | **FX rate** | **Foreign currency 3-month interest rate** | **US dollar 3-month interest rate** | **Implied Forward rate** |
| Euro | 0.978 | 3.378 % | 1.824% | 0.975 |
| Yen | 0.008 | 0.068 % | 1.824 % | 0.008 |
| Sterling | 1.563 | 4.003 % | 1.824 % | 1.555 |
| Canadian dollar | 0.631 | 2.883 % | 1.824 % | 0.630 |
| Swedish Kroner | 0.106 | 4.470 % | 1.824 % | 0.105 |
| Swiss Franc | 0.673 | 0.822 % | 1.824 % | 0.675 |

Given these FX rates and interest rates, the current USDX index level is equal to 107.306, as derived in Section II. The assumption of no uncertainty in exchange rates implies that the current 3-month forward FX rate will exactly equal the realized FX rate in three months. Therefore, the USDX index in three months will be equal to

$$\begin{aligned} USDX_t^{forward} &= 50.14348112 \times (0.975)^{-0.576} \times (0.008)^{-0.136} \times (1.555)^{-0.119} \times (0.630)^{-0.091} \\ &\quad \times (0.105)^{0.042} \times (0.675)^{-0.036} \\ &= 107.598 \end{aligned}$$

Therefore, the fair futures value without uncertainty would be equal to 107.598.

Now let's consider the more realistic case with uncertainty in the level of future exchange rates.

*B. When the spot rate equals the forward rate on average, but with volatility: The effect of geometric averaging*

Consider as a base case the exchange rates as listed in Table 1. Suppose that the FX rate for the pound sterling appreciates, but the FX rate for the Canadian dollar depreciates, so that the resulting USDX index rate is unchanged. In particular, the pound sterling appreciates by 100% and the Canadian dollar depreciates by 59.6%.

Suppose an investor is long \$1 million in the six constituent currencies of the USDX index, in the proper weights (57.6% in euro, 13.5% in yen, etc.).[2] Investors who are long the foreign currencies and use the USDX futures contract to hedge their exposure will go long \$1 million of the futures contracts. If the exchange rates change such that the pound sterling appreciates by 100% while the Canadian dollar depreciates by 59.6%, the "hedged" investor will experience an unexpected P&L from the combined

[2] The examples in this paper ignore important real-world considerations such as the bid-ask spread, brokerage commissions, and futures mark-to-market margin payments.

cash-market and futures-market position. In particular, the futures position will not change in value at all, since the index level would not changed, but the cash position will change. The pound sterling position will change by $119,000 (the amount of the change (100%) times the size of the position (11.9% of $1 million)) while the Canadian dollar position will change by $54,236 (the amount of the change (59.6%) times the size of the position (9.1% of $1 million)).

Why is the dollar change in the investors' positions not reflected in the USDX index, and therefore not in the UDSX index futures contract? The difference stems from fact that the value of the position is determined by the (weighted) sum of the individual components of the trading strategy, while the index is determined by the (weighted) product of the individual components of the index. In other words, the portfolio performance is an arithmetic average, while the index performance (and therefore the futures contract performance) is a geometric average. Mathematically, for any given change in the component exchange rates, the geometric average will always be less than the arithmetic average.

This difference between arithmetic and geometric averaging is the source of the divergence between the index (and therefore futures contract) performance and the portfolio performance. The larger the divergence of performance of the different currencies, the larger the divergence between the geometric average and the arithmetic average. Therefore, as Eytan, Harpaz, and Krull (1988) point out, the divergence between the geometric and arithmetic averages depend on both the volatilities of the individual currencies and their co-movements (sometimes referred to as their "correlations").

In fact, the only time that the geometric average and the arithmetic average yield the same return is if all of the individual components of the index change by the same amount. However, the second feature of the USDX index – where the USDX index is quoted such that a dollar appreciation leads to an increase in the index – prevents the standard cost-of-carry model from pricing the USDX index futures contract, even when all of the individual currencies change by the same proportion. We turn to this example next.

### *C. Pricing the USDX futures contract with a simple structure to the uncertainty: The effect of pricing in European terms*

Now consider a situation where all of the exchange rates are perfectly correlated. Specifically, suppose that we knew that all exchange rates shifted by the same percentage, denoted $x$.

In this situation, consider how an investor who has a long cash position in the foreign currencies would hedge that exposure. Suppose that the investor were long $Z_i$ units of currency $i$ and the exchange rates (in US dollar per foreign currency units) were denoted $FX_i$, so that the current value of their holdings (in U.S. dollars) would be

$$F_0 = \sum_{i=1}^{6} FX_i Z_i \ .$$

We denote $U_0$ as the value of the holdings of the futures contract, so that, for a hedged portfolio, $F_0 = U_0$.

Let us consider the value of foreign-exchange holdings and the futures contract if all six currencies have a proportional shift of $x$. In this scenario, the value of the foreign exchange holdings, denoted $F_1$, will be equal to

$$\begin{aligned} F_1 &= \sum_{i=1}^{6} (1+x) FX_i Z_i \\ &= (1+x) \sum_{i=1}^{6} FX_i Z_i \\ &= (1+x) F_0 \end{aligned}$$

With the shift in exchange rates of $x$, the index level, and therefore the futures contract, will also shift by the amount, $x$:

$$\begin{aligned} U_1 &= K \prod_{i=1}^{N} (FX_0 (1+x))^{-w_i} \\ &= K \prod_{i=1}^{N} (FX_0)^{-w_i} (1+x)^{-w_i} \\ &= (1+x)^{-1} K \prod_{i=1}^{N} (FX_0)^{-w_i} \\ &= (1+x)^{-1} U_0 \\ &= U_0 /(1+x) \end{aligned}$$

In this situation, the change in the value of the cash holdings would be

$$\begin{aligned} F_1 - F_0 &= (1+x)F_0 - F_0 \\ &= xF_0 \end{aligned}$$

and the change in the value of the futures position would be

$$\begin{aligned} U_1 - U_0 &= \frac{U_0}{1+x} - U_0 \\ &= \frac{-xU_0}{1+x} \end{aligned}$$

The combined gain or loss would then be the sum of these values:

$$(F_1 - F_0) + (U_1 - U_0) = xF_0 - \frac{xU_0}{1+x}$$

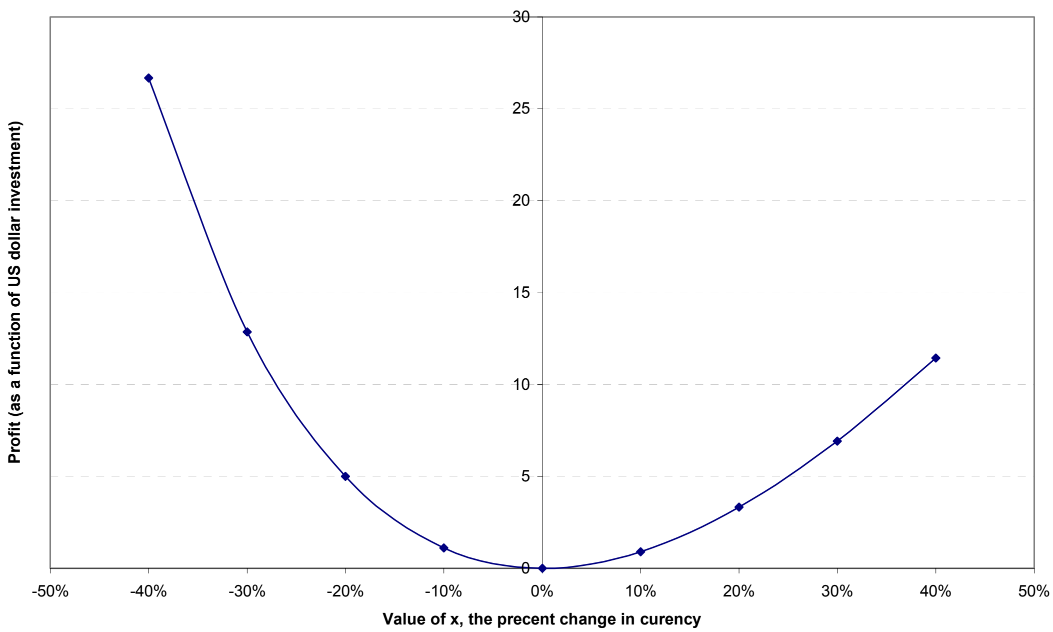
Since $F_0 = U_0$, this can be simplified to

$$xF_0 - \frac{xU_0}{1+x} = xF_0 - \frac{xF_0}{1+x}$$
$$= \frac{x^2}{(1+x)} F_0$$

Therefore, the proportional change in the initial investment of $F_0$ is $x^2/(1+x)$.

Figure 2 shows how the P&L from this combined cash and futures position changes as a function of the level of $x$. When $x$ is zero, as in the no-uncertainty case of section III.A, we have a zero P&L – consistent with a hedged position. As $x$ deviates from zero, the combined cash and futures position always yields a positive profit – regardless of whether $x$ is positive or negative.

**Figure 2: Profit Function: $x^2/(1+x)$**




Therefore, if uncertainty takes the form of proportional shifts in all foreign exchange rates, denoted by $x$, then the fair futures price will depend on the magnitude of

$x$. The fair futures price in this case will depend on the probability of a large move upward or a large movement downward.[3]

### *D. Pricing the USDX future contract under more general uncertainty conditions*

The cases of no uncertainty, reviewed in Section III.A, and of simple structures of uncertainty, reviewed in Section III.B and Section III.C, are clearly implausible models of reality. They do provide, however, the intuition for how a model for the USDX futures contract must deviate from the standard cost-of-carry model. In general, the fair futures price of the USDX futures contract depends on the foreign exchange volatility structure that the market (i.e., the traders in the market) believes. From the previous sections, though, we do have some restrictions about how an accurate pricing model will price the futures contract. First, if the markets have no foreign exchange uncertainty, the USDX futures contract pricing model should provide the same result as the simple cost of carry model. In fact, the lower the volatility in the market, the closer the market-determined futures price should be to the standard cost-of-carry model of futures prices. Second, the more dispersion that markets expect in the performance of the different currencies in the index, the more the market-determined futures price should deviate from the standard cost-of-carry model of futures prices. Third, if the markets have only the simple uncertainty associated with proportional shifts in all foreign exchange rates, the pricing model should return a futures price that declines in the level of that shift, which we have parameterized with the variable $x$.

Those who are interested in an exact pricing formula for the USDX futures contract, based on assumed distributions of the USDX index component foreign exchange rates, should consult Eytan, Harpaz, and Krull (1988). The formulas given in their paper need to be adjusted for the change in the index from ten currencies to six currencies, with the replacement of five European currencies with the Euro.

[3] The dependence of the difference between the geometric average and arithmetic average on the volatility of the exchange rates is likely the sources of the term "volatility premium" in describing the adjustment factor in determining the fair futures price of the USDX futures contract.

# Appendix A

# USDX Futures Contract Specifications

**U.S. Dollar Index (USDX) Futures Specifications (as of June 30, 2002):**

| | |
|---|---|
| **Contract size:** | $1000 times the USDX index. |
| **Trading hours:** | 3:00 a.m. to 8:00 a.m. and 8:05 a.m. to 3:00 p.m. |
| **Contract months:** | March, June, September, December |
| **Ticker symbol:** | DX |
| **Price quotation:** | The U.S. dollar index is quoted as a percent of its value as of March 1973, calculated to two decimal places (e.g., on July 31, 2002, the USDX index officially closed at 107.41) |
| **Minimum price fluctuation:** | The minimum price fluctuation, or "tick size" for the USDX index is 0.01 USDX point, which is equivalent to $10.00 per futures contract. |
| **Limit on daily price move:** | 200 ticks above & below prior day's settlement, except during last 30 minutes of any trading session when no limit applies. Should the price reach the limit and remain within 100 ticks of the limit for 15 minutes, then new limits will be established 200 ticks above and below the previous price limit |
| **Position limits:** | None |
| **Last day of trading**: | 2nd business day prior to the 3rd Wednesday of the expiring month. On the last trading day, trading ceases at 10:16 a.m. |
| **Settlement procedure:** | Contracts held to expiration are settled in cash, based on the value of the USDX index at 10am (New York time) on the last day of trading for an expiring contract. The USDX settlement value is computed by Reuters LTD, in accordance to New York Cotton Exchange regulations. |

## References

Eytan, T. Hanan, Giora Harpaz, Steven Krull (1988), "The Pricing of Dollar Index Futures Contracts," *Journal of Futures Markets,* Volume 8, Number 2, April 1988, pages 127-139.

Redfield, Corey B. (1986), "A Theoretical Analysis of the Volatility Premium in the Dollar Index Contract," *Journal of Futures Markets,* Volume 6, Number 4, August 1986, pages 619-627.