Examining the Nelson-Siegel Class of Term Structure Models*

In-sample fit versus out-of-sample forecasting performance

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September 23, 2007

Abstract

In this paper I examine various extensions of the Nelson and Siegel (1987) model with the purpose of fitting and forecasting the term structure of interest rates. As expected, I find that using more flexible models leads to a better in-sample fit of the term structure. However, I show that the out-of-sample predictability improves as well. A four-factor model, which adds a second slope factor to the three-factor Nelson-Siegel model, forecasts particularly well. Especially with a one-step state-space estimation approach the four-factor model produces accurate forecasts and outperforms competitor models across maturities and forecast horizons. Subsample analysis shows that this outperformance is also consistent over time.

Keywords: Term structure of interest rates, Nelson-Siegel, Svensson, Forecasting, State-space model

JEL classification codes: E4, C5, C32

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*I want to thank Torben Andersen, Martin Martens, Francesco Ravazzolo and Dick van Dijk for helpful discussions and comments. I also thank seminar participants at the Tinbergen Lunch Seminar series for useful comments. All remaining errors are mine alone.

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1 Introduction

Accurate estimates of the current term structure of interest are of crucial importance in many areas of finance. Equally important is the ability to forecast the future term structure. It is not surprising therefore that substantial research effort has been devoted to the questions of how to optimally estimate, model and forecast the term structure of interest rates. One class of models that has the potential of providing satisfactory answers to these questions is that of the Nelson-Siegel models.

Nelson and Siegel (1987) proposed to fit the term structure using a flexible, smooth parametric function. They demonstrated that their advocated model is capable of capturing many of the typically observed shapes that the yield curve assumes over time. Since then various extensions have been proposed that incorporate additional flexibility with a popular extension being the Svensson (1994) model. Despite the drawback that they lack theoretical underpinnings, the Bank of International Settlements (BIS, 2005) reports that currently nine out of thirteen central banks which report their curve estimation methods to the BIS use either the Nelson-Siegel or the Svensson model to construct zero-coupon yield curves. As the Nelson-Siegel model is also widely used among practitioners, this ranks it among the most popular term structure estimation methods.

Recently, Diebold and Li (2006) have shown that the three-factor Nelson-Siegel model can also be used to construct accurate term structure forecasts. By using a straightforward two-step estimation procedure they demonstrate that the model performs well, relative to competing models, especially for longer forecast horizons. Mönch (2006a) partially confirms these results and Fabozzi, Martellini, and Priaulet (2005) show that the Nelson-Siegel model produces forecasts that are not only statistically accurate but also economically meaningful by showing how these can be used to generate substantial investment returns.

Due to these successes it is not surprising that the Nelson-Siegel model is increasingly being used in other applications as well. For example, Diebold, Rudebusch, and Aruoba (2006b) use the model to study the interactions between the macro economy and the yield curve (see also Diebold, Piazzesi, and Rudebusch, 2005) whereas Diebold, Ji, and Li (2006a) apply it to identify systematic risk sources and to construct a generalized duration measure.

Much of this research focuses, however, solely on the original three-factor Nelson-Siegel model. Extensions such as the Svensson model have not yet been investigated for their out-of-sample performance whereas extensions like those of Björk and Christensen (1999) have been left nearly unexamined altogether. This paper tries to fill this gap. In particular, I examine several models within the Nelson-Siegel class for their in-sample fitting and out-of-sample forecasting performance. Diebold and Li (2006) show that the dynamic three-factor Nelson-Siegel model had the potential of performing well in both areas. This motivates a closer examination of the various extended Nelson-Siegel models. It is unclear, however, if, or to what extent, models that are capable of parsimoniously fitting the term structure in-sample should also necessarily render accurate out-of-sample forecasts. No-arbitrage models for example typically fit the term structure quite accurately but forecast poorly (Duffee, 2002). An important task is therefore to try and evaluate the trade-off between in-sample fit and out-of-sample forecasting performance. More flexible models will most likely improve the in-sample fit but the question thus is if and to what extent these can also produce better out-of-sample results. In order to address this question for the class of Nelson-Siegel models I use a sample of U.S. Treasury zero-coupon bond yields consisting of twenty years of monthly data. I determine which features of the extended models help to improve the term structure fit. To gauge the out-of-sample performance I construct yield forecasts for short and long-term horizons and compare these with forecasts from several competitor models. In addition to looking at different Nelson-Siegel specifications I also examine in detail the benefits of alternative model estimation techniques, discuss several potential estimation and identification issues and propose solutions on how to tackle these.

The results can be summarized as follows. First of all I show that the more flexible models fit the term structure more accurately than the three-factor Nelson and Siegel (1987) model. This is not a surprising result in itself. What is interesting though is that a similar fit can be obtained as that of the popular Svensson (1994) model by extending the three-factor model with a second slope factor as in Björk and Christensen (1999). The advantage of this particular four-factor model is that it is easier to estimate than the Svensson model as it is less hampered by potential non-identification issues when estimating the factors.

In addition to an improved in-sample fit, I also demonstrate that the four-factor model produces accurate out-of-sample forecasts. In fact, the four-factor model outperforms the random walk benchmark and AR and VAR competitor models as well as all other Nelson-Siegel specifications, including the three-factor model. The best results are obtained by simultaneously taking into account cross-sectional and time-series information about yields when estimating the model and using an AR specification for the factor dynamics. The four-factor model forecasts increasingly well for all maturities when the forecast horizon lengthens. The outperformance relative to the random walk is substantial as it reduces the RMSPE by often as much as 10% or more. Subsample analysis shows that, unlike the performance of for example the three-factor model, the four-factor model is consistently producing highly accurate forecasts.

The remainder of this paper is structured as follows. In Section 2 I give a short review of term structure estimation methods. Section 3 discusses the various Nelson-Siegel models in detail and Section 4 is devoted to the estimation of these models. Section 5 describes the data. The in-sample results are presented in Section 6 whereas Section 7 shows the out-of-sample forecast results. Section 8 concludes and offers some directions for further research.

2 Term structure estimation methods

The term structure of interest rates describes the relationship between interest rates and time to maturity. The standard way of measuring the term structure of interest rates is by means of the spot rate curve, or yield curve[1], on zero-coupon bonds. The reason behind this is that yields-to-maturity on coupon-bearing bonds suffer from the 'coupon-effect' (see Caks, 1977) which implies that two bonds which are identical in every respect except for bearing different coupon-rates can have a different yield-to-maturity. The problem with zero-coupon yields on the other hand, is that these can only be directly observed from Treasury Bills which have maturities of twelve months or less. Longer maturity zero-coupon yields need to be derived from coupon-bearing Treasury Notes and Bonds. In practice, we can therefore not observe the entire term structure of interest rates directly. We need to estimate it using approximation methods[2]. Term structure estimation methods are designed for the purpose of approximating one of three equivalent representations of the term structure: the spot rate curve, discount curve and forward rate curve. Once we have a representation for one of these we can automatically derive the other representations. In the remainder of this section I briefly discuss the three curves and fix notation[3]. For convenience, I assume throughout that all rates are continuously compounded.

[1]The yield-to-maturity and the spot rate on a zero-coupon bond are the same. Because in this paper I focus solely on zero-coupon bond interest rates I use both terms interchangeably.

[2]In the U.S. zero-coupon rates are to a certain extent more directly available through the use of STRIPS. The Treasury STRIPS program, which started in 1985, allows an investor to split coupon-bearing Treasury Notes and Bonds into a basket of zero-coupon securities, see Sack (2000) for a discussion. Due to a limited investor demand for short- and medium-term zero-coupon securities there are some concerns, however, about the liquidity of shorter-term STRIPS and it is therefore still more common practice to estimate the yield curve using coupon-bearing bonds.

[3]For a more elaborate discussion see, e.g., Svensson (1994).

The forward rate curve characterizes forward rates as a function of maturity. A forward rate $f_t(\tau, \tau^*)$ is the interest rate of a forward contract on an investment which is initiated τ periods in the future and which matures τ^* periods beyond the start date of the contract. We obtain the *instantaneous* forward rate $f_t(\tau)$ by letting the maturity of such a forward contract go to zero:

$$\lim_{\tau^* \downarrow 0} f_t(\tau, \tau^*) = f_t(\tau) \tag{1}$$

The instantaneous-maturity forward rate curve represent forward rates on infinitesimal-maturity forward contracts which are initiated τ periods in the future for $\tau \in [0, \infty)$.

Given the forward rate curve, we can determine the spot rate (or yield) on a zero-coupon bond with τ periods to maturity, denoted by $y_t(\tau)$, by taking the equally weighted average over the forward rates:

$$y_t(\tau) = \frac{1}{\tau} \int_0^{\tau} f_t(m) \mathrm{d}m \tag{2}$$

The discount curve, $P_t(\tau)$, which denotes the present value of a zero-coupon bond that pays out a nominal amount of \$1 after τ periods, can in turn be obtained from the spot rate curve by

$$P_t(\tau) = \exp\left[-\tau y_t(\tau)\right] \tag{3}$$

The final relationship we have links forward rates directly to the discount curve and is given by

$$f_t(\tau) = -\frac{1}{P_t(\tau)} \frac{\mathrm{d}P_t(\tau)}{\mathrm{d}\tau} = y_t(\tau) + \tau \frac{\mathrm{d}y_t(\tau)}{\mathrm{d}\tau} \tag{4}$$

We can move from one curve to the other by using the relationships specified in (2)-(4).

Various methods have been proposed to estimate the term structure from (quoted) bond prices. A popular approach is the bootstrapping procedure by Fama and Bliss (1987) which consists of sequentially extracting forward rates from bond prices with successively longer maturities. The Fama and Bliss (1987) approach exactly prices all bonds included in the procedure and assumes that the forward rate between observed maturities is constant. The dataset I analyze in this paper consists of Fama-Bliss interest rates. Other term structure estimation methods use for example cubic splines (McCulloch, 1975), exponential splines (Vasicek and Fong, 1982), polynomials functions (Chambers *et al.*, 1984), parametric methods (Nelson-Siegel, see e.g. Bliss, 1997) or non-parametric methods (Linton *et al.*, 2001). Studies such as Bliss (1997), Ferguson and Raymar (1998) and Jeffrey *et al.* (2006) compare several different estimation methods and demonstrate the pros and cons of the various methods.

Once the decision has been made as to which method to use to construct an estimate of the term structure, the next step is to build a model to describe the evolution of the term structure over time. Popular models are no-arbitrage affine models, e.g. the one-factor models by Vasicek (1977) and Cox *et al.* (1985) or multi-factor models as specified and analyzed in Duffie and Kan (1996), Dai and Singleton (2000) and De Jong (2000). In this study I focus solely on the class of Nelson-Siegel models. Diebold and Li (2006) show that the Nelson-Siegel model not only provides a good in-sample fit of the term structure but also produces accurate out-of-sample interest rate forecasts for a 6- and 12-month forecast horizon. Diebold and Li (2006) only consider the original three-factor Nelson and Siegel (1987) model, however. The purpose of this paper is to examine a broader class of Nelson-Siegel models. This includes for example the four-factor specifications proposed by Svensson (1994) and Björk and Christensen (1999).

3 Nelson-Siegel class of models

3.1 Three-factor base model

Nelson and Siegel (1987) suggest to fit the forward rate curve at a given date with a mathematical class of approximating functions. The functional form they advocate uses Laguerre functions which consist of the product between a polynomial and an exponential decay term. The resulting Nelson-Siegel approximating forward curve can be assumed to be the solution to a second order differential equation with equal roots for spot rates[4]

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t} \exp\left(-\frac{\tau}{\lambda_t}\right) + \beta_{3,t} \left(\frac{\tau}{\lambda_t}\right) \exp\left(-\frac{\tau}{\lambda_t}\right) \tag{5}$$

The parameters $\beta_{t,1},\ \beta_{t,2}$ and $\beta_{t,3}$ are determined by initial conditions and λ_t is a constant associated with the equation. By averaging over forward rates, as in (2), we obtain the spot rate curve

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t} \left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\left(\frac{\tau}{\lambda_t}\right)}\right] + \beta_{3,t} \left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\left(\frac{\tau}{\lambda_t}\right)} - \exp\left(-\frac{\tau}{\lambda_t}\right)\right] \tag{6}$$

There are several reasons why the Nelson-Siegel model is such a popular term structure estimation method. First of all, it provides a parsimonious approximation of the yield curve

[4]For the specification of the Nelson-Siegel model I follow Fabozzi *et al.* (2005), Diebold and Li (2006) and Diebold, Rudebusch, and Aruoba (2006b) although I specify the decay parameter(s) the same way as in Nelson and Siegel (1987).

using only a small number of parameters (contrary to for example spline methods). Together, the three components $[1, \frac{1-\exp(-\tau/\lambda_t)}{(\tau/\lambda_t)}, \frac{1-\exp(-\tau/\lambda_t)}{(\tau/\lambda_t)} - \exp(-\tau/\lambda_t)]$, give the model enough flexibility to capture a range of monotonic, humped and S-type shapes typically observed in yield curve data. Second of all, the model produces forward and yield curves which have the desirable property of starting off from an easily computed instantaneous short rate value of $\beta_{1,t} + \beta_{2,t}$ and levelling off at a finite infinite-maturity value of $\beta_{1,t}$ which is constant[5]:

$$\lim_{\tau \downarrow 0} y_t(\tau) = \beta_{1,t} + \beta_{2,t}; \qquad \lim_{\tau \to \infty} y_t(\tau) = \beta_{1,t} \tag{7}$$

Finally, the three Nelson-Siegel components have a clear interpretation as short, medium and long-term components. These labels are the result of each element's contribution to the yield curve. Figure 1[a] depicts the value of each component as a function of maturity. The *long-term* component is the component on $\beta_{1,t}$ because it is constant at 1 and therefore the same for every maturity. The component on $\beta_{2,t}$ is $\left[\frac{1-\exp(-\tau/\lambda_t)}{(\tau/\lambda_t)}\right]$ and is designated as the *short-term* component. It starts at 1 but then decays to zero at an exponential rate. The rate of decay is determined by the parameter λ_t. Smaller values for λ_t induce a faster decay to zero. The *medium-term* component is $\left[\frac{1-\exp(-\tau/\lambda_t)}{(\tau/\lambda_t)} - \exp(-\tau/\lambda_t)\right]$ which starts at 0, increases for medium maturities and then decays to zero again thereby creating a hump-shape. The decay parameter λ_t determines at which maturity this component reaches its maximum.

- Insert Figure 1 around here -

Although the Nelson-Siegel model was in essence designed to be a static model which does not account for the intertemporal evolution of the term structure, Diebold and Li (2006) show that the coefficients $\beta_{1,t}, \beta_{2,t}$ and $\beta_{3,t}$ can be interpreted as three latent dynamic factors[6]. Moreover, the authors show that the labels level, slope and curvature are appropriate for these factors. The long-term factor $\beta_{1,t}$ governs the level of the yield curve whereas $\beta_{2,t}$ and $\beta_{3,t}$ govern its slope and curvature respectively.

By casting the Nelson-Siegel model into a dynamic framework, Diebold and Li (2006) further show that the model is capable of replicating the main empirical facts of the term

[5]The limiting behavior of the spot curve and the forward curve are the same. The Nelson-Siegel discount curve, which follows from combining (6) with (3), is given by

$$P_t(\tau) = \exp\{(-\beta_{1,t}\tau - \beta_{2,t}\lambda_t[1 - \exp(-\lambda_t\tau)] - \beta_{3,t}\lambda_t[1 - \exp(-\lambda_t\tau) - \tau\exp(-\lambda_t\tau)]\},$$

The discount curve starts at 1 and converges to zero when maturity goes to infinity as required.

[6]The short, medium and long-term components can therefore also be interpreted as factor loadings.

structure of interest rates over time: the average curve is upward sloping and concave, yield dynamics are highly persistent with long maturity rates being more persistent than short-maturity rates, and interest rate volatility is decreasing for longer maturities. Due to its attractive properties and its widespread use by central banks and practitioners I regard the three-factor model in (6) as the Nelson-Siegel base model. Note that Diebold and Li (2006) as well as for example Dolan (1999), Fabozzi *et al.* (2005) and Mönch (2006a) first fix λ_t to a pre-specified value and then proceed with analyzing the three-factor model. Here, I estimate λ_t as well as fixing it.

Although the base model can already capture a wide range of shapes, it cannot handle all the shapes that the term structure assumes over time. As an attempt to remedy this problem, several more flexible Nelson-Siegel specifications have been proposed in the literature to better fit more complicated shapes, mainly shapes with multiple minima and/or maxima. These extended Nelson-Siegel models achieve the increase in flexibility by introducing either additional factors, further decay parameters, or by a combination of both. In the remainder of this section I discuss which of these specifications I will examine for their in-sample fit and out-of-sample predictive accuracy.

3.2 Alternative Nelson-Siegel specifications

3.3 Two-factor model

The first model I consider is a restriction rather than an extension of the three-factor model. Litterman and Scheinkman (1991), among many other studies, show that the variation in interest rates can be explained by only a small number of underlying common factors. Typically, the first three principal component factors are already sufficient since these explain the bulk of interest rate variance but also because they have the intuitive interpretation as level, slope and curvature factors from the manner in which these factors affect the yield curve. The third factor has usually very little to add, however, (typically only a few percentage points) to the amount of interest rate variance that is already captured by the first two factors[7]. For this reason, authors such as Bomfim (2003) and Rudebusch and Wu (2003) consider two-factor affine models to explain interest rate dynamics whereas Diebold, Piazzesi, and Rudebusch (2005) examine a two-factor Nelson-Siegel model. Compared to the three-factor

[7] For the dataset I use here, the first and second factor explain 95.6% and 4% each of the variance in yield levels. The third factor explains only an additional 0.23%.

Nelson-Siegel model, the two-factor model only contains the level and slope factor:

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t} \left[\frac{1 - \exp\left(-\frac{\tau}{\lambda_t}\right)}{\left(\frac{\tau}{\lambda_t}\right)} \right] \tag{8}$$

Diebold, Piazzesi, and Rudebusch (2005) argue that since the first two principal components explain nearly all variation in interest rates, a two-factor model may suffice to forecast the term structure. They also argue, however, that two factors will most likely not be enough to accurately fit the entire yield curve[8].

3.4 Björk and Christensen (1999) four-factor model

The three-factor Nelson-Siegel model can be extended in various ways to increase its flexibility. From an estimation point of view, the easiest approach is to introduce additional factors. Björk and Christensen (1999) propose to add a fourth factor to the approximating forward curve in (5):

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t} \exp\left(-\frac{\tau}{\lambda_t}\right) + \beta_{3,t} \left(\frac{\tau}{\lambda_t}\right) \exp\left(-\frac{\tau}{\lambda_t}\right) + \beta_{4,t} \exp\left(-\frac{2\tau}{\lambda_t}\right) \tag{9}$$

The four-factor Nelson-Siegel yield curve is then given by

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t} \left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\left(\frac{\tau}{\lambda_t}\right)} \right] + \beta_{3,t} \left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\left(\frac{\tau}{\lambda_t}\right)} - \exp\left(-\frac{\tau}{\lambda_t}\right) \right] + \beta_{4,t} \left[\frac{1-\exp\left(-\frac{2\tau}{\lambda_t}\right)}{\left(\frac{2\tau}{\lambda_t}\right)} \right] \tag{10}$$

The fourth component, $\left[\frac{1-\exp(-2\tau/\lambda_t)}{(2\tau/\lambda_t)}\right]$, resembles the second component as it also mainly affects short-term maturities. The difference is that it decays to zero at a faster rate which can be seen from Figure 1[b]. The factor $\beta_{4,t}$ can therefore be interpreted as a second slope factor. As a result, the four-factor Nelson-Siegel model captures the slope of the term structure by the (weighted) sum of $\beta_{2,t}$ and $\beta_{4,t}$. The instantaneous short rate in (7) is for the four-factor model therefore equal to $y_t(0) = \beta_{1,t} + \beta_{2,t} + \beta_{4,t}$. Diebold, Rudebusch, and Aruoba (2006b) report that the four-factor model marginally improves the in-sample fit of the term structure but they do not consider out-of-sample forecasting.

[8]Diebold, Piazzesi, and Rudebusch (2005) show how to impose no-arbitrage restrictions on this two-factor Nelson-Siegel model. As argued by Diebold *et al.* (2006b) no-arbitrage restrictions are likely to be, at least approximately, satisfied in the U.S. Treasury data analyzed here. Therefore, imposing no-arbitrage restrictions are unlikely to improve the performance of the two-factor model reported in Section 6 and 7. Its poor performance seems primarily due to the limited number of only two factors.

Björk and Christensen (1999) also consider a five-factor model:

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{\tau}{\lambda_t}\right) + \beta_{3,t}\exp\left(-\frac{\tau}{\lambda_t}\right) + \beta_{4,t}\left(\frac{\tau}{\lambda_t}\right)\exp\left(-\frac{\tau}{\lambda_t}\right) + \beta_{5,t}\exp\left(-\frac{2\tau}{\lambda_t}\right)$$

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{\tau}{2\lambda_t}\right) + \beta_{3,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\left(\frac{\tau}{\lambda_t}\right)}\right] + \beta_{4,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_t}\right)}{\left(\frac{\tau}{\lambda_t}\right)} - \exp\left(-\frac{\tau}{\lambda_t}\right)\right] + \beta_{5,t}\left[\frac{1-\exp\left(-\frac{2\tau}{\lambda_t}\right)}{\left(\frac{2\tau}{\lambda_t}\right)}\right]$$

and Diebold, Rudebusch, and Aruoba (2006b) report that adding two additional factors again only leads to a negligible improvement in in-sample fit. The problem with the five-factor model, however, is that it contains a component which is linear in τ. Consequently, the model implies linearly increasing long-maturity spot and forward rates. This is problematic and I therefore do not consider the five-factor model here.

3.5 Bliss (1997) three-factor model

A second option to make the Nelson-Siegel more flexible is through relaxing the restriction that the slope and curvature component are governed by the same decay parameter λ_t. Bliss (1997) estimates the term structure of interest rates with the three-factor Nelson-Siegel model but allows for two different decay parameters $\lambda_{1,t}$ and $\lambda_{2,t}$[9]. The forward curve and spot rate curves are then given by

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t}\exp\left(-\frac{\tau}{\lambda_{1,t}}\right) + \beta_{3,t}\left(\frac{\tau}{\lambda_{2,t}}\right)\exp\left(-\frac{\tau}{\lambda_{2,t}}\right) \tag{11}$$

and

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_{1,t}}\right)}{\left(\frac{\tau}{\lambda_{1,t}}\right)}\right] + \beta_{3,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_{2,t}}\right)}{\left(\frac{\tau}{\lambda_{2,t}}\right)} - \exp\left(-\frac{\tau}{\lambda_{2,t}}\right)\right] \tag{12}$$

Obviously, the Bliss Nelson-Siegel model will only be different from the base model if $\lambda_{1,t} \neq \lambda_{2,t}$.

Nelson and Siegel (1987) also consider an approximating forward curve with different decay parameters[10]. The forward curve is again derived as the solution to a second-order

[9] Bliss (1997) calls this the 'Extended Nelson-Siegel' but as several more extensions are considered here as well I simply refer to it as the 'Bliss' model.

[10] Nelson and Siegel (1987) try to fit this model to their sample of yields which only consists of maturities up until one year. They report that the model is over-parameterized and therefore use the forward curve in (5). Bliss (1997) remarks that over-parametrization should not pose any problem when also longer-maturity yields are fitted, which is also the case here.

differential equation but now with real and unequal roots. Their forward rate curve is given by

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t} \exp\left(-\frac{\tau}{\lambda_{1,t}}\right) + \beta_{3,t}\exp\left(-\frac{\tau}{\lambda_{2,t}}\right)$$

We need the additional factor $\left(\frac{\tau}{\lambda_{2,t}}\right)$ in (11) to obtain the curvature factor. Otherwise, the model contains two slope factors which are different only if the decay parameters are different. The model would then closely resemble the two-factor model.

3.6 Svensson (1994) four-factor model

A popular term-structure estimation method among central banks (see BIS, 2005) is the four-factor Svensson (1994) model. Svensson (1994) proposes to increase the flexibility and fit of the Nelson-Siegel model by adding a second hump-shape factor with its own separate decay parameter. The resulting four-factor forward curve is given by:

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t} \exp\left(-\frac{\tau}{\lambda_{1,t}}\right) + \beta_{3,t}\left(\frac{\tau}{\lambda_{1,t}}\right)\exp\left(-\frac{\tau}{\lambda_{1,t}}\right) + \beta_{4,t}\left(\frac{\tau}{\lambda_{2,t}}\right)\exp\left(-\frac{\tau}{\lambda_{2,t}}\right) \tag{13}$$

The resulting equation for the zero-coupon yield curve is then

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_{1,t}}\right)}{\left(\frac{\tau}{\lambda_{1,t}}\right)}\right] + \beta_{3,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_{1,t}}\right)}{\left(\frac{\tau}{\lambda_{1,t}}\right)} - \exp\left(-\frac{\tau}{\lambda_{1,t}}\right)\right] + \beta_{4,t}\left[\frac{1-\exp\left(-\frac{\tau}{\lambda_{2,t}}\right)}{\left(\frac{\tau}{\lambda_{2,t}}\right)} - \exp\left(-\frac{\tau}{\lambda_{2,t}}\right)\right] \tag{14}$$

The fourth component, $\left[\frac{1-\exp(-\tau/\lambda_{2,t})}{(\tau/\lambda_{2,t})} - \exp(-\tau/\lambda_{2,t})\right]$, introduces a second medium-term component to the model which is depicted by the dash-dotted line in Figure 1[c]. The Svensson Nelson-Siegel model can more easily fit term structure shapes with more that one local maximum or minimum along the maturity spectrum. As the fourth component mainly affects medium-term maturities, the limiting results in (7) also hold for the Svensson model.

3.7 Adjusted Svensson (1994) four-factor model

A potential problem with the Svensson model is that it is highly non-linear which can make the estimation of the model difficult, see Bolder and Stréliski (1999) for a discussion. A multicolinearity problem arises when the decay parameters $\lambda_{1,t}$ and $\lambda_{2,t}$ assume similar values. When this happens, the Svensson model reduces to the three-factor base model but with a curvature factor equal to the sum of $\beta_{3,t}$ and $\beta_{4,t}$. Only the sum of these parameters can then still be estimated efficiently, not the individual parameters[11].

One way to try and cure this multicolinearity problem is to make sure that the two medium-term components are different when $\lambda_{1,t} \simeq \lambda_{2,t}$. I therefore propose an 'Adjusted' Svensson model which is given by the following forward and zero curves:

$$f_t(\tau) = \beta_{1,t} + \beta_{2,t} \exp\left(-\frac{\tau}{\lambda_{1,t}}\right) + \beta_{3,t} \left(\frac{\tau}{\lambda_t}\right) \exp\left(-\frac{\tau}{\lambda_{1,t}}\right) + \beta_{4,t} \left[\exp\left(-\frac{\tau}{\lambda_{2,t}}\right) + \left(\frac{2\tau}{\lambda_{2,t}} - 1\right) \exp\left(-\frac{2\tau}{\lambda_{2,t}}\right)\right] \tag{15}$$

and

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t} \left[\frac{1-\exp\left(-\frac{\tau}{\lambda_{1,t}}\right)}{\left(\frac{\tau}{\lambda_{1,t}}\right)}\right] + \beta_{3,t} \left[\frac{1-\exp\left(-\frac{\tau}{\lambda_{1,t}}\right)}{\left(\frac{\tau}{\lambda_{1,t}}\right)} - \exp\left(-\frac{\tau}{\lambda_{1,t}}\right)\right] + \beta_{4,t} \left[\frac{1-\exp\left(-\frac{\tau}{\lambda_{2,t}}\right)}{\left(\frac{\tau}{\lambda_{2,t}}\right)} - \exp\left(-\frac{2\tau}{\lambda_{2,t}}\right)\right] \tag{16}$$

The adjustment to the second curvature component ensures that multicolinearity is no longer an issue. The adjusted component also starts at 0 but then increases for medium maturities at a faster rate than the first curvature component and returns to zero faster as well. The dash-dotted line in Figure 1[d] depicts the fourth component as a function of maturity. The difference between the two additional curvature components in the Svensson and Adjusted Svensson model can be seen by comparing Figure 1[c] with Figure 1[d].

3.8 General specification

The different Nelson-Siegel specifications that I examine are all nested and can therefore be captured in one general model set-up. In particular, consider the following state-space representation:

$$Y_t = X_t \beta_t + \varepsilon_t \tag{17}$$

$$\beta_t = \mu + \Phi \beta_{t-1} + \nu_t \tag{18}$$

The measurement equations in (17) specify the vector of yields, which contains N different maturities, $Y_t = [y_t(\tau_1) \dots y_t(\tau_N)]'$, as the sum of a Nelson-Siegel spot rate curve, $X_t\beta_t$, plus a vector of yield errors which are assumed to be independent across maturities but with

[11] One example of this multicolinearity effect can be seen in Gimeno and Nave (2006). When applying the Svensson model to estimate the zero-yield curve from Spanish Treasury Bonds, Gimeno and Nave report that $\beta_{3,t}$ and $\beta_{4,t}$ display clear structural streaks and often take on large values but with opposite signs. The sum of the two parameters is stable across time, however (see Figure 3[a] in Gimeno and Nave, 2006). The reason for this becomes apparent from their Figure 2 in which it is shown that the extreme factor estimates correspond to samples for which the estimated values for $\lambda_{1,t}$ and $\lambda_{2,t}$ are very similar.

different variance terms, $\sigma^2(\tau_i)$. The Nelson-Siegel spot rate curves are those discussed in the previous sections with β_t being the $(K \times 1)$ vector of factors and X_t the $(N \times K)$ matrix of factor loadings which are potentially time-varying if the decay parameter(s) are estimated alongside the factors. Each of the Nelson-Siegel models in sections 3.1-3.7 is a special case of (17) with a different number of factors and/or a different specification for the factor loadings.

If we are only interested in fitting the term structure then the measurement equations are sufficient. However, in order to construct term structure forecasts we also need a model for the factor dynamics. I follow the dynamic frame-work of Diebold and Li (2006) and Diebold, Rudebusch, and Aruoba (2006b) by specifying first-order autoregressive processes for the factors as in the state equations (18). These can be either individual AR(1) processes or one multivariate VAR(1) process[12]. The vector μ and matrix Φ have dimensions $(K \times 1)$ and $(K \times K)$ respectively. The model is completed by assuming that the measurement equation and state equation error vectors are orthogonal and normally distributed:

$$\begin{bmatrix} \varepsilon_t \\ \nu_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0_{N\times 1} \\ 0_{K\times 1} \end{bmatrix}, \begin{bmatrix} H & 0 \\ 0 & Q \end{bmatrix} \right) \tag{19}$$

where H is a $N \times N$ matrix which I assume to be diagonal throughout the analysis. For the state equation covariance matrix Q I make the assumption that it is either a diagonal $(K \times K)$ matrix or a full matrix, depending on the estimation procedure which I discuss next.

4 Estimation procedures

There are several approaches to estimating the latent factors and parameters in the Nelson-Siegel state-space representation. These approaches depend crucially on whether the measurement and state equations are estimated separately or simultaneously and on the assumptions regarding the decay parameters[13].

The most straightforward approach is used in for example Fabozzi *et al.* (2005) and Diebold and Li (2006) and consists of a two-step procedure. In the first step the measurement equations are treated as a cross-sectional model and Least Squares is used to estimate the

[12] Here I use a straightforward linear specification of the measurement and state equations. More complex specifications, such as a Markov Switching approach, are used in for example Bernadell *et al.* (2005).

[13] I only use frequentist maximum likelihood techniques to estimate parameters. Mönch (2006b) and De Pooter *et al.* (2007) consider Bayesian estimation of the three-factor model. Whereas a Bayesian approach would also account for parameter uncertainty, I do not pursue it here and leave this for further research.

parameters for every month separately. In the second step time series models are specified and fitted for the factors. A second, somewhat more demanding estimation approach is a one-step procedure in which all the parameters in the state-space system are estimated simultaneously. This approach uses the Kalman filter to estimate the factors and is proposed in Diebold, Rudebusch, and Aruoba (2006b). Here I use the one-step as well as the two-step estimation procedures and in this section I discuss both techniques in detail. Specific details regarding the estimation are given in Appendix A.

4.1 Two-step approach with a fixed decay parameter

Diebold and Li (2006) suggest to fix λ_t in the three-factor model to a pre-specified value which is the same for every t, instead of treating it as an unknown parameter. By doing so, the nonlinear measurement equations become linear in the state vector which can then be estimated using straightforward cross-sectional OLS. The decay parameter λ_t determines the (medium-term) maturity at which the factor loading on the curvature factor $\beta_{3,t}$ is at its maximum. The value of 16.42 that Diebold and Li (2006) use for λ_t is such that this maximum is reached at a 30-month maturity. Larger values for λ_t produces slower decaying factor loadings with the curvature factor achieving it maximum at a longer maturity and vice versa. Although other authors have used different values as well, I follow Diebold and Li (2006) and set λ_t equal to 16.42.

The first step of the estimation procedure produces time-series of estimated values for each of the K factors; $\{\beta_{i,t}\}_{t=1}^{T}$ for $i = 1, \ldots, K$. The next step is to estimate the factor dynamics of the state equations. I estimate separate AR(1) models for each factor, thus assuming that Φ and Q are both diagonal, as well as a joint VAR(1) by assuming that Φ and Q are full matrices instead.

I apply the two-step estimation approach with a fixed decay parameter only to estimate the two, three and four-factor Nelson-Siegel specifications. The remaining models have two decay parameters and would therefore require finding two appropriate values to choose for $\lambda_{1,t}$ and $\lambda_{2,t}$, which is difficult. I use the notation 'NS2' to indicate the two-step estimation procedure. I denote the two, three and four-factor models by **NS2-2**, **NS2-3** and **NS2-4** respectively and add suffixes '-AR' and '-VAR' to indicate the time-series model specification for the state equations.

4.2 Two-step approach with estimated decay parameters

When the decay parameters are estimated alongside the factors, the estimation of the now nonlinear measurement equations in the first step becomes more challenging and requires nonlinear least squares. However, the increased flexibility of the model as a result of the additional parameter(s) can nevertheless make this a worthwhile exercise to undertake. I therefore also estimate the two, three and four-factor models when treating λ_t as a parameter and I denote these by **NS2-2-$\boldsymbol{\lambda}$**, **NS2-3-$\boldsymbol{\lambda}$** and **NS2-4-$\boldsymbol{\lambda}$** with suffixes '-AR' and '-VAR'. The Bliss and (Adjusted) Svensson models all have two decay parameters, $\lambda_{1,t}$ and $\lambda_{2,t}$, which should even further improve the fit of the Nelson-Siegel model due to the increased flexibility of the factor loadings. The two-step estimation procedure can also be applied to these models and I use the notation **NS2-B**, **NS2-S** and **NS2-AS** for the Bliss, Svensson and Adjusted Svensson model respectively. Note that in the second step I do not model the dynamics of the decay parameters explicitely. Instead, in order to construct forecasts I use the median of their in-sample estimated values, see Section 7.1 for further details.

4.3 Restrictions on the decay parameters

The nonlinear estimation procedure can result in factor estimates which can sometimes be very extreme. An example is shown and discussed in Gimeno and Nave (2006) for the Svensson model. Gimeno and Nave report extreme (and often offsetting) values for factor estimates. Bolder and Stréliski (1999) also address numerical problems and estimation issues when estimating the Svensson model.

The nonlinear model structure seems to pose serious difficulties for optimization procedures to arrive at reasonable estimates. An additional reason, which to the best of my knowledge seems to have been overlooked in the literature surprisingly, is the behavior of the factor loadings when the decay parameters take on extreme values. When this happens multicolinearity problems can occur and some of the factors are then no longer uniquely identified. To understand why this is the case we need to examine the factor loadings as functions of λ_t. We have the following straightforward limiting results

$$\lim_{\lambda_t \downarrow 0} \left[\frac{1-\exp(-\tau/\lambda_t)}{(\tau/\lambda_t)} \right] = 0; \qquad \lim_{\lambda_t \downarrow 0} \left[\frac{1-\exp(-\tau/\lambda_t)}{(\tau/\lambda_t)} - \exp(-\tau/\lambda_t) \right] = 0 \tag{20}$$

$$\lim_{\lambda_t \to \infty} \left[\frac{1-\exp(-\tau/\lambda_t)}{(\tau/\lambda_t)} \right] = 1; \qquad \lim_{\lambda_t \to \infty} \left[\frac{1-\exp(-\tau/\lambda_t)}{(\tau/\lambda_t)} - \exp(-\tau/\lambda_t) \right] = 0 \tag{21}$$

The results in (20) imply that for very small values of λ_t the slope and curvature factors will be near non-identification which can result in extreme estimates[14]. For large values of λ_t, as indicated by (21), curvature factors are nearly non-identified. Furthermore, the level and slope factors are jointly identified, but no longer identified separately and can therefore take on extreme, offsetting, values[15].

If we are only interested in fitting the term structure at a given point in time in step one, these non-identification issues do not necessarily cause problems. Although the factor estimates can be extreme, the models still accurately fit the term structure. The real problem occurs when estimating dynamics in step two as the time-series of the factors can potentially be plagued by outliers. In order to prevent extreme factor estimates, I impose restrictions on the decay parameters. By only allowing the curvature factor loading to reach its maximum for maturities between one and five years, the decay parameters are restricted to lie in the interval [6.69, 33.46][16]. I impose one additional restriction on the Svensson and Adjusted Svensson models separately.

For the Svensson model I restrict the loading on the second curvature factor, $\beta_{4,t}$, to reach its maximum for a maturity which is at least twelve months shorter than the corresponding maturity for the first curvature loading. Specifically it comes down to the following minimum distance restriction: $\lambda_{1,t} \geq \lambda_{2,t} + 6.69$. This restriction prevents the case where $\beta_{3,t}$ and $\beta_{4,t}$ are only jointly identified but not individually. Note that the two curvature components in the Svensson model, and therefore $\lambda_{1,t}$ and $\lambda_{2,t}$, as well as their role in the restriction, are interchangeable. In the Adjusted Svensson model the curvature factor loadings are different so there is no need to impose any minimum distance between the two decay parameters. I do, however, again force the first curvature hump to be to the right of the second curvature hump by imposing the restriction $\lambda_{1,t} \geq \lambda_{2,t}$.

[14]Note that in the Bliss and (Adjusted) Svensson models non-identification issues arise when either $\lambda_{1,t}$ or $\lambda_{2,t}$ tends to zero and even more so when both parameters tend to zero.

[15]This explains the peaks with opposite signs in the level and slope estimates in Figure 1 of Gimeno and Nave (2006).

[16]Recall that Diebold and Li (2006) fix the decay parameter such that the maximum is reached at a maturity of two and a half years. Note that Gürkanyak, Sack, and Wright (2006) do not impose restrictions on the Svensson model when estimating the U.S. Treasury yield curve. They find that the second hump is located at much longer maturities (beyond twenty years). However, Gürkanyak *et al.* (2006) estimate the term structure using bonds with maturities up to thirty years. I only use maturities up to ten years and the domain of the curvature humps of one to five years seems therefore reasonable and sufficiently wide in order not to be too restrictive. Some experimentation with using wider domains indeed resulted in factor estimates that were more ‘extreme’.

4.4 One-step state-space approach

The alternative to the two-step approach is to estimate all parameters simultaneously. By using the prediction-error decomposition of the likelihood we can estimate parameters by maximum likelihood and apply the Kalman filter to obtain optimal factor estimates. The likelihood for the state-space system in (17)-(18) is given by

$$\mathcal{L}(\Theta) = \sum_{t=1}^{T} \left[-\frac{1}{2} \ln (2\Pi) - \frac{1}{2} \ln (|f_{t|t-1}|) - \frac{1}{2} \eta'_{t|t-1} f^{-1}_{t|t-1} \eta_{t|t-1} \right] \tag{22}$$

which is a function of the parameter set $\Theta = (\lambda_1, \lambda_2, \beta_t, \mu, \Phi, H, Q)$. The likelihood is comprised of the $(N \times 1)$ yield prediction error vector; $\eta_{t|t-1} \equiv y_t - y_{t|t-1}$ where $y_{t|t-1}$ is the vector of in-sample yield forecasts given information up to time $t-1$, and of the $(N \times N)$ conditional covariance matrix of the prediction errors; $f_{t|t-1} \equiv \mathbb{E}[\eta_{t|t-1}\eta'_{t|t-1}]$, see Kim and Nelson (1999) for further details. Note that the decay parameters are assumed to be constant over time[17]. As these can now be estimated using information from both the cross-section as well as the time-series of yields it is much less likely that they will take on extreme values. Furthermore, because the dynamics of the factors are explicitly taken into account when optimizing the likelihood, it does not seem necessary anymore to impose the earlier restrictions on the decay parameters. I estimate all the models using this one-step procedure with the decay parameters being estimated alongside the factors and the remaining parameters and allowing for Q to be a full matrix. I denote the results for the different models using this approach by **NS1-2**, **NS1-3**, **NS1-4**, **NS1-B**, **NS1-S** and **NS1-AS**.

Diebold, Rudebusch, and Aruoba (2006b) favor the one-step over the two-step estimation approach because parameters are estimated simultaneously which ensures that the uncertainty of all parameters is taken into account at the same time. The drawback, however, is that the number of parameters to estimate is substantial in the state-space model. For example, for the four-factor Svensson model with a VAR(1) specification for the state equations, the total number of parameters for the dataset used here equals 49 (two decay parameters, four parameters in μ, 16 parameters in Φ, $N = 17$ parameters in H, four variance and six covariance terms in Q). In order to reduce the number of parameters I therefore also try two alternative specifications for the system of state equations in (18). Apart from specifying VAR(1) dynamics for the factors by assuming Φ to be a full matrix as in Diebold,

[17] Huse (2007), on the contrary, argues that for the three-factor model the decay parameter should also be treated as a dynamic factor and that it should be modelled accordingly. I do not consider this approach here.

Rudebusch, and Aruoba (2006b) I also consider AR(1) dynamics using a diagonal Φ. Additionally I specify random walk factor dynamics by setting μ equal to zero and Φ equal to the identity matrix[18]. I distinguish the different dynamics by using the suffixes '-VAR', '-AR' and '-RW' for respectively VAR(1), AR(1) and random walk dynamics. An overview of all model abbreviations used is given in Table 1.

- Insert Table 1 around here -

5 Data

The dataset available here consists of end-of-month continuously compounded U.S. zero-coupon bond forward rates. I compute constant maturity spot rates by averaging these forwards rates as in (2). The forward rates are constructed from filtered average bid-ask price quotes on U.S. Treasury securities using the Fama and Bliss (1987) bootstrap method as outlined in Bliss (1997)[19]. The price quotes are taken from the CRSP government bond files. CRSP filters the available quotes by taking out illiquid bonds and bonds with option features. Similar to Diebold and Li (2006), Diebold, Rudebusch, and Aruoba (2006b) and Mönch (2006a), I use unsmoothed Fama-Bliss yields[20].

I estimate the class of Nelson-Siegel models using data for the sample period 1984:1 - 2003:12 ($T = 240$ observations) and I use the following $N = 17$ maturities in the estimation: τ = 3, 6, 9, 12, 15, 18, 21, 24 and 30 months as well as 3, 4,..., 10 years. I start my dataset after the Volcker period to allow for a fair comparison with the results in Diebold and Li (2006) and Mönch (2006a). Note that the forecasting results reported by De Pooter *et al.* (2007) for the three-factor Nelson-Siegel model with both the two-step and one-step estimation procedure are based on a much longer span of data (1970:1 - 2003:12).

Figure 2 shows time-series plots for a subset of the maturities and illustrates how yield levels and spreads vary substantially throughout the sample. For example, for the period from 1994 onwards, which is the period I use to evaluate the models' forecasting performance,

[18]Diebold and Li (2006) find for the three-factor model that the null of a unit root in the factor dynamics cannot be rejected for $\beta_{1,t}$ and $\beta_{2,t}$. Fabozzi *et al.* (2005) find similar results and therefore model first differences of the level and slope factors.

[19]I kindly thank Robert Bliss for providing me with the unsmoothed Fama-Bliss forward rates and the programs to construct the spot rates.

[20]The reason for using unsmoothed Fama-Bliss yields is that Bliss (1997) finds, using parametric and nonparametric tests, that the Fama-Bliss method does best overall in terms of estimating the term structure in comparison with other popular estimation methods.

we can distinguish a stable period (mid 1990s till the end of 2000) but also a period where short term interest rates fell by roughly 4%, resulting in a sharp increase in the term spread (the last three years of the sample). It is clear from Figure 3 that not only the level of the term structure fluctuates over time but also its slope and curvature. The curve takes on

- Insert Figures 2, 3 and Table 2 around here -

various forms ranging from nearly flat to (inverted) S-type shapes. Table 2 reports summary statistics for yield levels for various maturities. The stylized facts common to yield curve data are clearly present: the sample average curve is upward sloping and concave, volatility is decreasing with maturity, autocorrelations are very high and increasing with maturity. The null of normality is rejected for medium and longer term maturities due to positive skewness and excess kurtosis but cannot be rejected for shorter maturities. Correlations between yields of different maturities are high (80% or above), especially for close-together maturities.

6 In-sample fit results

6.1 In-sample fit

In this section I discuss the results of fitting the term structure using the class of Nelson-Siegel models. I only focus on the fit from step one of the two-step estimation procedure due to the fact that the one-step procedure potentially also uses (future) time-series information which is unavailable if we want to fit the term structure at a given point in time. We can expect that more flexible models result in a better fit. However, as the increased flexibility can be obtained both by additional decay parameters as well as by additional factors, the question is which of the two modelling options improves the fit more.

Figure 4 shows that all models accurately fit the average curve. The only exception is the two-factor model with fixed decay parameter, shown in Panel [a], most likely because it lacks a curvature component. Nevertheless, freeing up the decay parameter seems to provide

- Insert Figure 4 around here -

sufficient additional flexibility as the two-factor average curve now becomes virtually indistinguishable from the three and four-factor models (Panel [b]).

Whereas the average fit may be nearly identical across the different models, Figure 5 on the other hand shows that the fit in individual months can be quite different. Shown in

- Insert Figure 5 around here -

Figure 5 are the actual term structures in four specific months of the sample. These four months are an example of the various different term structure shapes that occur in the data. Whereas for November 1995 and September 2000 the shape is respectively S-shaped and downward sloping, for June 1989 and August 1998 the shapes are more difficult to describe. The lines in each panel show the fit of various models. The two-factor model in particular has difficulties fitting the more complex curves, but the three-factor model also does not seem flexible enough judging from, for example, Panel [c]. Graphically, the best fit is obtained with the four-factor model and the (Adjusted) Svensson models which give very similar fitted curves.

Table 3 reports detailed in-sample results for all models, which have been estimated with the restrictions on the decay parameters in place. The best fitting models, as judged by a

- Insert Table 3 around here -

number of standard criteria given in the table (standard deviation of yield errors, root mean squared fit error, mean absolute fit error, minimum and maximum fit error) are represented by the bold numbers. The results can be summarized by making the following observations.

The models that achieve the best fit overall are indeed the most flexible models, in particular the (Adjusted) Svensson model. For nearly every maturity shown in the table, the Svensson models are the most accurate on all criteria, including having the lowest persistence in yield errors. Except from the two-factor model, all models perform relatively similar, however, which agrees with the results in Dahlquist and Svensson (1996) and Diebold, Rudebusch, and Aruoba (2006b) who demonstrate that the three-factor model fits the term structure well compared to more elaborate models. It is nonetheless interesting to examine how the results of the remaining models compare to those of the Svensson models, but in particular how they compare amongst each other. For the two and three-factor models we can judge which extension yields the largest gain; estimating λ_t or adding a factor. From columns two to six in Table 3 it becomes clear that for the two-factor model adding a (curvature) factor improves the in-sample fit much more than by estimating λ_t alongside the level and slope factors. The results for the three-factor model lead to the same conclusion although the improvement when going from the three to the four-factor model is much less substantial than going from the two to the three-factor model. Estimating λ_t instead of using the fixed value of 16.42 improves the fit for each model although in absolute terms the benefits are minor (tens of basis points). Another comparison to make is that between the three-factor model with estimated λ_t and the Bliss model as the latter does not impose that the slope and curvature factor are determined by the same decay parameter. For every maturity the Bliss model marginally improves the fit. However, the four-factor model with estimated λ_t is always more accurate than the Bliss model, showing that it is more beneficial to introduce the second slope factor than separate decay parameters.

In fact, the four-factor model with an estimated λ_t fits the term structure only marginally worse than the best fitting model with the maximum overall difference being no larger than 0.6 basis points (for a 3-month maturity based on both the root mean squared error and the mean absolute error). This means that, compared to the three-factor base model, it does not seem to make much difference whether a second curvature factor with a separate decay parameter is added (the Svensson model) or just a second slope factor which has the same λ_t as the first three factors. Comparing the Svensson model with its Adjusted alternative shows that the latter fits marginally better.

To summarize, the best fitting models in an absolute sense are indeed the models which allow for the most amount of flexibility which are the Svensson and Adjusted Svensson models. However, the four-factor model provides a fit which is nearly as accurate and has the benefit of being easier to estimate because the nonlinearities in the model are due to only one decay parameter instead of two. The interesting question now is whether the additional slope factor, in addition to improving the in-sample fit, can also help to improve the out-of-sample performance.

Before I turn to discussing the forecast results I first address the effect of imposing the restrictions on the decay parameters. Restricting these will most likely mean that some of the in-sample fit performance is sacrificed. The question is, however, to what extent this actually is the case. To assess the effect on in-sample fit I report in Table 4 the in-sample fit of those specifications that require estimating one or two decay parameters when no restrictions are

- Insert Table 4 around here -

imposed. Comparing Table 3 and Table 4 shows that in absolute terms the unrestricted models indeed fit the term structure more accurately. However, the differences are only substantial for the two-factor model which is explainable as with only two factors, having an additional parameter can make quite a difference. For example, for the 3-month maturity, the root mean squared error goes down from 18 to 12 basis points. For all other models, differences are, however, marginal with criteria such as standard deviation and mean absolute error being only 0.5 basis points worse for the restricted models. Furthermore, the bold numbers in each panel, which highlight the best fitting models per maturity, show almost negligible differences.

The results indicate that whether or not imposing restrictions does not matter much in terms of in-sample fit. Nevertheless, the reason why the restrictions are useful becomes apparent when we examine the time-series of the estimated factors. As these are modelled in the second step of the two-step procedure it is important that these series are relatively 'well-behaved'. That this not necessarily needs to be the case using the unrestricted estimation procedure is due to potential non-identification issues, as discussed earlier. Not imposing restrictions can result in extreme factor estimates. An example is given in Figure 6 in which

- Insert Figure 6 around here -

the solid and dotted lines represent respectively the restricted and unrestricted estimates of the level, slope and curvature factors in the three-factor model. For most of the sample the restricted and unrestricted estimates are all but identical, except for a small number of months. For each of these months the unrestricted λ_t is substantially higher than the upper limit of 33.46. In particular for May 1986 this is the case with $\widehat{\lambda}_t$ equaling 65.61 as a result of which the level and slope factors are estimated at $\widehat{\beta}_{1,t} = 2.94$ and $\widehat{\beta}_{2,t} = 3.26$. Only the sum of these is somewhat close to the true level of the curve of 7.86% (using as proxy the 10-year yield) whereas with the restrictions in place the level estimate is $\widehat{\beta}_{1,t} = 7.34$.

6.2 Factor estimates

Time-series of the factor estimates, obtained with the two-step procedures are represented by the solid lines in Figures 7-9. Comparing the subgraphs within each row and across figures shows that the different models all give rather similar estimates for the level, slope and curvature factors. The estimates differ nevertheless in magnitude, mainly for the four-factor model. The time-series for the latter seems to suffer somewhat from outliers, in particular

- Insert Figures 7 - 9 around here -

when the decay parameter is fixed (Figure 7) with some of the spikes in the slope and curvature factors disappearing when the decay parameter is estimated as well (Figure 8). Panels [h]-[k] of Figure 9 show that the two curvature factor estimates for the Adjusted Svensson model are more stable than those for the Svensson model. The latter still exhibit severe spikes, despite the restrictions on $\lambda_{1,t}$ and $\lambda_{2,t}$.

As an indication of the differences in the resulting factor estimates between the one and two-step estimation methods, Figures 8 and 9 also show the Kalman filter factor estimates for the full sample by means of the dotted lines. Whereas in general the time-series are quite close, there are certainly differences, mainly for the more complex models like the Svensson models. Using cross-sectional yield data as well as information concerning the evolution of yields over time smoothes out the factor estimates.

Table 5 presents detailed full-sample summary statistics for the two-step factor estimates. Statistics for empirical level, slope, curvature estimates, which have been constructed from the yields directly, are shown in the last rows of the table. The estimated factors mimic the empirical factors quite closely which is also clear from the italicized numbers in the last

- Insert Table 5 around here -

three columns showing the correlations between the estimated and empirical factors. All factors are highly autocorrelated and there is also substantial cross-correlation across factors[21]. The importance of accounting for this cross correlation from a forecasting perspective will be assessed by comparing the results between the AR and VAR specifications for the factor dynamics.

7 Out-of-sample forecasting results

For the out-of-sample performance I run a similar horse-race between the different models as for the in-sample fit. However, now there is no clear-cut conjecture how models will perform as there may be a trade-off between in-sample and out-of-sample performance. The models that provide a better in-sample do not necessary have to perform well out-of-sample because of the risk of overfitting. This will especially be the case when the models are estimated with the two-step procedure as the fitting process in the first step does not take into account the dynamics of the factors in the second step, the latter being crucial for the out-of-sample performance.

[21] Especially the two slope factors in the four-factor model are very strongly, negatively, correlated. Panels [f] and [i] of Figures 7 and 8 also indicate that to a certain extent the slope factors seem to offset each other, giving rise to a potential multicolinearity problem. However, the results in this and the following section show that adding the second slope factor helps to improve not only the in-sample fit but also the out-of-sample forecasting accuracy.

7.1 Forecast procedure

I assess the forecasting performance of the Nelson-Siegel models by dividing the full data sample into the initial estimation period 1984:1 - 1993:12 (120 observations) and the forecasting period 1994:1 - 2003:12 (120 observations). Next to gauging the models' predictive accuracy over the full sample I also consider two subsamples: 1994:1 - 2000:12 (84 observations) and 2001:1 - 2003:12 (36 observations). The first subsample is the out-of-sample period used by Diebold and Li (2006) and allows me to directly compare the performance of the alternative Nelson-Siegel specifications with that of the three-factor factor model results of Diebold and Li (2006). The second subsample starts in 2001 when the Federal Reserve lowered the target rate from 6.5% to 6% in a first of eleven subsequent decreases, resulting in a drop of short-term interest rates by 4% and a strong widening of spreads. Mönch (2006a) and De Pooter *et al.* (2007) both show that predictability is scarce in 2001-2003 and it will be interesting to see how the Nelson-Siegel models perform in this period.

All the models are estimated recursively with an expanding data window. Interest rate forecasting is carried out by constructing factor predictions using the state equations and subsequently substituting these predictions in the measurement equations to obtain the interest rate forecasts. I consider four forecast horizons, $h = 1$ month as well as 3, 6 and 12 months ahead. The h-month ahead factor forecasts, $\widehat{\beta}_{T+h}$, are iterated forecasts which follow from forward iteration of the state equations in (18) as follows

$$\widehat{\beta}_{T+h} = \left[I_K - \widehat{\Phi}^h\right]\left[I_K - \widehat{\Phi}\right]^{-1}\widehat{\mu} + \widehat{\Phi}^h\beta_T$$

where I_K is the $(K \times K)$ identity matrix, $\widehat{\mu}$ and $\widehat{\Phi}$ the state equation parameter estimates and β_T the last available factor estimates. With the one-step estimation method I use the in-sample decay parameter estimates to compute the factor loadings. With the two-step method I use the median value of the time-series of decay parameter estimates[22].

[22]Experimentation with alternative choices (using the most recent decay parameter estimate and using the mean estimate) revealed that using the median gives more stable results. Note that Nelson and Siegel (1987) who estimate λ_t alongside the factors in the three-factor model also report fit results when imposing the median λ_t estimate. They find that the in-sample fit is not degraded much when doing so.

7.2 Competitor models

Random walk

I consider three competitor models against which to judge the predictive accuracy of the Nelson-Siegel models. The first is the benchmark Random Walk model

$$y_t(\tau_i) = y_{t-1}(\tau_i) + \varepsilon_t(\tau_i), \qquad \varepsilon_t(\tau_i) \sim \mathcal{N}\left(0, \sigma^2(\tau_i)\right) \tag{23}$$

Many other studies that consider interest rate forecasting all show that consistently outperforming the random walk is difficult. The Random Walk h-month ahead forecast is equal to the last observed value; $\widehat{y}_{T+h}(\tau_i) = y_T(\tau_i)$.

AR(1) model

The second competitor model is a first-order univariate autoregressive model which allows for mean-reversion

$$y_t(\tau_i) = \mu(\tau_i) + \phi(\tau_i) y_{t-1}(\tau_i) + \varepsilon_t(\tau_i), \qquad \varepsilon_t(\tau_i) \sim \mathcal{N}\left(0, \sigma^2(\tau_i)\right) \tag{24}$$

VAR(1) model

The third and final competitor model is an unrestricted VAR(1) model for yield levels. A well-known shortcoming of using VAR models for yield forecasting is that only maturities that are included in the model can be forecasted. To keep down the number of parameters I therefore estimate a VAR(1) model in which the lagged yields are replaced by their first three common factors. The reason is that these factors explain over 99% of the total variation and also because of their Litterman and Scheinkman (1991) interpretation as level, slope and curvature factors. I extract the factor matrix, denoted by F_{t-1}, by applying static principal component analysis on the panel of lagged yields (using data up until month $t-1$) which consist of 13 maturities: $\tau = 1$, 3 and 6 months and 1, 2, ..., 10 years[23]. The VAR(1) model is then given by

$$Y_t = \mu + \Phi F_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \Sigma) \tag{25}$$

with $Y_t = [y_t^{(1m)}, ..., y_t^{(10y)}]'$, μ a (13×1) vector, Φ a (13×3) matrix and Σ a full (13×13) matrix.

[23]Note that similar to Diebold and Li (2006), I do not use the 1-month maturity in the Nelson-Siegel models. I do include it here in order to also assess the forecasts for this short maturity.

One important difference between the Nelson-Siegel and VAR models is that the latter does not impose a specific parametric form on the right hand side of the measurement equations. The VAR(1) model can therefore be used to determine whether the exponential factor loading structure of the Nelson-Siegel class of model is beneficial for forecasting yields. For the AR and VAR model I similarly construct iterated forecasts.

7.3 Forecast evaluation

I use a number of standard forecast error evaluation criteria to assess the quality of the out-of-sample forecasts. In particular, I report the Root Mean Squared Prediction Error (RMSPE) for the individual maturities as well as the Trace Root Mean Squared Prediction Error (TRMSPE). The latter combines the forecast errors of all maturities and summarizes the performance per model, thereby allowing for a direct comparison between models[24]. Significant differences between the forecast performance of the random walk and each of the models are tested for using the White (2000) 'reality check' test which I implement using the stationary bootstrap method of Politis and Romano (1994) with 1000 block-bootstraps of the forecast error series and an average block-length of 12 months.

7.4 Forecast results

The results for the full sample period 1994:1 - 2003:12 are presented in Tables 6-9. The first line in each table show the (T)RMSPEs for the random walk. All other entries are relative (T)RMSPEs with respect to the random walk, including those in lines two and three which show the results for the competitor AR(1) and VAR(1) models. Bold numbers indicate outperformance with respect to the random walk. The results for this sample are directly comparable to those of Mönch (2006a) as he uses an almost identical forecasting sample (1994:1 - 2003:9) and also reports results for the NS2-3-AR and NS2-3-VAR specifications. Although the random walk statistics are all but identical, for the Nelson-Siegel three factor model I find somewhat better statistics for longer forecast horizons than Mönch. This is most likely caused by my use of iterated forecasts whereas Mönch uses direct forecasts,

[24]Results of other evaluation criteria such as the Mean Prediction Error (MPE), Mean Absolute Prediction Error (MAPE) and forecast regression R^2-s are not reported here but are available upon request. For more details regarding the TRMSPE, see Christoffersen and Diebold (1998). I compute the TRMSPE over the following maturities for which I compute forecasts: τ = 1, 3, 6 and 12 months and 2,...,10 years. Tables 6-13 show results for individual maturities for only a subset of these thirteen maturities.

the different sets of maturities used and by the small differences in estimation and forecast periods.

For a 1-month horizon there only seems to be a certain degree of predictability for maturities of less than one year. With the two-step estimation method it is mainly the four-factor model that does well for the one and three-month maturities. The 19% outperformance for the one-month maturity relative to the random walk is significant at the 5% level according to the White reality check test. A VAR(1) specification for the factor dynamics clearly works better than separate AR(1) models per factor. For the one-step models, it is in terms of

- Insert Tables 6 - 9 around here -

TRMSPE less clear which specification for the factor dynamics is to be preferred. However, allowing for a full coefficient matrix clearly produces the most accurate short-maturity forecasts, although the best model is NS1-4-AR with a TRMSPE of 0.98.

Table 7 shows for the 3-month horizon that for all models the best results are obtained for short maturities using a VAR specification for the factors, instead of using an AR or RW specification. The second and third line of the table shows this to hold also for the VAR model using yield levels directly. From Panel B is it clear that the one-step estimation procedure yields the most accurate results. It is interesting that the four-factor and (Adjusted) Svensson models not only fit the term structure very well, but that they also produce accurate short-maturity forecasts. For the one, three, six and twelve-month maturity they all outperform the random walk by 30%, 23%, 13% and 6% respectively. The four-factor model yields the most accurate results with the one-step procedure. The four-factor model is also the most accurate using the two-step procedure, with NS2-4-λ-VAR doing marginally better than NS2-4-VAR due to being more accurate for long maturities. The TRMSPEs of NS2-4-λ-VAR and NS1-4-AR are very similar but the latter model outperforms the random walk also for long maturities. In fact, from Panel B it seems that whereas the VAR specification works well for short maturities, the AR specification in general produces better forecasts for long maturities.

This pattern becomes more evident for the 6-month horizon. For the one-step procedure the VAR specification outperforms the random walk for short maturities, with the outperformance being strongly statistically significant, but for long maturities the performance is poor whereas the exact opposite pattern is visible for the AR specification. The NS1-4-AR again is the only model that forecasts well across the entire maturity spectrum, clearly giving it the lowest TRMSPE of 0.93. The closest competitor with the two-step procedure is still the

NS2-4-λ-VAR model although the NS2-3-λ-VAR is a close second with a relative TRMSPE with is only 1% lower.

Finally, for the 12-month horizon, shown in Table 9, all the two-step VAR specifications outperform the random walk by up to 15% for short maturities but at the same time produce very poor forecasts for long maturities. Only the NS2-3-λ-VAR model is more accurate than the random walk for all maturities except the 10-year maturity. The VAR model for yield levels has an increasingly worse performance for longer maturities and the relative RMSPEs I report in the third row of the table are higher than those given in Mönch (2006a). This is most likely due to my use of a larger set of maturities and because I construct iterated forecasts. The Svensson and Adjusted Svensson models are again able to forecast both short and long maturities with the one-step estimation procedure, although not consistently with either AR or VAR factor dynamics. The NS1-4-AR model is still the most accurate model with a relative TRMSPE of 0.90 and significant outperformance for individual maturities up to ten years.

Overall the full-sample results can be summarized as follows. With both the one-step and two-step estimation procedures, using VAR factor dynamics is typically optimal for constructing short-maturity forecasts, irrespective of the forecast horizon. It is, however, only the one-step estimation procedure that also produces increasingly accurate forecasts when the forecast horizon lengthens, more specifically with the assumption of AR factor dynamics. With the two-step procedure such an improvement is lacking. This strongly suggests to simultaneously use cross-sectional and time-series information when the purpose of using the Nelson-Siegel model is that of forecasting the term structure. The best overall performing model is the four-factor model which is the only model that accurately forecasts the entire maturity spectrum, especially for the 6 and 12-month horizons. It is interesting that adding a second slope factor not only improves the in-sample fit but also the out-of-sample performance. In fact, adding factors in general seems to benefit the forecasting performance as the (Adjusted) Svensson model also predicts reasonably well compared to for the example the three-factor model. Whether freeing up decay parameters is helpful is somewhat ambiguous for the three and four-factor model. The Bliss model, however, does not forecast well.

Although in general it holds that imposing the Nelson-Siegel exponential structure on the factor loadings certainly helps compared to the AR(1) and VAR(1) yield level models, the three-factor model, and in particular the two-factor model, forecasts rather disappointingly. As Diebold and Li (2006) report very accurate forecasts for the NS2-AR model it will be interesting to see how well the performance of the four-factor model holds up for the 1994-2000 period.

7.5 Subsample results

Sample 1994:1 - 2000:12

Except from an initial surge during 1994, interest rates are fairly stable during this period which is characterized by a substantial amount of predictability as shown in Tables 10 and 11. Although for a one-month horizon this is not case, it is certainly true for longer horizons. The results I find for the NS2-AR and NS2-VAR are very similar to those reported in Diebold and Li (2006) and Mönch (2006b). For the 3-month horizon, the two-step NS2-3-AR model produces accurate forecasts and performs better than the NS2-3-VAR model. Although NS2-2-λ-AR, NS2-3-λ-AR and NS2-AS-AR have a lower TRMSPEs (0.91 vs 0.94) all these

- Insert Tables 10 and 11 around here -

models forecast well mainly for medium and long maturities whereas the NS2-3-AR model outperforms the random walk for all maturities. The same holds for the NS1-3-AR model which for short maturities is outperformed by the NS1-4-AR model which is the best one-step model. The VAR specifications still deliver the more accurate short-maturity forecasts but the AR specifications now also do well for short maturities as well as reasonably well for longer maturities.

The relative TRMSPE numbers for the 6 and 12-month horizons in Table 11 show that AR dynamics, either in the two-step or one-step estimation approach, clearly outperform VAR dynamics. The two-step three-factor model with the Diebold and Li (2006) approach of fixing λ_t indeed forecasts well. However, estimating the decay parameter alongside the factors seems worthwhile. The results for the NS2-3-λ-AR show that doing so improves forecasts for short and medium maturities, with strong statistical outperformance relative to the random walk, but that is also leads to a decrease in accuracy for long maturities. With the one-step approach, however, the random walk is outperformed fairly evenly for each maturity. The results for the one-step four-factor, Bliss and (Adjusted) Svensson models are all very similar.

Sample 2001:1 - 2003:12

Mönch (2006b) examines the performance of the NS2-AR and NS2-VAR models for the sample 2000:1 - 2003:9 and finds that it is much worse than for the Diebold and Li (2006) out-of-sample period 1994:1 - 2000:12. This leads him to conclude that "...some of the strong forecast performance of the Nelson-Siegel model documented by Diebold and Li may be due to their choice of forecast period". The results for the second subsample that I examine shed more light on this claim. In fact, the results in Tables 12 and 13 support Mönch's conclusion

- Insert Tables 12 and 13 around here -

as the NS2-AR and NS2-VAR models are shown to forecast poorly for all horizons[25]. For the 1 and 3-month horizon there is some predictability again for short maturities using the one-step estimation methods with VAR dynamics for the factors. The four-factor performance is reasonable for the 3-month horizon with RMSPEs below one for short and long maturities although medium maturities are predicted poorly. For the 6 and 12-month horizon the VAR specification again beats the AR specification due to accurate short-maturity forecasts.

The only model that forecasts well in this period with its downward trend in short term interest rates and strong increase in interest spreads is the NS1-4-AR model. For the 1 and 3-month horizon the model has difficulties forecasting the two and five-year maturities. However, for longer horizons, the model produces increasingly accurate forecasts and does so consistently across all maturities. Especially for the 12-month horizon NS1-4-AR reduces RMSPEs relative to the random walk by at least 11% for maturities up to seven years resulting in an overall relative TRMSPE of 0.85. Mönch (2006a) compares the forecasting performance of a number of competing models, among which are the NS2-3-AR and NS2-3-VAR specifications, for a very similar period (2000:1 - 2003:9) and finds that his proposed Factor Augmented VAR model is the only model capable of accurately forecasting the term structure for longer forecast horizons. However, Table 13 shows that the four-factor model is a strong competitor for Mönch's FAVAR model.

8 Conclusion

In this paper I compare the in-sample fit and out-of-sample performance of a range of different Nelson-Siegel specifications. The in-sample results show that more elaborate models which incorporate multiple decay parameters and additional slope or curvature factors improve the fit of the original Nelson and Siegel (1987) three-factor functional form. The four-factor model performs qualitatively similar as the popular Svensson (1994) model but has the

[25]See also the subsample analysis in De Pooter *et al.* (2007) who arrive at the same conclusion that the forecasting performance of the three-factor Nelson-Siegel varies substantially across subperiods.

advantage that it is easier to estimate as is it less affected by potential multicolinearity problems.

Besides an improved in-sample fit relative to the three-factor model I also document a better out-of-sample performance with the four-factor model. More specifically, using the Kalman filter to estimate the latent factors over time and assuming AR(1) factor dynamics produces forecasts which consistently outperform those of benchmark models such as the random walk and unrestricted VAR models. This outperformance holds across the maturity spectrum and is most prominent for longer forecast horizons.

The analysis of this paper can be extended in a number of ways. Firstly, I have only judged the forecast performance of the various Nelson-Siegel models by considering statistical accuracy by means of the (T)RMSPE. Fabozzi *et al.* (2005) use the slope and curvature forecasts of the three-factor model to implement systematic trading strategies and assess the returns of these strategies. It will be interesting to conduct a similar type of analysis for the models used here to evaluate forecasts from an economic point of view. Secondly, the use of Bayesian inference techniques will be interesting to examine. Mönch (2006a) and De Pooter *et al.* (2007) both use MCMC methods to draw inference on the parameters and latent factors in the three-factor model. Explicitly taking into account parameter uncertainty may further improve the predictive accuracy of especially the more complex models. Finally, the use of macroeconomic variables and/or macroeconomic factors as in Diebold, Rudebusch, and Aruoba (2006b) can potentially further improve forecasts compared to the yields-only approach that I have used here. All these topics are part of ongoing research.

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Appendix A: Estimation details

A.1 Estimation details

General specification

The general specification which captures all the different Nelson-Siegel specifications is given by

$$
\begin{aligned}
Y_t &= X_t\beta_t + \varepsilon_t && \text{(A-26)}\\
\beta_t &= \mu + \Phi\beta_{t-1} + \nu_t && \text{(A-27)}
\end{aligned}
$$

with Y_t the $(N \times 1)$ vector of yields, $X_t\beta_t$ the Nelson-Siegel spot rate curve, β_t the $(K \times 1)$ vector of factors and X_t the $(N \times K)$ matrix of factor loadings. The latter are time-varying if the decay parameter(s) are estimated alongside the factors in the two-step procedure.

Two-step procedure

I estimate the parameters in step one of the two-step estimation procedure by minimizing the sum of squared yield errors, $\sum_{i=1}^{N}\left[y_t(\tau_i) - \widehat{y}_t(\tau_i)\right]^2$. When the decay parameter is fixed in the two, three and four-factor models I apply OLS. When decay parameters are estimated alongside the factors I use NLS to find optimal parameter estimates. In the latter case, the parameters of the two, three and four-factor models are initialized at the Diebold and Li (2006) value for λ_t and the OLS estimates for the factors. For the Bliss model both $\lambda_{1,t}$ and $\lambda_{2,t}$ are initialized at 16.42. Determining starting values for the (Adjusted) Svensson model is somewhat more complex as there is the additional restriction on $\lambda_{1,t}$ and $\lambda_{2,t}$. As starting values for the level, slope, curvature factor and $\lambda_{1,t}$ I use the optimal factor estimates and the λ_t estimate from the three-factor model. The fourth factor, $\beta_{4,t}$, is initialized to zero. If $\widehat{\lambda}_{1,t}$ is larger than twice the minimum allowed value of 6.69 then $\lambda_{2,t}$ is initialized to $0.5\widehat{\lambda}_{1,t}$. When $\widehat{\lambda}_{1,t}$ is smaller than 13.38 then $\lambda_{1,t}$ and $\lambda_{2,t}$ are initialized to 13.38 and 6.69 respectively. By doing so all the restrictions on $\lambda_{1,t}$ and $\lambda_{2,t}$ are satisfied. Because the minimum distance restriction is only imposed for the Svensson model, I initialize $\lambda_{2,t}$ in the Adjusted Svensson model to $\frac{1}{2}(6.69+\widehat{\lambda}_{1,t})$. As the two-step estimation procedure is numerically challenging because of the nonlinearity in the factor loadings, whenever possible I use the analytical gradient and hessian which are given in the web appendix to this paper[26].

One-step procedure

For the one-step state-space estimation method, which is only used to construct forecasts, I maximize the likelihood given in (22). It is of particular importance to start the optimization procedure with accurate starting values because of the large number of parameters. For the two, three and four-factor and Bliss models I initialize the parameters as follows. The decay parameters are set to 16.42 and the factors to their two-step OLS estimates. The equation parameters μ and Φ in the state equations are initialized with the estimates from either a VAR model or AR models for the factors. The variance parameters in H and Q are initialized to one and the optimization is performed using standard deviations to ensure positive variance estimates. The covariance parameters in Q are initially set to zero. The Kalman filter is started with the unconditional mean and variance of the factor estimates and the first twelve observations are discarded when computing the likelihood in (22). The approach for the (Adjusted) Svensson model is the same except for the fact that I use the optimal factor estimates from the two-step procedure as starting values. Furthermore, $\lambda_{1,t}$ and $\lambda_{2,t}$ are initialized by using the median of the two-step estimates.

[26]The web appendix is available on `http://www.depooter.net`

Table 1: Model abbreviations

Two-step models	**model description**	**factor dynamics**
NS2-2-AR	2-factor NS model with λ fixed to 16.42	AR(1) per factor
NS2-2-VAR	2-factor NS model with λ fixed to 16.42	VAR(1) for factors
NS2-3-AR	3-factor NS model with λ fixed to 16.42	AR(1) per factor
NS2-3-VAR	3-factor NS model with λ fixed to 16.42	VAR(1) for factors
NS2-4-AR	4-factor NS model with λ fixed to 16.42	AR(1) per factor
NS2-4-VAR	4-factor NS model with λ fixed to 16.42	VAR(1) for factors
NS2-2-λ-AR	2-factor NS model treating λ as parameter	AR(1) per factor
NS2-2-λ-VAR	2-factor NS model treating λ as parameter	VAR(1) for factors
NS2-3-λ-AR	3-factor NS model treating λ as parameter	AR(1) per factor
NS2-3-λ-VAR	3-factor NS model treating λ as parameter	VAR(1) for factors
NS2-4-λ-AR	4-factor NS model treating λ as parameter	AR(1) per factor
NS2-4-λ-VAR	4-factor NS model treating λ as parameter	VAR(1) for factors
NS2-B-AR	3-factor Bliss NS model treating λ_1, λ_2 as parameters	AR(1) per factor
NS2-B-VAR	3-factor Bliss NS model treating λ_1, λ_2 as parameters	VAR(1) for factors
NS2-S-AR	4-factor Svensson NS model treating λ_1, λ_2 as parameters	AR(1) per factor
NS2-S-VAR	4-factor Svensson NS model treating λ_1, λ_2 as parameters	VAR(1) for factors
NS2-AS-AR	4-factor Adjusted Svensson model treating λ_1, λ_2 as parameters	AR(1) per factor
NS2-AS-VAR	4-factor Adjusted Svensson model treating λ_1, λ_2 as parameters	VAR(1) for factors

One-step models	**model description**	**factor dynamics**
NS1-2-RW	2-factor NS model	random walk per factor
NS1-2-AR	2-factor NS model	AR(1) per factor
NS1-2-VAR	2-factor NS model	VAR(1) for factors
NS1-3-RW	3-factor NS model	random walk per factor
NS1-3-AR	3-factor NS model	AR(1) per factor
NS1-3-VAR	3-factor NS model	VAR(1) for factors
NS1-4-RW	4-factor NS model	random walk per factor
NS1-4-AR	4-factor NS model	AR(1) per factor
NS1-4-VAR	4-factor NS model	VAR(1) for factors
NS1-B-RW	3-factor Bliss NS model	random walk per factor
NS1-B-AR	3-factor Bliss NS model	AR(1) per factor
NS1-B-VAR	3-factor Bliss NS model	VAR(1) for factors
NS1-S-RW	4-factor Svensson NS model	random walk per factor
NS1-S-AR	4-factor Svensson NS model	AR(1) per factor
NS1-S-VAR	4-factor Svensson NS model	VAR(1) for factors
NS1-AS-RW	4-factor Adjusted Svensson NS model	random walk per factor
NS1-AS-AR	4-factor Adjusted Svensson NS model	AR(1) per factor
NS1-AS-VAR	4-factor Adjusted Svensson NS model	VAR(1) for factors

Notes: The table gives model abbreviations used in the subsequent tables and graphs. 'NS' stands for Nelson-Siegel model. For the two-step models (top panel), the factors are estimated in a first step using least squared applied to the cross-section of yields in each month. In the second step, the dynamics of the estimated factors from the first step are estimated. For the one-step models (bottom panel), all parameters are estimated simultaneously as a state-space model using the Kalman filter. The table shows model abbreviations in the first column, model descriptions in the second column and the specification for the factor dynamics in the third column.

Table 2: Summary statistics

maturity	mean	stdev	skew	kurt	min	max	JB-p	ρ_1	ρ_{12}	ρ_{24}
1-month	5.051	2.090	0.045	2.864	0.794	10.727	0.845	0.968	0.552	0.121
3-month	5.286	2.175	-0.005	2.843	0.876	10.905	0.849	0.977	0.563	0.148
6-month	5.434	2.226	0.024	2.951	0.958	11.169	0.962	0.977	0.559	0.162
1-year	5.707	2.290	0.051	3.056	1.040	11.928	0.946	0.976	0.562	0.190
2-year	6.083	2.281	0.234	3.351	1.299	12.777	0.201	0.974	0.555	0.229
3-year	6.365	2.211	0.357	3.465	1.618	13.115	0.031	0.973	0.561	0.268
4-year	6.589	2.168	0.481	3.524	1.999	13.268	0.003	0.972	0.572	0.297
5-year	6.711	2.125	0.599	3.613	2.351	13.410	0.000	0.972	0.574	0.319
6-year	6.878	2.108	0.666	3.574	2.663	13.493	0.000	0.973	0.589	0.335
7-year	6.967	2.061	0.761	3.727	3.003	13.554	0.000	0.972	0.577	0.333
8-year	7.058	2.019	0.751	3.650	3.221	13.596	0.000	0.972	0.590	0.359
9-year	7.106	2.001	0.753	3.584	3.389	13.529	0.000	0.973	0.599	0.371
10-year	7.102	1.982	0.783	3.598	3.483	13.595	0.000	0.973	0.600	0.373

Notes: The table shows summary statistics for end-of-month unsmoothed continuously compounded U.S. zero-coupon yields. The results shown are for annualized yields (expressed in precentages). The sample period is January 1984 - December 2003 (240 observations). Reported are the mean, standard deviation, skewness, kurtosis, minimum, maximum, the p-value of the Jarque-Bera test statistic for normality and the 1$^{\text{st}}$, 12$^{\text{th}}$ and 24$^{\text{th}}$ sample autocorrelation.

Table 3: In-sample fit: restricted decay parameters

maturity	NS2-2	NS2-3	NS2-4	NS2-2-λ	NS2-3-λ	NS2-4-λ	NS2-B	NS2-S	NS2-AS
					Mean Error				
3-month	6.55	-0.97	1.47	5.06	-0.26	1.65	1.09	1.50	1.36
6-month	3.06	-0.93	-0.86	1.67	-0.75	-0.97	-0.86	-0.92	-0.93
1-year	-0.23	0.50	-0.98	-0.52	0.22	-1.07	-0.70	-0.98	-0.87
2-year	-6.52	-2.17	-2.34	-4.30	-2.42	-2.23	-2.34	-2.31	-2.28
5-year	-5.50	-4.36	-2.86	-3.55	-4.13	-2.95	-3.10	-2.92	-3.12
10-year	2.53	-2.40	-3.97	-2.82	-2.44	-3.87	-3.75	-3.94	-3.69
					Standard Deviation				
3-month	32.06	8.59	3.12	17.09	5.25	2.74	3.98	2.59	**2.23**
6-month	15.88	4.12	4.11	9.07	4.20	3.94	4.05	3.96	**3.91**
1-year	9.23	8.11	5.49	8.26	6.70	5.46	6.17	5.42	**5.33**
2-year	16.53	4.72	4.70	8.57	4.77	4.45	4.57	**4.40**	4.45
5-year	7.21	5.90	4.74	6.14	5.40	4.62	5.11	4.67	**4.60**
10-year	20.07	7.12	5.15	10.26	6.08	4.93	5.30	**4.73**	4.89
					Root Mean Squared Error				
3-month	32.72	8.64	3.45	17.82	5.26	3.20	4.12	3.00	**2.62**
6-month	16.17	4.23	4.19	9.22	4.27	4.06	4.14	4.07	**4.02**
1-year	9.24	8.12	5.58	8.28	6.70	5.56	6.21	5.51	**5.40**
2-year	17.77	5.20	5.25	9.59	5.35	4.98	5.14	**4.96**	5.00
5-year	9.07	7.34	5.54	7.09	6.80	**5.48**	5.98	5.50	5.55
10-year	20.23	7.52	6.50	10.64	6.55	6.27	6.50	6.16	**6.12**
					Mean Absolute Error				
3-month	25.56	6.39	2.76	12.64	3.71	2.55	3.06	2.34	**1.93**
6-month	12.19	3.06	3.03	6.19	3.10	2.95	3.00	**2.94**	2.97
1-year	7.45	6.37	4.51	6.58	5.24	4.42	4.84	4.35	**4.25**
2-year	13.71	3.70	3.75	6.81	3.75	3.58	3.63	**3.56**	3.64
5-year	7.45	6.13	4.33	5.88	5.53	4.21	4.75	**4.19**	4.29
10-year	15.71	5.91	5.25	8.31	5.21	5.06	5.14	4.98	**4.94**
					Minimum Error				
3-month	-86.81	-34.51	-11.65	-49.15	-22.38	-7.83	-16.67	-5.98	**-4.13**
6-month	-40.00	-14.11	-13.82	-20.69	-13.29	-12.66	-13.02	**-12.63**	-12.72
1-year	-20.74	-18.33	-17.91	-20.75	-16.61	-20.70	-20.31	-20.07	**-19.69**
2-year	-46.90	-19.05	-20.10	-27.83	-21.67	-18.32	-19.22	-17.15	**-16.75**
5-year	-27.39	-19.89	-20.23	-18.38	-17.20	-23.54	**-17.20**	-23.59	-23.46
10-year	-37.58	-25.57	-18.38	-41.31	-18.39	**-18.37**	-18.39	-19.09	-19.28
					Maximum Error				
3-month	75.52	21.75	10.60	54.67	12.43	9.03	12.44	8.97	**8.97**
6-month	44.64	21.81	22.10	34.28	22.16	22.09	22.20	21.22	**18.55**
1-year	28.83	26.69	13.48	22.14	21.99	12.03	17.86	11.96	**11.89**
2-year	36.97	**16.64**	16.97	22.28	18.98	16.98	18.67	18.91	18.33
5-year	20.39	18.62	12.37	19.48	13.39	**10.20**	13.39	10.87	11.53
10-year	53.93	16.41	7.96	24.75	17.02	7.97	9.25	8.05	**7.45**
					$\widehat{\rho}_1$				
3-month	0.907	0.754	0.483	0.817	0.689	0.483	0.473	0.417	0.435
6-month	0.875	0.276	0.270	0.688	0.248	0.278	0.271	0.244	0.322
1-year	0.659	0.582	0.386	0.615	0.510	0.390	0.417	0.378	0.369
2-year	0.913	0.649	0.628	0.759	0.613	0.615	0.625	0.597	0.622
5-year	0.805	0.740	0.644	0.746	0.696	0.642	0.606	0.602	0.609
10-year	0.889	0.627	0.488	0.706	0.550	0.442	0.408	0.405	0.438
					$\widehat{\rho}_{12}$				
3-month	0.347	0.087	0.102	0.281	0.023	0.192	0.052	0.174	0.119
6-month	0.430	0.203	0.188	0.304	0.159	0.239	0.215	0.220	0.240
1-year	0.347	0.296	0.370	0.311	0.338	0.356	0.330	0.353	0.356
2-year	0.295	0.129	0.132	0.091	0.129	0.100	0.104	0.102	0.108
5-year	0.099	0.046	-0.092	-0.060	-0.099	-0.112	-0.116	-0.122	-0.121
10-year	0.394	0.297	0.305	0.190	0.205	0.298	0.215	0.264	0.250

Notes: The table show in-sample fit error statistics for the full sample 1984:1-2003:12 (240 observations). The statistics are expressed in basis points. Results are shown for the models with λ_t fixed to 16.42 [NS2-2, NS2-3, NS-4], with λ estimated (but restricted) [NS-2-λ, NS-3-λ, NS-4-λ], the Bliss extension [NS2-B] and the adjusted Svensson model [NS2-(A)S]. The statistics $\widehat{\rho}_1$ and $\widehat{\rho}_{12}$ represent the 1st and 12th autocorrelation of the yield errors. For selected statistics, bold numbers indicate the best performing model.

Table 4: In-sample fit: unrestricted decay parameters

maturity	NS2-2-λ	NS2-3-λ	NS2-4-λ	NS2-B	NS2-S	NS2-AS
			Mean Error			
3-month	4.36	-0.41	1.67	1.52	1.33	0.85
6-month	0.34	-0.82	-0.98	-0.88	-0.37	0.10
1-year	-1.30	0.27	-1.10	-0.99	-1.15	-1.10
2-year	-3.45	-2.33	-2.23	-2.31	-2.31	-2.43
5-year	-1.95	-4.32	-2.91	-2.91	-2.94	-2.83
10-year	-5.42	-2.15	-3.93	-3.82	-3.49	-3.63
			Standard Deviation			
3-month	11.18	5.12	2.82	2.96	2.32	**2.29**
6-month	6.03	4.14	3.97	3.93	3.70	**3.45**
1-year	8.14	6.57	5.41	5.65	5.33	**5.15**
2-year	6.43	4.67	4.55	4.40	4.40	**4.37**
5-year	6.37	5.11	4.55	4.52	**4.37**	4.39
10-year	8.63	5.67	4.94	4.38	**4.20**	4.33
			Root Mean Squared Error			
3-month	12.00	5.13	3.28	3.33	2.67	**2.45**
6-month	6.04	4.22	4.09	4.03	3.72	**3.45**
1-year	8.25	6.58	5.52	5.73	5.45	**5.27**
2-year	7.30	5.22	5.06	**4.96**	**4.96**	5.00
5-year	6.66	6.69	5.41	5.38	5.26	**5.22**
10-year	10.19	6.07	6.31	5.81	**5.46**	5.65
			Mean Absolute Error			
3-month	8.72	3.58	2.62	2.58	1.91	**1.58**
6-month	4.54	3.03	2.97	2.90	2.75	**2.59**
1-year	6.73	5.16	4.46	4.57	4.24	**4.14**
2-year	5.42	3.67	3.65	**3.56**	**3.56**	3.67
5-year	5.40	5.39	4.21	4.16	**3.95**	3.96
10-year	7.89	4.88	5.12	4.79	**4.45**	4.63
			Minimum Error			
3-month	-41.06	-22.44	-8.14	-7.63	**-3.96**	-12.38
6-month	-19.68	-13.33	-12.80	-12.90	**-12.72**	-12.77
1-year	-20.75	**-16.69**	-20.78	-20.40	-20.32	-20.27
2-year	-21.66	-21.67	-18.01	**-17.15**	**-17.15**	-17.17
5-year	-17.24	-17.25	-23.13	**-16.94**	-23.46	-23.31
10-year	-41.02	-17.03	-16.49	-17.09	**-15.66**	-18.51
			Maximum Error			
3-month	36.29	11.32	9.15	9.58	**9.02**	9.17
6-month	17.93	22.18	22.03	22.19	15.79	**9.46**
1-year	21.98	22.00	**11.81**	12.01	19.90	11.90
2-year	18.08	18.89	17.32	18.91	18.91	19.20
5-year	19.47	9.32	8.84	9.37	**8.62**	8.66
10-year	20.54	16.97	**7.79**	7.86	7.93	8.32
			$\widehat{\rho}_1$			
3-month	0.748	0.654	0.477	0.505	0.385	0.346
6-month	0.483	0.242	0.270	0.283	0.241	0.245
1-year	0.609	0.510	0.393	0.389	0.399	0.416
2-year	0.688	0.607	0.617	0.603	0.618	0.614
5-year	0.769	0.692	0.644	0.638	0.601	0.608
10-year	0.668	0.551	0.483	0.402	0.389	0.398
			$\widehat{\rho}_{12}$			
3-month	0.305	0.010	0.139	0.146	0.100	0.122
6-month	0.266	0.154	0.216	0.236	0.241	0.230
1-year	0.343	0.339	0.358	0.349	0.354	0.360
2-year	0.058	0.127	0.109	0.090	0.061	0.080
5-year	0.136	-0.133	-0.099	-0.175	-0.181	-0.182
10-year	0.231	0.217	0.264	0.243	0.241	0.215

Notes: The table show in-sample fit error statistics for the full sample 1984:1-2003:12 (240 observations). The statistics are expressed in basis points. Results are shown for models with unrestricted decay parameter(s) [λ(s)]. Error statistics are given for the two, three and four-factor specification [NS2-2-λ, NS2-3-λ, NS-4-λ], the Bliss extension [NS2-B] and the adjusted Svensson model [NS2-(A)S]. The statistics $\widehat{\rho}_1$ and $\widehat{\rho}_{12}$ represent the 1^{st} and 12^{th} autocorrelation of the yield errors. For selected statistics, bold numbers indicate the best performing model.

Table 5: Factor summary statistics

		summary statistics				**correlations estimated factors**				**correlations yield factors**		
	factor	mean	stdev	ρ_1	ρ_{12}	β_1	β_2	β_3	β_4	L	S	C
NS2-2	β_1	7.403	2.115	0.970	0.571	1	-	-	-	*0.996*	-0.195	0.544
	β_2	-2.387	1.625	0.966	0.420	-0.144	1	-	-	-0.089	*0.990*	0.472
NS2-3	β_1	7.531	1.896	0.972	0.635	1	-	-	-	*0.973*	-0.324	0.353
	β_2	-2.394	1.639	0.967	0.426	-0.280	1	-	-	-0.079	*0.988*	0.485
	β_3	-0.571	2.171	0.923	0.376	0.347	0.510	1	-	0.539	0.415	*0.992*
NS2-4	β_1	7.599	1.911	0.971	0.620	1	-	-	-	*0.978*	-0.312	0.390
	β_2	-1.103	4.774	0.857	0.291	0.022	1	-	-	0.063	*0.409*	0.470
	β_3	-1.536	3.199	0.831	0.178	0.136	-0.615	1	-	0.264	0.204	*0.335*
	β_4	-1.426	4.591	0.816	0.202	-0.125	-0.942	0.763	1	-0.104	*-0.080*	-0.348
NS2-2-λ	β_1	7.610	1.944	0.969	0.616	1	-	-	-	*0.974*	-0.326	0.379
	β_2	-2.585	1.766	0.965	0.446	-0.253	1	-	-	-0.050	*0.983*	0.509
NS2-3-λ	β_1	7.534	1.861	0.962	0.632	1	-	-	-	*0.968*	-0.318	0.351
	β_2	-2.414	1.599	0.944	0.445	-0.280	1	-	-	-0.074	*0.981*	0.467
	β_3	-0.686	2.362	0.856	0.454	0.349	0.515	1	-	0.534	0.441	*0.861*
NS2-4-λ	β_1	7.586	1.871	0.969	0.613	1	-	-	-	*0.969*	-0.318	0.380
	β_2	-1.115	4.505	0.868	0.306	0.045	1	-	-	0.080	*0.431*	0.490
	β_3	-1.420	3.284	0.796	0.185	0.140	-0.558	1	-	0.304	0.199	*0.339*
	β_4	-1.412	4.348	0.828	0.197	-0.156	-0.933	0.724	1	-0.115	*-0.082*	-0.358
NS2-B	β_1	7.599	1.888	0.961	0.627	1	-	-	-	*0.967*	-0.326	0.363
	β_2	-2.519	1.649	0.937	0.405	-0.315	1	-	-	-0.103	*0.978*	0.413
	β_3	-0.412	2.685	0.880	0.352	0.394	0.349	1	-	0.571	0.307	*0.937*
NS2-S	β_1	7.596	1.864	0.955	0.605	1	-	-	-	*0.962*	-0.321	0.366
	β_2	-2.530	1.634	0.931	0.382	-0.294	1	-	-	-0.074	*0.966*	0.412
	β_3	-0.961	2.818	0.746	0.288	0.236	0.450	1	-	0.449	0.270	*0.586*
	β_4	0.679	1.968	0.721	0.199	0.172	-0.132	-0.474	1	0.113	0.042	*0.345*
NS2-AS	β_1	7.585	1.883	0.962	0.617	1	-	-	-	*0.963*	-0.321	0.358
	β_2	-2.515	1.643	0.937	0.381	-0.298	1	-	-	-0.083	*0.968*	0.409
	β_3	-0.895	2.593	0.805	0.329	0.284	0.500	1	-	0.487	0.372	*0.703*
	β_4	0.277	0.946	0.715	0.099	0.113	-0.221	-0.330	1	0.077	-0.073	*0.250*
yield factors	L	7.102	1.982	0.973	0.600	-	-	-	-	-	-	-
	S	-1.815	1.217	0.958	0.387	-	-	-	-	-	-	-
	C	-0.222	0.823	0.922	0.417	-	-	-	-	-	-	-

Notes: The table shows summary statistics of estimated factors for different Nelson-Siegel specifications. Statistics are shown for the two, three and four-factor model specification with λ_t fixed to 16.42 [NS2-2, NS2-3, NS-4], with λ_t estimated (but restricted) [NS-2-λ, NS-3-λ, NS-4-λ], the Bliss extension [NS2-B] and the adjusted Svensson model [NS2-(A)S]. Columns 1-4 represent the mean and standard deviation of the factors and their 1^{st} and 12^{th} order sample autocorrelation. Columns 5-8 show the correlation matrix of the factors within a given model whereas the final columns give the correlation of each factor with the empirical level (L, [10-year yield]), (negative of the) slope (S, -[10-year yield - 3-month yield]) and curvature (C, [2*2-year yield-10-year yield-3-month yield]). Statistics are calculated over the sample 1984:1 - 2003:12 (240 observations).

Table 6: Forecast results for the sample 1994:1 - 2003:12, 1-month horizon

	TRMSPE	**RMSPE**							
Maturity	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y
RW	101.59	30.12	21.18	21.82	25.71	29.12	30.48	29.30	27.95
AR	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00
VAR	1.01	**0.81**	**0.95**^{10}	**0.99**	1.03	1.07	1.01	1.02	1.07
Panel A: two-step models									
NS2-2-AR	1.18	1.55	1.44	1.05	1.07	1.27	1.04	1.05	1.26
NS2-2-VAR	1.19	1.62	1.55	1.12	1.05	1.24	1.03	1.07	1.28
NS2-3-AR	1.02	**0.97**	**0.98**	1.07	1.07	1.07	1.02	1.01	1.03
NS2-3-VAR	1.01	**0.95**^{5}	**0.87**^{5}	**0.97**^{10}	1.02	1.06	1.03	1.03	1.03
NS2-4-AR	1.07	**0.97**	1.26	1.23	1.18	1.13	1.04	1.03	1.01
NS2-4-VAR	**0.99**	**0.81**^{5}	**0.85**	**0.94**	**0.98**	1.06	1.02	1.03	1.03
NS2-2-λ-AR	1.68	1.00	1.01	1.38	1.88	2.17	1.85	1.58	1.45
NS2-2-λ-VAR	1.66	1.05	1.05	1.34	1.83	2.12	1.82	1.57	1.44
NS2-3-λ-AR	1.23	1.01	1.26	1.41	1.36	1.33	1.25	1.19	1.15
NS2-3-λ-VAR	1.21	**0.87**^{10}	1.05	1.26	1.30	1.33	1.26	1.21	1.19
NS2-4-λ-AR	1.10	**0.97**	1.26	1.26	1.20	1.18	1.09	1.06	1.06
NS2-4-λ-VAR	1.00	**0.83**^{5}	**0.90**	**0.97**	1.00	1.08	1.03	1.02	1.06
NS2-B-AR	1.18	1.04	1.30	1.36	1.23	1.16	1.22	1.20	1.19
NS2-B-VAR	1.14	**0.90**	1.06	1.16	1.12	1.12	1.19	1.20	1.21
NS2-S-AR	1.30	1.04	1.36	1.53	1.47	1.40	1.32	1.27	1.30
NS2-S-VAR	1.11	**0.84**^{5}	**0.98**	1.12	1.12	1.13	1.14	1.16	1.22
NS2-AS-AR	1.29	1.08	1.42	1.52	1.45	1.40	1.31	1.25	1.20
NS2-AS-VAR	1.24	**0.92**	1.18	1.30	1.30	1.33	1.28	1.25	1.22
Panel B: one-step models									
NS1-2-RW	1.29	2.22	2.31	1.67	1.06	1.05	1.00	1.02	1.09
NS1-2-AR	1.30	2.23	2.31	1.68	1.09	1.09	1.04	1.03	1.11
NS1-2-VAR	1.32	2.25	2.35	1.72	1.10	1.08	1.05	1.04	1.13
NS1-3-RW	1.01	1.07	1.00	**0.99**	1.01	1.04	1.01	1.01	1.01
NS1-3-AR	1.02	1.09	1.04	1.05	1.06	1.07	1.02	1.01	1.02
NS1-3-VAR	1.04	1.07	**0.97**^{5}	**0.98**^{5}	1.01	1.07	1.07	1.05	1.07
NS1-4-RW	1.00	**0.93**	1.01	1.01	**0.97**	1.04	1.01	1.01	1.02
NS1-4-AR	**0.98**	**0.89**	**0.97**	**0.97**	**0.96**	1.03	1.00	1.00	1.02
NS1-4-VAR	**0.99**	**0.82**^{10}	**0.86**	**0.90**^{10}	**0.94**^{10}	1.04	1.02	1.03	1.08
NS1-B-RW	1.01	1.05	1.05	1.01	**0.98**	1.03	1.01	1.01	1.04
NS1-B-AR	1.02	1.05	1.07	1.04	1.01	1.05	1.02	1.01	1.04
NS1-B-VAR	1.02	**0.92**^{5}	**0.88**^{5}	**0.96**^{5}	1.00	1.06	1.06	1.05	1.10
NS1-S-RW	1.01	1.01	1.06	1.01	**0.97**	1.04	1.01	1.02	1.01
NS1-S-AR	1.02	1.02	1.10	1.04	1.00	1.07	1.02	1.02	1.04
NS1-S-VAR	**0.99**	**0.84**	**0.87**	**0.90**^{10}	**0.94**^{10}	1.03	1.02	1.03	1.07
NS1-AS-RW	1.00	1.00	1.06	1.01	**0.97**	1.04	1.01	1.02	1.01
NS1-AS-AR	1.02	1.01	1.09	1.03	**0.99**	1.06	1.02	1.01	1.03
NS1-AS-VAR	**0.99**	**0.84**	**0.87**	**0.90**^{10}	**0.94**^{10}	1.03	1.02	1.03	1.07

Notes: Bold numbers indicate outperformance relative to the random walk (RW) whereas $()^{10}$, $()^{5}$ and $()^{1}$ indicate significant outperformance at the 90%, 95% and 99% level respectively according to the White (2000) reality check using 1000 block-bootstraps and an average block-length of 12.

Table 7: Forecast results for the sample 1994:1 - 2003:12, 3-month horizon

	TRMSPE	**RMSPE**							
Maturity	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y
RW	197.03	54.05	48.59	51.06	55.68	60.20	57.56	53.78	50.08
AR	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01
VAR	1.01	**0.74**^{10}	**0.89**	**0.98**	1.05	1.07	1.03	1.03	1.08
Panel A: two-step models									
NS2-2-AR	1.03	**0.90**^{1}	**0.89**^{1}	**0.97**^{5}	1.05	1.10	1.02	1.03	1.13
NS2-2-VAR	1.04	**0.97**^{5}	**0.96**^{5}	**0.99**^{10}	1.03	1.08	1.03	1.05	1.16
NS2-3-AR	1.01	**0.93**	1.01	1.07	1.08	1.06	1.02	1.00	1.00
NS2-3-VAR	1.00	**0.80**^{1}	**0.85**^{1}	**0.96**^{10}	1.03	1.06	1.04	1.03	1.03
NS2-4-AR	1.08	1.05	1.22	1.20	1.18	1.13	1.04	1.01	1.00
NS2-4-VAR	**0.99**	**0.72**^{1}	**0.81**^{5}	**0.92**^{10}	**0.99**	1.04	1.03	1.03	1.05
NS2-2-λ-AR	1.20	**0.82**^{1}	**0.95**	1.13	1.29	1.38	1.27	1.17	1.15
NS2-2-λ-VAR	1.18	**0.86**^{1}	**0.96**^{10}	1.11	1.25	1.33	1.25	1.17	1.16
NS2-3-λ-AR	1.07	1.04	1.17	1.22	1.20	1.13	1.05	1.01	1.02
NS2-3-λ-VAR	**0.99**	**0.82**^{1}	**0.93**	1.03	1.06	1.03	**0.99**	1.00	1.05
NS2-4-λ-AR	1.08	1.01	1.16	1.17	1.17	1.14	1.06	1.03	1.02
NS2-4-λ-VAR	**0.95**	**0.71**^{5}	**0.79**	**0.89**	**0.96**	1.01	**0.99**	**0.99**	1.04
NS2-B-AR	1.11	1.06	1.19	1.23	1.19	1.13	1.11	1.09	1.10
NS2-B-VAR	1.03	**0.82**^{10}	**0.91**	1.00	1.03	1.03	1.06	1.08	1.14
NS2-S-AR	1.27	1.14	1.32	1.41	1.43	1.37	1.25	1.20	1.22
NS2-S-VAR	1.00	**0.77**^{1}	**0.85**^{10}	**0.95**	**0.99**	1.02	1.02	1.05	1.14
NS2-AS-AR	1.16	1.11	1.26	1.31	1.29	1.22	1.13	1.08	1.08
NS2-AS-VAR	1.00	**0.84**	**0.94**	1.01	1.04	1.03	1.01	1.02	1.08
Panel B: one-step models									
NS1-2-RW	1.03	1.20	1.11	1.04	**0.99**	1.02	1.00	1.01	1.06
NS1-2-AR	1.07	1.25	1.15	1.10	1.04	1.06	1.04	1.04	1.10
NS1-2-VAR	1.09	1.26	1.18	1.11	1.04	1.06	1.06	1.06	1.13
NS1-3-RW	**0.99**	**0.91**^{5}	**0.94**^{10}	**0.99**	1.00	1.02	1.01	1.00	1.00
NS1-3-AR	1.00	**0.95**	**0.99**	1.05	1.06	1.05	1.01	**0.99**	**0.99**
NS1-3-VAR	1.05	**0.85**^{1}	**0.89**^{1}	**0.99**^{5}	1.04	1.09	1.11	1.10	1.12
NS1-4-RW	1.00	**0.94**	**0.99**	1.00	**0.99**	1.02	1.01	1.01	1.01
NS1-4-AR	**0.96**	**0.81**	**0.88**	**0.93**	**0.95**	**0.99**	**0.99**	**0.98**	**0.99**
NS1-4-VAR	**0.99**	**0.68**^{1}	**0.76**^{5}	**0.87**^{5}	**0.94**^{10}	1.02	1.05	1.06	1.11
NS1-B-RW	1.00	**0.98**	1.00	1.00	**0.99**	1.02	1.00	1.00	1.02
NS1-B-AR	1.01	**0.99**	1.03	1.05	1.03	1.04	1.00	1.00	1.02
NS1-B-VAR	1.04	**0.78**^{1}	**0.84**^{1}	**0.96**^{5}	1.03	1.09	1.10	1.10	1.14
NS1-S-RW	1.00	**0.99**	1.01	1.00	**0.99**	1.02	1.01	1.01	1.00
NS1-S-AR	1.01	1.01	1.04	1.04	1.02	1.05	1.02	1.00	1.01
NS1-S-VAR	**0.99**	**0.69**^{5}	**0.77**^{5}	**0.87**^{5}	**0.94**^{10}	1.02	1.05	1.06	1.10
NS1-AS-RW	1.00	**0.99**	1.01	1.00	**0.99**	1.02	1.01	1.01	1.00
NS1-AS-AR	1.01	1.00	1.04	1.03	1.01	1.04	1.01	**0.99**	1.00
NS1-AS-VAR	**0.99**	**0.70**	**0.77**	**0.87**	**0.94**	1.02	1.05	1.06	1.11

Notes: Bold numbers indicate outperformance relative to the random walk (RW) whereas $()^{10}$, $()^{5}$ and $()^{1}$ indicate significant outperformance at the 90%, 95% and 99% level respectively according to the White (2000) reality check using 1000 block-bootstraps and an average block-length of 12.

Table 8: Forecast results for the sample 1994:1 - 2003:12, 6-month horizon

	TRMSPE	**RMSPE**							
Maturity	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y
RW	295.47	84.53	82.84	84.95	87.81	90.76	84.03	77.10	70.68
AR	1.01	1.02	1.00	1.00	1.01	1.01	1.01	1.01	1.02
VAR	1.12	**0.92**	1.03	1.10	1.17	1.16	1.11	1.12	1.19
Panel A: two-step models									
NS2-2-AR	1.02	**0.89**^{1}	**0.92**^{1}	1.00	1.05	1.06	1.03	1.03	1.11
NS2-2-VAR	1.03	**0.91**^{5}	**0.93**^{5}	**0.99**	1.03	1.05	1.05	1.06	1.15
NS2-3-AR	1.02	**0.99**	1.02	1.07	1.08	1.05	1.01	**0.99**	1.01
NS2-3-VAR	1.02	**0.86**^{1}	**0.89**^{1}	**0.98**^{10}	1.05	1.07	1.05	1.04	1.06
NS2-4-AR	1.04	1.02	1.09	1.10	1.13	1.09	1.02	**0.99**	1.00
NS2-4-VAR	**0.99**	**0.78**^{1}	**0.84**^{5}	**0.93**^{10}	1.00	1.04	1.04	1.04	1.08
NS2-2-λ-AR	1.09	**0.92**^{1}	**0.99**	1.08	1.16	1.18	1.12	1.08	1.10
NS2-2-λ-VAR	1.08	**0.93**^{1}	**0.98**	1.06	1.13	1.15	1.11	1.09	1.12
NS2-3-λ-AR	1.09	1.06	1.12	1.17	1.18	1.13	1.06	1.04	1.07
NS2-3-λ-VAR	**0.97**	**0.85**^{1}	**0.92**	**0.99**	1.02	**0.99**	**0.96**	**0.99**	1.08
NS2-4-λ-AR	1.05	**0.98**	1.05	1.08	1.12	1.10	1.05	1.03	1.06
NS2-4-λ-VAR	**0.96**	**0.75**^{1}	**0.81**^{10}	**0.89**	**0.95**	**0.98**	**0.99**	1.02	1.09
NS2-B-AR	1.13	1.07	1.13	1.17	1.19	1.15	1.13	1.12	1.16
NS2-B-VAR	1.02	**0.82**^{10}	**0.89**	**0.97**	1.02	1.02	1.05	1.09	1.18
NS2-S-AR	1.27	1.14	1.24	1.32	1.38	1.34	1.26	1.23	1.28
NS2-S-VAR	1.02	**0.80**^{1}	**0.86**^{10}	**0.95**	1.01	1.02	1.04	1.09	1.20
NS2-AS-AR	1.15	1.10	1.18	1.24	1.27	1.21	1.12	1.09	1.12
NS2-AS-VAR	**0.97**	**0.85**	**0.91**	**0.97**	1.00	**0.98**	**0.96**	**0.99**	1.09
Panel B: one-step models									
NS1-2-RW	1.00	**0.97**^{1}	**0.95**^{1}	**0.98**^{1}	1.00	1.00	**0.99**	1.01	1.06
NS1-2-AR	1.06	1.07	1.04	1.07	1.08	1.07	1.05	1.05	1.12
NS1-2-VAR	1.08	1.06	1.02	1.06	1.06	1.07	1.08	1.10	1.19
NS1-3-RW	1.00	**0.96**^{5}	**0.96**^{5}	**0.99**^{10}	1.01	1.01	1.01	1.00	1.01
NS1-3-AR	1.00	1.00	1.01	1.05	1.06	1.03	**0.98**	**0.97**	**0.98**
NS1-3-VAR	1.10	**0.92**^{1}	**0.94**^{1}	1.03	1.09	1.13	1.16	1.16	1.21
NS1-4-RW	1.00	**0.99**	**0.99**	1.00	1.00	1.01	1.01	1.00	1.02
NS1-4-AR	**0.93**	**0.83**^{10}	**0.86**	**0.91**	**0.93**	**0.95**	**0.95**	**0.95**	**0.98**
NS1-4-VAR	1.01	**0.73**^{1}	**0.79**^{1}	**0.88**^{5}	**0.97**	1.03	1.08	1.10	1.18
NS1-B-RW	1.00	1.01	1.00	1.00	1.00	1.01	1.00	1.00	1.03
NS1-B-AR	1.01	1.03	1.03	1.05	1.05	1.03	**0.99**	**0.99**	1.02
NS1-B-VAR	1.10	**0.86**^{1}	**0.90**^{1}	1.01	1.09	1.14	1.16	1.17	1.24
NS1-S-RW	1.01	1.02	1.00	1.00	1.00	1.01	1.01	1.01	1.01
NS1-S-AR	1.00	1.04	1.03	1.03	1.03	1.02	**0.98**	**0.97**	**0.99**
NS1-S-VAR	1.01	**0.74**^{1}	**0.79**^{1}	**0.88**^{5}	**0.96**	1.03	1.08	1.10	1.18
NS1-AS-RW	1.00	1.02	1.00	1.00	1.00	1.01	1.01	1.01	1.01
NS1-AS-AR	**0.99**	1.03	1.02	1.02	1.01	1.01	**0.97**	**0.96**	**0.99**
NS1-AS-VAR	1.01	**0.74**^{1}	**0.79**^{1}	**0.88**^{5}	**0.96**	1.03	1.08	1.10	1.18

Notes: Bold numbers indicate outperformance relative to the random walk (RW) whereas $()^{10}$, $()^{5}$ and $()^{1}$ indicate significant outperformance at the 90%, 95% and 99% level respectively according to the White (2000) reality check using 1000 block-bootstraps and an average block-length of 12.

Table 9: Forecast results for the sample 1994:1 - 2003:12, 12-month horizon

	TRMSPE	**RMSPE**							
Maturity	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y
RW	435.08	138.26	140.53	142.35	140.73	133.83	114.30	103.36	94.12
AR	1.01	1.00	**0.99**	**0.99**	**0.99**	1.01	1.03	1.03	1.05
VAR	1.37	1.17	1.23	1.26	1.32	1.36	1.41	1.47	1.60
Panel A: two-step models									
NS2-2-AR	1.04	**0.99**^{5}	**0.99**	1.02	1.05	1.06	1.06	1.06	1.11
NS2-2-VAR	1.04	**0.97**^{5}	**0.97**^{5}	**0.99**	1.02	1.04	1.07	1.09	1.15
NS2-3-AR	1.03	1.01	1.01	1.03	1.05	1.06	1.04	1.02	1.04
NS2-3-VAR	1.05	**0.96**^{5}	**0.97**	1.01	1.05	1.08	1.08	1.07	1.10
NS2-4-AR	1.00	**0.88**	**0.92**	**0.95**	1.01	1.04	1.03	1.02	1.06
NS2-4-VAR	1.02	**0.89**^{1}	**0.92**^{5}	**0.97**	1.01	1.04	1.05	1.07	1.13
NS2-2-λ-AR	1.08	**0.99**	1.00	1.03	1.07	1.11	1.11	1.10	1.14
NS2-2-λ-VAR	1.06	**0.99**^{5}	**0.99**	1.02	1.05	1.07	1.10	1.10	1.15
NS2-3-λ-AR	1.09	1.01	1.02	1.05	1.09	1.11	1.12	1.12	1.17
NS2-3-λ-VAR	**0.96**	**0.91**^{10}	**0.93**	**0.95**	**0.97**	**0.95**	**0.94**^{5}	**0.98**^{5}	1.08
NS2-4-λ-AR	1.03	**0.86**	**0.90**	**0.94**	1.00	1.05	1.09	1.10	1.17
NS2-4-λ-VAR	**0.98**	**0.85**^{1}	**0.88**^{5}	**0.92**	**0.96**	**0.99**	1.02	1.06	1.16
NS2-B-AR	1.14	1.00	1.03	1.06	1.11	1.15	1.20	1.21	1.29
NS2-B-VAR	1.05	**0.90**	**0.94**	**0.97**	1.02	1.05	1.10	1.14	1.25
NS2-S-AR	1.24	1.06	1.10	1.15	1.22	1.26	1.30	1.32	1.41
NS2-S-VAR	1.04	**0.90**^{5}	**0.92**	**0.97**	1.01	1.03	1.08	1.13	1.25
NS2-AS-AR	1.13	1.02	1.05	1.09	1.13	1.15	1.16	1.16	1.22
NS2-AS-VAR	**0.97**	**0.90**	**0.92**	**0.95**	**0.98**	**0.97**	**0.97**^{10}	1.01	1.12
Panel B: one-step models									
NS1-2-RW	1.00	1.01	**0.99**	1.01	1.01	1.00	**0.99**	1.01	1.06
NS1-2-AR	1.10	1.12	1.09	1.10	1.10	1.10	1.08	1.09	1.17
NS1-2-VAR	1.11	1.08	1.05	1.07	1.07	1.08	1.13	1.16	1.27
NS1-3-RW	1.00	1.00	**0.99**	1.00	1.00	1.01	1.00	1.00	1.01
NS1-3-AR	1.00	1.02	1.01	1.02	1.03	1.02	**0.98**	**0.96**	**0.98**
NS1-3-VAR	1.16	1.03	1.03	1.07	1.12	1.17	1.22	1.24	1.32
NS1-4-RW	1.00	1.02	1.00	1.00	1.00	1.01	1.00	1.00	1.02
NS1-4-AR	**0.90**	**0.85**^{5}	**0.86**^{5}	**0.88**^{5}	**0.90**^{10}	**0.90**^{10}	**0.91**^{5}	**0.92**^{5}	**0.98**
NS1-4-VAR	1.06	**0.86**^{1}	**0.88**^{1}	**0.94**^{5}	1.00	1.06	1.14	1.19	1.31
NS1-B-RW	1.00	1.03	1.00	1.00	1.00	1.01	**0.99**	1.00	1.03
NS1-B-AR	1.02	1.03	1.02	1.03	1.04	1.04	1.00	**0.99**	1.02
NS1-B-VAR	1.18	1.00	1.02	1.08	1.14	1.21	1.26	1.28	1.37
NS1-S-RW	1.01	1.03	1.00	1.00	1.00	1.01	1.01	1.00	1.01
NS1-S-AR	**0.98**	1.02	1.00	**0.99**	1.00	**0.99**	**0.95**	**0.94**	**0.98**
NS1-S-VAR	1.05	**0.85**^{1}	**0.87**^{1}	**0.93**^{5}	**0.99**	1.05	1.13	1.18	1.30
NS1-AS-RW	1.01	1.03	1.00	1.00	1.00	1.01	1.00	1.00	1.01
NS1-AS-AR	**0.96**	1.01	**0.99**	**0.98**	**0.98**	**0.97**	**0.94**	**0.93**^{10}	**0.97**
NS1-AS-VAR	1.05	**0.85**^{1}	**0.87**^{1}	**0.93**^{5}	**0.99**	1.05	1.13	1.18	1.30

Notes: Bold numbers indicate outperformance relative to the random walk (RW) whereas ()^{10}, ()^{5} and ()^{1} indicate significant outperformance at the 90%, 95% and 99% level respectively according to the White (2000) reality check using 1000 block-bootstraps and an average block-length of 12.

Table 10: Forecast results for the sample 1994:1 - 2000:12, 1 & 3-month horizons

	$h=1$									$h=3$								
	TRMSPE	**RMSPE**								**TRMSPE**	**RMSPE**							
Maturity	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y
RW	92.65	29.82	17.87	19.30	23.95	26.84	27.48	26.40	25.31	184.76	45.82	36.70	41.99	50.42	57.46	55.79	53.25	49.22
AR	1.00	**0.99**	1.01	1.00	1.00	1.00	1.00	1.00	1.00	**0.99**	**0.97**	1.02	1.01	**0.99**	**0.99**	**0.99**	**0.99**	1.00
VAR	1.00	**0.82**	1.01	**0.93**	**0.97**	1.07	1.04	1.03	1.06	**0.97**	**0.69**	**0.86**	**0.90**	**0.95**	**0.99**	1.00	1.00	1.06
Panel A: two-step models																		
NS2-2-AR	1.08	1.30	1.25	1.00	**0.98**	1.04	1.02	1.06	1.20	**0.98**	**0.85**^{5}	**0.92**^{10}	**0.97**	**0.95**	**0.94**	**0.99**	1.03	1.13
NS2-2-VAR	1.08	1.28	1.25	1.03	1.01	1.05	1.03	1.07	1.21	1.01	**0.86**^{5}	**0.99**	1.03	1.00	**0.97**	1.01	1.05	1.15
NS2-3-AR	**0.98**	**0.90**	**0.91**	1.00	**0.99**	1.02	1.02	1.02	1.00	**0.94**	**0.73**^{5}	**0.90**	**0.97**	**0.96**	**0.96**	**0.96**	**0.96**	**0.97**
NS2-3-VAR	1.01	**0.99**	**0.93**	**0.96**	**0.98**	1.06	1.05	1.04	1.02	1.00	**0.80**^{10}	**0.88**	**0.98**	1.01	1.04	1.04	1.02	1.02
NS2-4-AR	1.01	**0.88**	1.33	1.20	1.07	1.02	**0.97**	1.03	**0.99**	1.00	1.07	1.41	1.23	1.07	**0.96**	**0.92**^{5}	**0.95**^{5}	**0.95**^{10}
NS2-4-VAR	**0.99**	**0.80**	**0.91**	**0.96**	**0.99**	1.07	1.02	1.04	1.04	1.00	**0.68**^{1}	**0.89**	**0.99**	1.02	1.05	1.02	1.02	1.04
NS2-2-λ-AR	1.16	**0.93**	1.11	1.30	1.41	1.37	1.16	1.12	1.10	**0.91**	**0.73**^{5}	**0.93**	1.00	**0.98**	**0.93**	**0.91**	**0.92**	**0.97**
NS2-2-λ-VAR	1.17	**0.93**	1.14	1.33	1.43	1.38	1.17	1.12	1.10	**0.95**	**0.78**^{1}	1.02	1.08	1.04	**0.97**	**0.93**	**0.94**	**0.99**
NS2-3-λ-AR	1.15	**0.87**	1.22	1.41	1.35	1.25	1.17	1.16	1.15	**0.91**	**0.75**	**0.96**	1.00	**0.95**	**0.88**	**0.91**	**0.93**	**0.97**
NS2-3-λ-VAR	1.16	**0.85**	1.17	1.41	1.40	1.30	1.16	1.14	1.19	**0.99**	**0.75**	1.00	1.09	1.09	1.02	**0.97**	**0.99**	1.07
NS2-4-λ-AR	1.02	**0.90**	1.34	1.23	1.06	1.02	1.01	1.04	1.05	**0.96**	1.00	1.29	1.14	**0.99**	**0.91**	**0.92**	**0.95**	**0.97**
NS2-4-λ-VAR	**0.99**	**0.85**	1.01	1.01	**0.98**	1.04	1.01	1.02	1.08	**0.96**	**0.69**	**0.86**	**0.93**	**0.95**	**0.98**	**0.98**	1.00	1.06
NS2-B-AR	1.17	**0.87**	1.17	1.15	1.01	1.05	1.28	1.30	1.28	**0.96**	**0.77**	**0.97**	**0.94**	**0.86**	**0.85**	1.00	1.04	1.09
NS2-B-VAR	1.20	**0.87**	1.08	1.11	1.07	1.12	1.30	1.34	1.35	1.05	**0.75**	**0.92**	**0.98**	**0.99**	**0.98**	1.08	1.13	1.21
NS2-S-AR	1.18	**0.81**	1.11	1.25	1.15	1.14	1.24	1.30	1.36	**0.97**	**0.78**	**0.92**	**0.93**	**0.88**	**0.89**	1.00	1.05	1.14
NS2-S-VAR	1.12	**0.81**	1.08	1.21	1.14	1.10	1.13	1.19	1.31	**0.99**	**0.74**^{10}	**0.90**	**0.97**	**0.97**	**0.97**	1.00	1.04	1.17
NS2-AS-AR	1.20	**0.90**	1.37	1.40	1.31	1.25	1.22	1.25	1.23	**0.91**	**0.84**	1.02	**0.97**	**0.89**	**0.86**	**0.91**	**0.94**	**0.98**
NS2-AS-VAR	1.19	**0.90**	1.35	1.39	1.34	1.28	1.18	1.20	1.23	**0.94**	**0.80**	**0.99**	**0.98**	**0.98**	**0.95**	**0.93**	**0.96**	1.04
Panel B: one-step models																		
NS1-2-RW	1.28	2.14	2.40	1.61	1.01	1.05	1.01	1.02	1.08	1.04	1.28	1.23	1.09	**0.99**	1.00	1.00	1.01	1.05
NS1-2-AR	1.33	2.21	2.51	1.71	1.05	1.07	1.06	1.04	1.13	1.08	1.42	1.37	1.18	1.02	1.02	1.03	1.03	1.11
NS1-2-VAR	1.32	2.18	2.47	1.68	1.05	1.08	1.07	1.05	1.14	1.09	1.36	1.33	1.17	1.03	1.03	1.05	1.05	1.13
NS1-3-RW	1.00	1.01	**0.99**	**0.98**	**0.99**	1.03	1.02	1.02	1.01	**0.99**	**0.76**^{1}	**0.92**	**0.99**	1.00	1.01	1.01	1.01	1.01
NS1-3-AR	1.01	1.04	1.01	1.01	1.01	1.04	1.02	1.02	1.01	**0.96**	**0.78**^{10}	**0.92**	**0.99**	**0.98**	**0.98**	**0.97**	**0.97**	**0.98**
NS1-3-VAR	1.06	1.15	1.11	1.00	**0.96**	1.05	1.10	1.08	1.08	1.06	**0.90**	**0.97**	1.02	1.01	1.05	1.11	1.09	1.13
NS1-4-RW	**0.99**	**0.87**	1.05	**0.99**	**0.96**	1.04	1.00	1.02	1.04	**0.99**	**0.81**^{1}	1.00	1.00	**0.98**^{10}	1.01	1.00	1.01	1.02
NS1-4-AR	**0.98**	**0.84**	**0.99**	**0.95**	**0.94**	1.02	1.00	1.03	1.05	**0.95**	**0.70**^{10}	**0.90**	**0.93**	**0.93**	**0.96**	**0.97**	**0.98**	1.01
NS1-4-VAR	1.01	**0.84**	**0.94**	**0.93**	**0.95**	1.05	1.04	1.06	1.13	1.02	**0.68**^{5}	**0.86**	**0.94**	**0.97**	1.03	1.05	1.07	1.15
NS1-B-RW	1.00	**0.91**	1.03	1.00	**0.95**	1.04	1.03	1.02	1.02	**0.99**	**0.82**^{1}	1.00	1.00	**0.98**^{10}	1.01	1.01	1.00	1.02
NS1-B-AR	**0.99**	**0.90**	1.02	1.00	**0.95**	1.03	1.03	1.02	1.02	**0.97**	**0.79**^{1}	**0.97**	**0.98**	**0.96**	**0.98**	**0.98**	**0.99**	1.00
NS1-B-VAR	1.03	**0.92**	**0.96**	**0.96**	**0.94**	1.07	1.11	1.08	1.10	1.06	**0.79**^{10}	**0.92**	1.00	1.01	1.07	1.11	1.10	1.14
NS1-S-RW	1.00	**0.91**	1.09	**0.99**	**0.95**	1.04	1.00	1.02	1.03	**0.99**	**0.85**^{1}	1.02	1.00	**0.98**^{5}	1.01	1.01	1.01	1.01
NS1-S-AR	1.01	**0.89**	1.08	1.00	**0.97**	1.05	1.02	1.04	1.06	**0.98**	**0.82**^{5}	1.00	**0.98**	**0.97**	**0.99**	**0.99**	**0.99**	1.01
NS1-S-VAR	1.01	**0.83**	**0.95**	**0.93**	**0.95**	1.05	1.04	1.07	1.12	1.02	**0.68**^{5}	**0.87**	**0.95**	**0.98**	1.02	1.05	1.07	1.14
NS1-AS-RW	1.00	**0.91**	1.09	**0.99**	**0.96**	1.04	1.00	1.02	1.03	**0.99**	**0.85**^{1}	1.02	1.00	**0.98**^{5}	1.01	1.01	1.01	1.01
NS1-AS-AR	1.00	**0.89**	1.08	**0.99**	**0.96**	1.05	1.02	1.04	1.05	**0.98**	**0.82**^{5}	1.00	**0.98**	**0.96**	**0.99**	**0.99**	**0.99**	1.01
NS1-AS-VAR	1.01	**0.83**	**0.95**	**0.93**	**0.95**	1.05	1.04	1.07	1.12	1.02	**0.68**^{5}	**0.87**	**0.95**	**0.98**	1.02	1.05	1.07	1.14

Notes: . Bold numbers indicate outperformance relative to the random walk (RW) whereas $()^{10}$, $()^{5}$ and $()^{1}$ indicate significant outperformance at the 90%, 95% and 99% level respectively according to the White (2000) reality check using 1000 block-bootstraps and an average block-length of 12.

Table 11: Forecast results for the sample 1994:1 - 2000:12, 6 & 12-month horizons

	$h = 6$									$h = 12$								
	TRMSPE	**RMSPE**								**TRMSPE**	**RMSPE**							
Maturity	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y
RW	271.12	63.55	59.67	65.57	74.29	83.88	82.10	77.99	73.00	366.08	94.51	93.83	97.71	101.96	108.91	107.22	102.70	98.50
AR	**0.98**	**0.95**	1.03	1.01	**0.98**	**0.97**	**0.97**	**0.98**	1.00	**0.97**	**0.92**^{1}	1.02	1.00	**0.96**^{1}	**0.94**^{1}	**0.95**^{5}	**0.97**	1.00
VAR	**0.97**	**0.68**^{5}	**0.79**	**0.85**^{10}	**0.91**	**0.92**	**0.99**	1.02	1.12	1.08	**0.70**^{1}	**0.73**^{1}	**0.77**^{1}	**0.81**^{1}	**0.88**	1.13	1.24	1.42
Panel A: two-step models																		
NS2-2-AR	**0.96**	**0.81**^{1}	**0.89**^{1}	**0.94**^{5}	**0.93**	**0.90**^{10}	**0.97**	1.01	1.09	**0.95**	**0.87**^{5}	**0.90**^{1}	**0.91**^{1}	**0.89**^{1}	**0.88**^{1}	**0.96**^{10}	**0.99**	1.05
NS2-2-VAR	1.01	**0.88**^{1}	1.00	1.04	1.01	**0.96**	1.01	1.04	1.12	1.01	1.01	1.06	1.05	1.01	**0.94**^{1}	**0.99**	1.02	1.08
NS2-3-AR	**0.92**	**0.80**^{5}	**0.90**	**0.94**	**0.93**	**0.90**^{5}	**0.93**^{5}	**0.93**^{1}	**0.94**	**0.90**	**0.85**^{5}	**0.88**^{5}	**0.90**^{1}	**0.89**^{1}	**0.86**^{1}	**0.91**^{1}	**0.92**^{1}	**0.95**^{5}
NS2-3-VAR	1.01	**0.83**^{1}	**0.91**^{5}	1.00	1.04	1.04	1.03	1.02	1.03	1.03	**0.95**	1.00	1.04	1.06	1.04	1.03	1.03	1.05
NS2-4-AR	**0.98**	1.15	1.28	1.17	1.06	**0.93**^{5}	**0.90**^{1}	**0.93**^{1}	**0.95**^{5}	**0.99**	1.06	1.11	1.07	1.02	**0.94**^{5}	**0.94**^{1}	**0.96**^{1}	1.01
NS2-4-VAR	1.01	**0.78**^{1}	**0.94**^{5}	1.01	1.05	1.03	1.02	1.02	1.05	1.03	**0.96**	1.02	1.05	1.06	1.03	1.01	1.02	1.06
NS2-2-λ-AR	**0.90**	**0.78**^{5}	**0.90**	**0.94**	**0.91**	**0.86**	**0.90**^{5}	**0.93**^{10}	**0.98**	**0.89**	**0.83**^{5}	**0.86**^{5}	**0.86**^{1}	**0.83**^{1}	**0.80**^{1}	**0.90**^{1}	**0.93**^{1}	**0.99**
NS2-2-λ-VAR	**0.95**	**0.88**^{1}	1.02	1.04	**0.99**	**0.91**	**0.93**	**0.96**	1.01	**0.94**	**0.98**	1.01	**0.99**	**0.93**^{1}	**0.85**^{1}	**0.91**^{1}	**0.94**^{5}	1.00
NS2-3-λ-AR	**0.87**	**0.73**	**0.86**	**0.88**	**0.85**	**0.80**^{5}	**0.88**^{5}	**0.92**^{10}	**0.98**	**0.88**	**0.72**^{1}	**0.77**^{1}	**0.79**^{1}	**0.78**^{1}	**0.77**^{1}	**0.92**^{1}	**0.97**^{5}	1.06
NS2-3-λ-VAR	**0.98**	**0.78**^{1}	**0.97**	1.03	1.03	**0.96**	**0.96**	1.00	1.10	**0.95**	**0.90**^{1}	**0.98**^{10}	**0.99**^{10}	**0.98**	**0.89**^{1}	**0.90**^{1}	**0.95**	1.06
NS2-4-λ-AR	**0.95**	1.03	1.14	1.04	**0.96**	**0.87**^{5}	**0.92**^{5}	**0.96**^{10}	1.01	**0.98**	**0.92**	**0.97**	**0.95**	**0.92**^{5}	**0.88**^{1}	**0.97**^{5}	1.02	1.10
NS2-4-λ-VAR	**0.98**	**0.72**^{1}	**0.86**	**0.93**	**0.96**	**0.96**	**0.99**	1.02	1.10	**0.99**	**0.85**^{5}	**0.92**	**0.94**	**0.95**^{10}	**0.94**^{1}	**0.99**	1.03	1.12
NS2-B-AR	**0.93**	**0.76**	**0.86**	**0.85**	**0.81**^{10}	**0.80**^{1}	**0.96**	1.01	1.09	**0.96**	**0.72**^{1}	**0.77**^{1}	**0.78**^{1}	**0.78**^{1}	**0.81**^{1}	1.02	1.08	1.18
NS2-B-VAR	1.03	**0.72**^{1}	**0.89**^{5}	**0.95**^{5}	**0.98**^{10}	**0.96**^{1}	1.05	1.11	1.22	1.07	**0.90**^{5}	1.00	1.02	1.03	**0.99**	1.06	1.12	1.24
NS2-S-AR	**0.96**	**0.76**	**0.83**	**0.84**	**0.83**	**0.84**^{5}	1.00	1.06	1.17	1.01	**0.70**^{1}	**0.73**^{1}	**0.77**^{1}	**0.79**^{1}	**0.86**^{1}	1.08	1.15	1.29
NS2-S-VAR	1.04	**0.77**^{1}	**0.89**^{5}	**0.96**	**0.99**	**0.99**	1.05	1.10	1.22	1.06	**0.91**	**0.96**	**0.99**	1.00	**0.98**	1.06	1.12	1.24
NS2-AS-AR	**0.88**	**0.81**	**0.90**	**0.87**	**0.83**	**0.79**^{1}	**0.88**^{5}	**0.93**^{10}	1.00	**0.91**	**0.74**^{1}	**0.78**^{5}	**0.78**^{1}	**0.78**^{1}	**0.78**^{1}	**0.95**^{1}	1.01	1.11
NS2-AS-VAR	**0.94**	**0.79**^{1}	**0.92**	**0.93**	**0.93**	**0.90**^{10}	**0.93**	**0.98**	1.08	**0.95**	**0.89**^{5}	**0.94**^{5}	**0.93**^{5}	**0.92**^{1}	**0.87**^{1}	**0.92**^{1}	**0.98**	1.09
Panel B: one-step models																		
NS1-2-RW	1.01	1.00	1.00	1.02	1.01	**0.99**	**0.99**	1.01	1.04	1.01	1.00	1.01	1.02	1.02	**0.98**^{5}	**0.99**^{10}	1.01	1.03
NS1-2-AR	1.05	1.19	1.13	1.10	1.03	1.00	1.03	1.04	1.10	1.04	1.13	1.08	1.06	1.01	**0.97**	1.01	1.03	1.11
NS1-2-VAR	1.06	1.12	1.10	1.09	1.05	1.02	1.05	1.06	1.14	1.06	1.09	1.07	1.07	1.03	**0.99**	1.04	1.07	1.16
NS1-3-RW	**0.99**	**0.84**^{1}	**0.95**^{5}	1.00	1.01	1.00	1.01	1.00	1.00	1.00	**0.95**	**0.99**	1.00	1.01	1.00	1.01	1.00	**0.99**
NS1-3-AR	**0.93**	**0.86**	**0.94**	**0.98**	**0.97**	**0.93**	**0.93**	**0.94**	**0.95**	**0.91**	**0.93**	**0.95**	**0.96**	**0.93**^{5}	**0.88**^{1}	**0.89**^{1}	**0.90**^{1}	**0.93**^{5}
NS1-3-VAR	1.10	**0.94**^{10}	**0.99**	1.05	1.06	1.07	1.13	1.13	1.18	1.13	1.04	1.06	1.08	1.08	1.07	1.15	1.17	1.25
NS1-4-RW	1.00	**0.92**^{1}	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	**0.99**	1.01	1.00	1.00	1.00	1.00	1.00	1.00
NS1-4-AR	**0.95**	**0.79**^{5}	**0.91**	**0.94**	**0.96**	**0.95**	**0.95**	**0.96**	**0.99**	**0.94**	**0.91**	**0.95**	**0.96**	**0.96**	**0.94**^{1}	**0.93**^{1}	**0.94**^{1}	**0.97**
NS1-4-VAR	1.07	**0.77**^{1}	**0.89**^{10}	**0.98**	1.03	1.05	1.09	1.12	1.20	1.13	**0.95**	**0.99**	1.04	1.07	1.08	1.14	1.19	1.29
NS1-B-RW	1.00	**0.93**^{5}	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00
NS1-B-AR	**0.95**	**0.89**^{10}	**0.97**	**0.97**	**0.96**	**0.95**^{5}	**0.96**^{10}	**0.96**	**0.98**	**0.93**	**0.92**^{10}	**0.95**^{10}	**0.94**^{5}	**0.94**^{1}	**0.92**^{1}	**0.92**^{1}	**0.93**^{1}	**0.94**
NS1-B-VAR	1.11	**0.88**^{5}	**0.96**	1.05	1.08	1.10	1.15	1.15	1.21	1.18	1.04	1.06	1.11	1.13	1.15	1.21	1.23	1.29
NS1-S-RW	1.00	**0.95**^{5}	1.02	1.00	1.00	1.00	1.01	1.01	1.00	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00
NS1-S-AR	**0.95**	**0.90**^{10}	**0.98**	**0.97**	**0.97**	**0.96**	**0.95**	**0.95**	**0.97**	**0.92**	**0.93**^{10}	**0.96**	**0.95**^{10}	**0.95**^{5}	**0.92**^{1}	**0.90**^{1}	**0.91**^{1}	**0.94**
NS1-S-VAR	1.07	**0.78**^{1}	**0.90**^{10}	**0.99**	1.04	1.05	1.09	1.12	1.19	1.13	**0.95**	1.00	1.05	1.08	1.08	1.14	1.19	1.28
NS1-AS-RW	1.00	**0.95**^{5}	1.01	1.00	1.00	1.00	1.01	1.01	1.01	1.00	1.00	1.01	1.00	1.00	1.00	1.00	1.00	1.00
NS1-AS-AR	**0.96**	**0.90**	**0.98**	**0.97**	**0.97**	**0.96**	**0.95**	**0.96**	**0.98**	**0.93**	**0.93**^{10}	**0.96**	**0.95**^{10}	**0.95**^{5}	**0.92**^{1}	**0.91**^{1}	**0.91**^{1}	**0.94**
NS1-AS-VAR	1.07	**0.78**^{1}	**0.90**^{10}	**0.99**	1.04	1.05	1.09	1.12	1.19	1.13	**0.95**	1.00	1.05	1.08	1.09	1.15	1.19	1.29

Notes: . Bold numbers indicate outperformance relative to the random walk (RW) whereas $()^{10}$, $()^{5}$ and $()^{1}$ indicate significant outperformance at the 90%, 95% and 99% level respectively according to the White (2000) reality check using 1000 block-bootstraps and an average block-length of 12.

Table 12: Forecast results for the sample 2001:1 - 2003:12, 1 & 3-month horizons

	$h = 1$									$h = 3$								
	TRMSPE	**RMSPE**								**TRMSPE**	**RMSPE**							
Maturity	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y
RW	119.88	30.79	27.39	26.81	29.40	33.84	36.55	35.16	33.31	222.46	69.23	68.35	67.30	66.12	66.01	61.41	54.99	51.99
AR	1.02	1.04	1.01	1.01	1.03	1.02	1.01	1.01	1.01	1.03	1.06	1.00	1.01	1.03	1.05	1.04	1.04	1.02
VAR	1.02	**0.76**	**0.90**	1.06	1.12	1.07	**0.98**	1.01	1.08	1.08	**0.78**	**0.92**	1.04	1.17	1.20	1.09	1.09	1.11
Panel A: two-step models																		
NS2-2-AR	1.31	1.98	1.60	1.11	1.20	1.55	1.06	1.05	1.34	1.09	0.94	**0.88**	**0.97**	1.17	1.33	1.09	1.04	1.13
NS2-2-VAR	1.33	2.18	1.80	1.22	1.12	1.48	1.04	1.07	1.37	1.09	1.07	**0.94**	**0.95**	1.07	1.24	1.08	1.06	1.17
NS2-3-AR	1.06	1.10	1.03	1.15	1.19	1.14	1.03	1.01	1.06	1.12	1.10	1.07	1.16	1.22	1.21	1.11	1.08	1.08
NS2-3-VAR	1.00	**0.85**	**0.81**	**0.98**	1.08	1.06	1.00	1.01	1.06	1.00	**0.80**	**0.82**	**0.95**	1.05	1.09	1.05	1.05	1.06
NS2-4-AR	1.14	1.13	1.19	1.27	1.32	1.28	1.13	1.04	1.04	1.19	1.03	1.06	1.17	1.31	1.38	1.24	1.14	1.09
NS2-4-VAR	**0.98**	**0.83**	**0.80**	**0.90**	**0.96**	1.03	1.02	1.01	1.02	**0.96**	**0.75**	**0.75**	**0.86**	**0.95**	1.04	1.05	1.04	1.05
NS2-2-λ-AR	2.21	1.15	**0.91**	1.46	2.44	2.98	2.47	2.04	1.81	1.54	**0.90**	**0.97**	1.23	1.61	1.91	1.75	1.58	1.45
NS2-2-λ-VAR	2.17	1.28	**0.95**	1.35	2.31	2.89	2.43	2.01	1.79	1.48	**0.93**	**0.93**	1.13	1.49	1.80	1.69	1.54	1.43
NS2-3-λ-AR	1.32	1.28	1.30	1.41	1.37	1.44	1.36	1.22	1.16	1.30	1.27	1.29	1.40	1.47	1.45	1.27	1.17	1.11
NS2-3-λ-VAR	1.28	**0.91**	**0.91**	1.06	1.12	1.37	1.38	1.30	1.20	1.00	**0.88**	**0.88**	0.98	1.03	1.07	1.03	1.02	**0.99**
NS2-4-λ-AR	1.19	1.10	1.17	1.29	1.39	1.39	1.19	1.08	1.08	1.23	1.02	1.07	1.20	1.37	1.44	1.28	1.17	1.11
NS2-4-λ-VAR	1.02	**0.79**	**0.78**	**0.92**	1.03	1.15	1.05	1.02	1.03	**0.94**	**0.72**	**0.74**	**0.86**	**0.97**	1.06	1.01	**0.98**	**0.99**
NS2-B-AR	1.21	1.34	1.42	1.58	1.51	1.31	1.13	1.04	1.04	1.33	1.29	1.32	1.44	1.53	1.49	1.29	1.19	1.14
NS2-B-VAR	1.05	**0.97**	1.03	1.22	1.20	1.11	1.03	1.00	**0.98**	1.00	**0.88**	**0.91**	1.02	1.09	1.10	**1.02**	**0.98**	**0.96**
NS2-S-AR	1.45	1.43	1.58	1.81	1.85	1.71	1.41	1.24	1.22	1.63	1.41	1.53	1.73	1.93	1.93	1.62	1.47	1.37
NS2-S-VAR	1.10	**0.88**	**0.87**	1.02	1.09	1.18	1.14	1.11	1.08	1.01	**0.80**	**0.82**	**0.94**	1.03	1.09	1.06	1.06	1.08
NS2-AS-AR	1.41	1.39	1.48	1.65	1.64	1.59	1.42	1.25	1.16	1.46	1.32	1.40	1.56	1.68	1.67	1.46	1.34	1.26
NS2-AS-VAR	1.30	**0.98**	**0.98**	1.16	1.23	1.41	1.39	1.31	1.19	1.09	**0.88**	**0.91**	1.03	1.11	1.16	1.15	1.14	1.14
Panel B: one-step models																		
NS1-2-RW	1.30	2.39	2.22	1.74	1.13	1.06	**0.98**	1.01	1.10	1.02	1.11	1.02	1.00	**0.99**	1.04	**0.99**	1.01	1.06
NS1-2-AR	1.27	2.26	2.09	1.65	1.13	1.11	1.01	1.01	1.10	1.05	1.04	**0.98**	1.02	1.07	1.13	1.05	1.04	1.08
NS1-2-VAR	1.32	2.40	2.23	1.76	1.16	1.10	1.02	1.03	1.13	1.09	1.15	1.06	1.06	1.06	1.12	1.08	1.09	1.14
NS1-3-RW	1.02	1.19	1.01	1.01	1.03	1.04	**0.99**	**0.99**	1.01	1.00	1.04	**0.96**	**0.99**	1.01	1.04	1.01	1.00	**0.99**^{5}
NS1-3-AR	1.05	1.20	1.06	1.10	1.13	1.11	1.02	1.00	1.02	1.07	1.08	1.04	1.10	1.15	1.15	1.07	1.02	1.01
NS1-3-VAR	1.02	**0.89**	**0.80**	**0.96**	1.09	1.10	1.03	1.02	1.06	1.04	**0.80**	**0.83**	**0.96**	1.08	1.15	1.12	1.10	1.10
NS1-4-RW	1.00	1.04	**0.97**	1.02	**0.99**	1.03	1.01	1.00	**0.98**^{5}	1.01	1.04	**0.98**	1.01	1.00	1.03	1.02	1.00	**0.98**^{5}
NS1-4-AR	**0.99**	1.00	**0.95**	**0.99**	**0.99**	1.04	1.00	**0.97**	**0.98**	**0.97**	**0.90**	**0.87**	**0.93**	**0.97**	1.05	1.02	**0.98**	**0.97**
NS1-4-VAR	**0.97**	**0.79**	**0.77**	**0.86**	**0.93**	1.02	1.01	**0.99**	1.00	**0.94**	**0.69**	**0.68**	**0.79**	**0.90**	1.02	1.05	1.03	1.03
NS1-B-RW	1.03	1.30	1.07	1.02	1.02	1.03	**0.98**	**0.99**	1.06	1.01	1.12	1.00	1.01	1.00	1.03	**0.99**	1.00	1.03
NS1-B-AR	1.05	1.31	1.11	1.10	1.10	1.07	1.00	**0.99**	1.06	1.07	1.15	1.07	1.10	1.12	1.12	1.04	1.02	1.04
NS1-B-VAR	1.01	**0.92**	**0.80**	**0.95**	1.08	1.06	1.01	1.02	1.10	1.03	**0.77**	**0.79**	**0.93**	1.06	1.12	1.09	1.10	1.15
NS1-S-RW	1.02	1.19	1.03	1.02	**0.98**	1.04	1.01	1.00	**0.99**	1.01	1.11	1.00	1.01	**0.99**	1.03	1.02	1.01	**0.97**^{10}
NS1-S-AR	1.04	1.26	1.11	1.09	1.05	1.09	1.02	**0.98**	1.01	1.07	1.17	1.07	1.08	1.09	1.13	1.06	1.01	1.01
NS1-S-VAR	**0.97**	**0.86**	**0.79**	**0.86**	**0.92**	1.02	1.00	**0.99**	1.00	**0.94**	**0.71**	**0.69**	**0.79**	**0.89**	1.02	1.04	1.03	1.04
NS1-AS-RW	1.02	1.18	1.03	1.02	**0.98**	1.04	1.01	1.00	**0.99**	1.01	1.11	1.00	1.01	**0.99**	1.03	1.02	1.01	**0.97**^{10}
NS1-AS-AR	1.04	1.25	1.10	1.08	1.04	1.08	1.01	**0.98**	1.00	1.05	1.16	1.06	1.07	1.07	1.11	1.04	1.00	**0.99**
NS1-AS-VAR	**0.97**	**0.85**	**0.79**	**0.86**	**0.92**	1.02	1.00	**0.99**	1.00	**0.94**	**0.71**	**0.69**	**0.79**	**0.89**	1.02	1.04	1.03	1.04

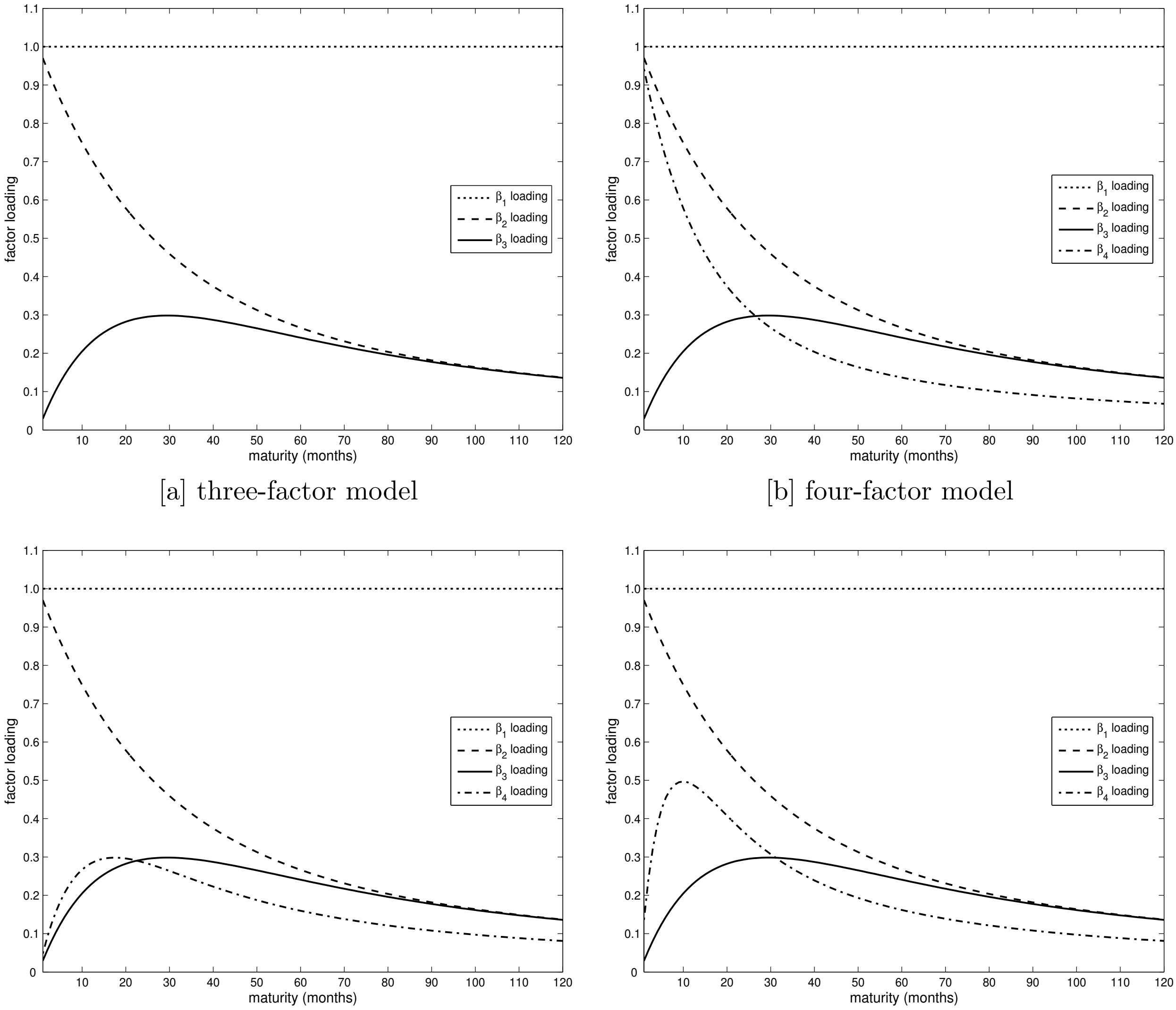
Notes: . Bold numbers indicate outperformance relative to the random walk (RW) whereas $()^{10}$, $()^{5}$ and $()^{1}$ indicate significant outperformance at the 90%, 95% and 99% level respectively according to the White (2000) reality check using 1000 block-bootstraps and an average block-length of 12.

Table 13: Forecast results for the sample 2001:1 - 2003:12, 6 & 12-month horizons

	$h=6$									$h=12$								
	TRMSPE	**RMSPE**								**TRMSPE**	**RMSPE**							
Maturity	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y	all	1-m	3-m	6-m	1-y	2-y	5-y	7-y	10-y
RW	342.90	118.17	118.78	116.69	111.89	104.27	88.12	75.10	65.31	548.98	199.40	204.79	204.91	197.20	173.72	127.44	104.69	84.53
AR	1.05	1.05	0.99	1.00	1.03	1.06	1.08	1.07	1.06	1.05	1.04	0.97	0.98	1.01	1.06	1.13	1.15	1.18
VAR	1.29	1.04	1.15	1.24	1.38	1.43	1.32	1.33	1.38	1.59	1.33	1.39	1.43	1.53	1.64	1.73	1.84	2.00
Panel A: two-step models																		
NS2-2-AR	1.09	**0.93**	**0.94**	1.03	1.16	1.25	1.12	1.07	1.16	1.12	1.03	1.03	1.06	1.12	1.19	1.19	1.18	1.27
NS2-2-VAR	1.06	**0.93**	**0.89**	**0.95**	1.05	1.16	1.12	1.11	1.23	1.07	**0.94**[1]	**0.93**[1]	**0.96**	1.02	1.11	1.18	1.21	1.34
NS2-3-AR	1.15	1.10	1.09	1.15	1.21	1.24	1.16	1.12	1.16	1.14	1.07	1.06	1.08	1.13	1.19	1.20	1.19	1.27
NS2-3-VAR	1.03	**0.87**	**0.88**	**0.97**	1.06	1.12	1.10	1.08	1.14	1.06	**0.97**	**0.96**	1.00	1.05	1.12	1.14	1.15	1.25
NS2-4-AR	1.13	**0.92**	**0.96**	1.05	1.19	1.29	1.20	1.13	1.12	1.01	**0.79**	**0.82**	**0.89**	1.00	1.11	1.15	1.14	1.20
NS2-4-VAR	**0.97**	**0.77**	**0.79**	**0.87**	**0.96**	1.04	1.07	1.08	1.16	1.01	**0.86**[10]	**0.87**	**0.92**	**0.98**	1.05	1.11	1.15	1.31
NS2-2-λ-AR	1.31	1.00	1.03	1.17	1.36	1.53	1.45	1.38	1.37	1.22	1.05	1.05	1.10	1.18	1.30	1.37	1.38	1.48
NS2-2-λ-VAR	1.24	**0.96**[10]	**0.97**	1.07	1.24	1.42	1.39	1.34	1.36	1.17	**0.99**	**0.99**	1.03	1.10	1.22	1.33	1.36	1.49
NS2-3-λ-AR	1.33	1.22	1.24	1.33	1.43	1.48	1.34	1.27	1.27	1.25	1.12	1.12	1.16	1.23	1.32	1.36	1.36	1.45
NS2-3-λ-VAR	**0.96**	**0.89**	**0.89**	**0.96**	1.00	1.02	**0.96**	**0.96**	1.03	**0.96**	**0.91**	**0.90**	**0.93**	**0.96**	1.00	**0.99**[5]	1.02	1.14
NS2-4-λ-AR	1.18	**0.95**	1.00	1.10	1.25	1.36	1.26	1.19	1.19	1.07	**0.84**	**0.87**	**0.94**	1.05	1.17	1.23	1.24	1.33
NS2-4-λ-VAR	**0.93**	**0.77**	**0.78**	**0.87**	**0.95**	1.01	1.00	1.00	1.09	**0.98**	**0.85**	**0.86**	**0.90**	**0.96**	1.02	1.06	1.11	1.26
NS2-B-AR	1.36	1.22	1.25	1.35	1.46	1.52	1.39	1.33	1.33	1.29	1.11	1.12	1.17	1.25	1.37	1.43	1.44	1.54
NS2-B-VAR	1.01	**0.88**	**0.90**	**0.98**	1.06	1.10	1.05	1.03	1.08	1.04	**0.91**	**0.91**	**0.95**	1.01	1.09	1.15	1.18	1.30
NS2-S-AR	1.60	1.33	1.41	1.57	1.76	1.84	1.64	1.56	1.54	1.42	1.19	1.22	1.29	1.40	1.51	1.56	1.59	1.71
NS2-S-VAR	**0.99**	**0.82**	**0.85**	**0.94**	1.02	1.06	1.04	1.06	1.16	1.03	**0.89**	**0.91**	**0.96**	1.02	1.07	1.10	1.15	1.30
NS2-AS-AR	1.45	1.25	1.32	1.44	1.58	1.64	1.47	1.39	1.38	1.29	1.12	1.14	1.20	1.29	1.38	1.40	1.41	1.49
NS2-AS-VAR	1.00	**0.89**	**0.91**	1.00	1.06	1.08	1.02	1.02	1.10	**0.99**	**0.90**	**0.92**	**0.96**	1.01	1.04	1.03	1.05	1.17
Panel B: one-step models																		
NS1-2-RW	**0.98**	**0.95**	**0.92**	**0.96**	**0.99**	1.02	**0.99**	1.01	1.12	1.00	1.02	**0.99**	1.00	1.00	1.02	**0.99**	1.01	1.13
NS1-2-AR	1.08	**0.99**	**0.98**	1.05	1.11	1.16	1.09	1.09	1.18	1.16	1.12	1.10	1.12	1.15	1.19	1.18	1.21	1.34
NS1-2-VAR	1.10	1.01	**0.98**	1.03	1.08	1.14	1.14	1.18	1.31	1.16	1.07	1.05	1.07	1.10	1.15	1.24	1.32	1.54
NS1-3-RW	1.00	1.02	**0.97**	**0.99**	1.00	1.03	1.01	1.00	1.04	1.00	1.03	**0.99**	**0.99**	1.00	1.02	1.00	**0.99**	1.06
NS1-3-AR	1.08	1.07	1.05	1.10	1.14	1.16	1.07	1.03	1.06	1.07	1.06	1.04	1.05	1.08	1.11	1.09	1.07	1.12
NS1-3-VAR	1.11	**0.90**	**0.92**	1.02	1.12	1.21	1.21	1.21	1.27	1.19	1.03	1.03	1.07	1.14	1.24	1.32	1.36	1.50
NS1-4-RW	1.01	1.03	**0.98**	1.00	1.00	1.02	1.02	1.00	1.02	1.01	1.03	1.00	1.00	1.00	1.02	1.01	1.00	1.05
NS1-4-AR	**0.91**	**0.85**	**0.84**	**0.88**	**0.91**	**0.96**	**0.94**	**0.92**	**0.97**	**0.85**	**0.82**	**0.81**	**0.84**	**0.86**	**0.87**	**0.87**	**0.89**	1.00
NS1-4-VAR	**0.93**	**0.71**	**0.72**	**0.81**	**0.90**	1.01	1.06	1.06	1.14	1.00	**0.81**[10]	**0.83**	**0.89**	**0.96**	1.04	1.12	1.19	1.36
NS1-B-RW	1.01	1.06	**0.99**	1.00	1.00	1.02	**0.99**	**0.99**	1.09	1.01	1.04	1.00	1.00	1.00	1.02	**0.98**[5]	**0.99**	1.10
NS1-B-AR	1.09	1.11	1.07	1.10	1.13	1.14	1.06	1.05	1.12	1.09	1.07	1.05	1.06	1.09	1.13	1.09	1.09	1.20
NS1-B-VAR	1.09	**0.84**	**0.87**	**0.98**	1.10	1.19	1.19	1.22	1.33	1.19	**0.98**	1.00	1.06	1.15	1.25	1.33	1.39	1.57
NS1-S-RW	1.01	1.06	1.00	1.00	1.00	1.02	1.02	1.01	1.02	1.01	1.04	1.00	1.00	1.00	1.02	1.01	1.00	1.04
NS1-S-AR	1.06	1.11	1.05	1.07	1.08	1.10	1.04	1.00	1.04	1.03	1.05	1.01	1.01	1.02	1.04	1.02	1.01	1.08
NS1-S-VAR	**0.93**	**0.71**	**0.72**	**0.80**	**0.89**	1.00	1.05	1.05	1.13	**0.98**	**0.80**[10]	**0.81**	**0.87**	**0.94**	1.02	1.10	1.17	1.34
NS1-AS-RW	1.01	1.06	1.00	1.00	1.00	1.02	1.02	1.01	1.02	1.01	1.04	1.00	1.00	1.00	1.02	1.01	1.00	1.04
NS1-AS-AR	1.03	1.10	1.04	1.05	1.06	1.08	1.01	**0.97**	1.01	1.00	1.04	1.00	**0.99**	1.00	1.01	**0.97**	**0.97**[10]	1.04
NS1-AS-VAR	**0.93**	**0.71**	**0.72**	**0.80**	**0.89**	1.00	1.05	1.05	1.14	**0.98**	**0.80**[10]	**0.81**	**0.87**	**0.94**	1.02	1.10	1.17	1.34

Notes: . Bold numbers indicate outperformance relative to the random walk (RW) whereas ()[10], ()[5] and ()[1] indicate significant outperformance at the 90%, 95% and 99% level respectively according to the White (2000) reality check using 1000 block-bootstraps and an average block-length of 12.

Figure 1: Nelson-Siegel factor loadings



[a] three-factor model

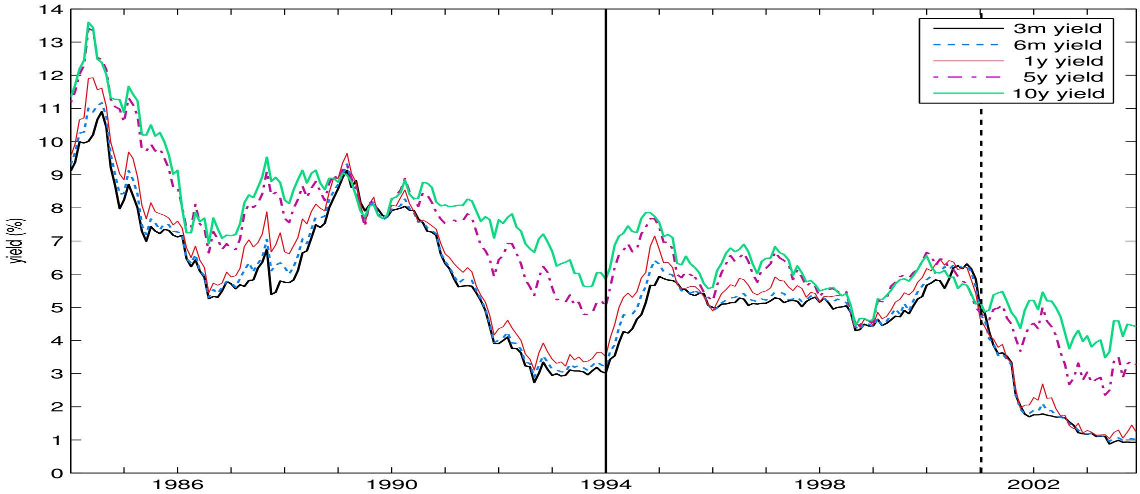
[b] four-factor model

[c] Svensson model

[d] Adjusted Svensson model

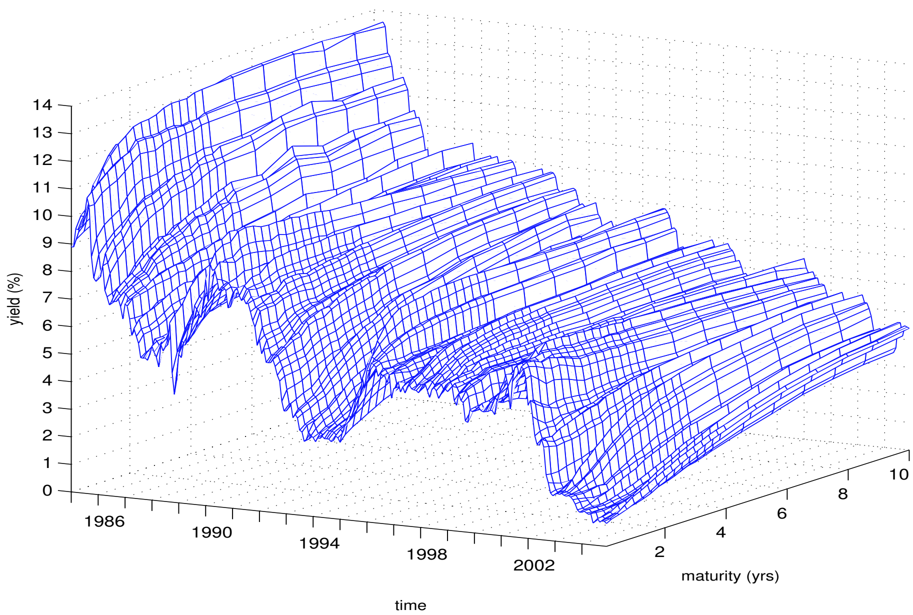
Notes: The graph depicts the factor loadings for β_1 (dotted line), β_2 (dashed line), β_3 (solid line) and β_4 (dash-dotted line) for the [a] three-factor, [b] four-factor, [c] Svensson and [d] Adjusted Svensson Nelson-Siegel model. The factor loadings are plotted using $\lambda_t = 16.42$ for the 3-factor and 4-factor models. For the (Adjusted) Svensson model it holds that $\lambda_{1,t} = 16.42$ and $\lambda_{2,t} = 9.73$ which ensures that the maturities at which the two curvature factors reach their maximum is at least twelve months apart.

Figure 2: U.S. zero-coupon yields



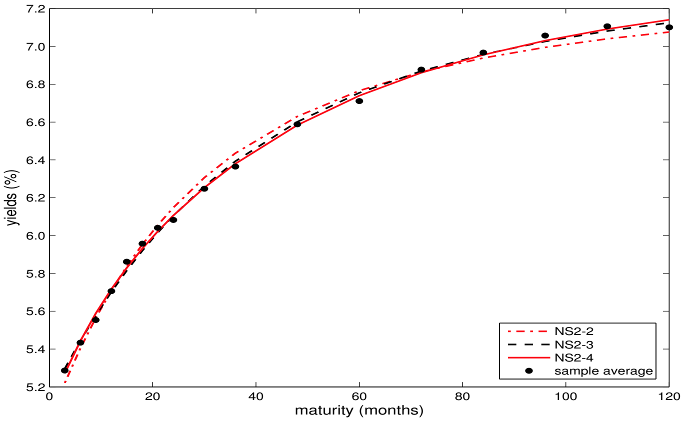
Notes: The figure shows time-series plots for a subset of maturities of end-of-month U.S. zero coupon yields constructed using the unsmoothed Fama and Bliss (1987) bootstrap method. Sample period is January 1984 - December 2003 (240 observations). The solid vertical line indicates the start of the forecasting sample (January 1994 - December 2003). The dotted line divides the forecast sample into two subsamples (January 1994 - December 2000 and January 2001 - December 2003).

Figure 3: U.S. zero-coupon yields

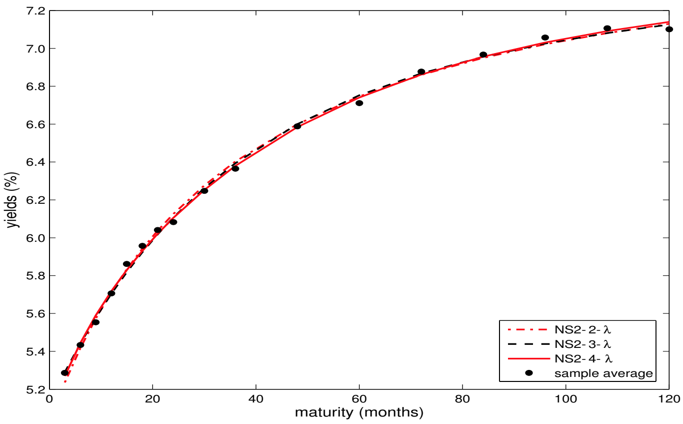


Notes: The figure shows a 3-dimension plot of the panel of end-of-month U.S. zero coupon yields constructed using the unsmoothed Fama and Bliss (1987) bootstrap method. Sample period is January 1984 - December 2003 (240 observations).

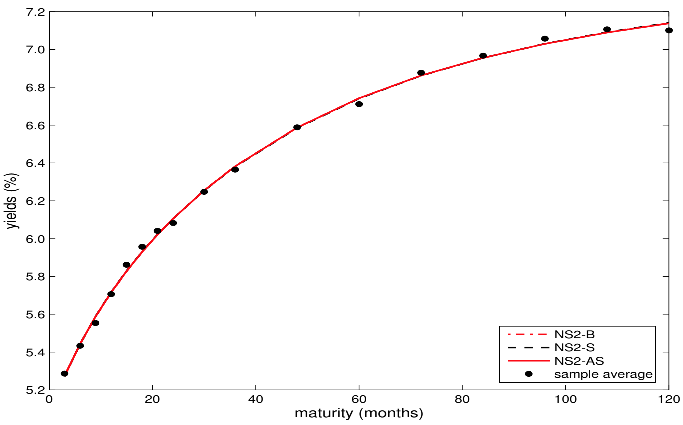
Figure 4: Fitted average yield curve



[a] NS2-2, NS2-3, NS2-4



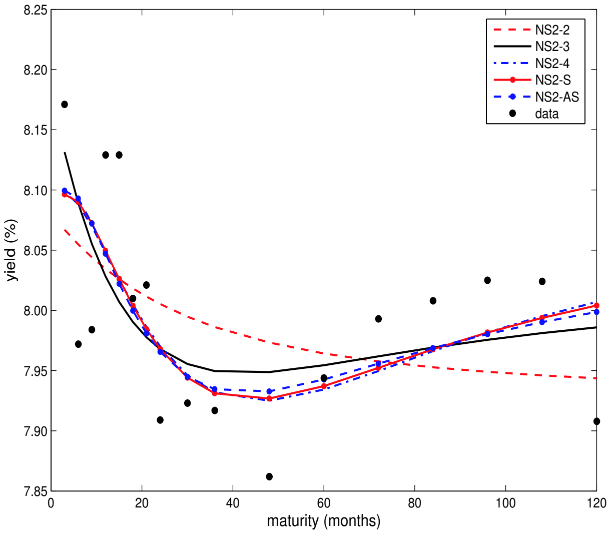
[b] NS2-2-λ, NS2-3-λ, NS2-4-λ



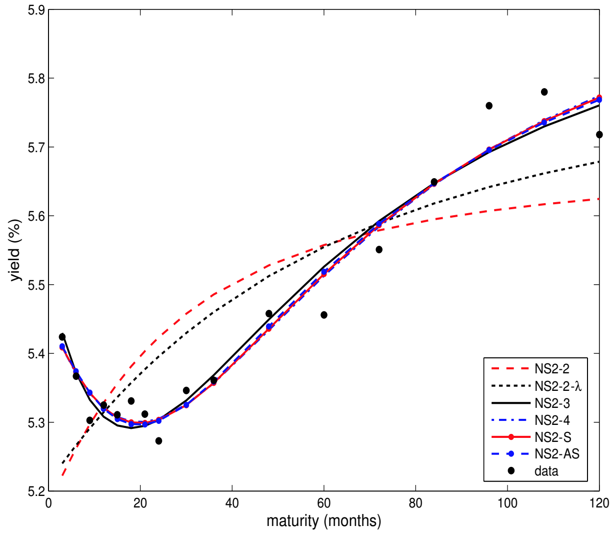
[c] NS2-B, NS2-S, NS2-AS

Notes: The graph shows the average fitted curve for different Nelson-Siegel models. Panel [a] shows the estimated average curve for the two-factor, three-factor and four-factor. Panel [b] shows the average curve for the same models but where now λ_t is estimated alongside (β_1 β_2 β_3 β_4). Finally, Panel [c] depicts the average curve for the three-factor Bliss model, the four-factor model Svensson extension (second column) and the Adjusted Svensson model. The dots in each graph are the actual sample averages. The solid and (dash-)dotted lines depict the fitted lines. The sample period is 1984:1 - 2003:12 (240 observations).

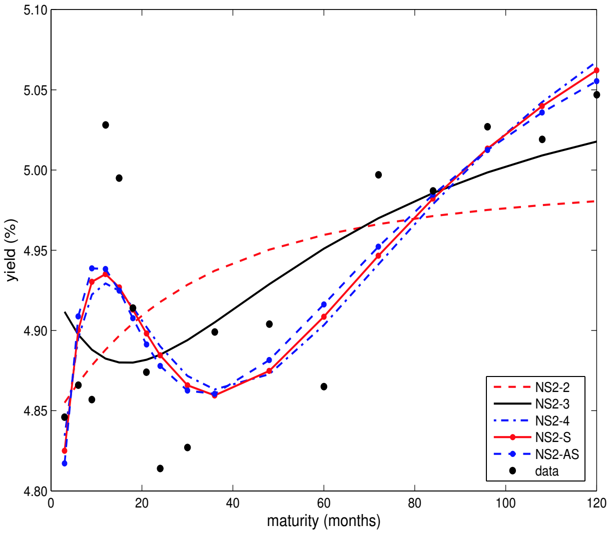
Figure 5: Fitted yield curve for specific months



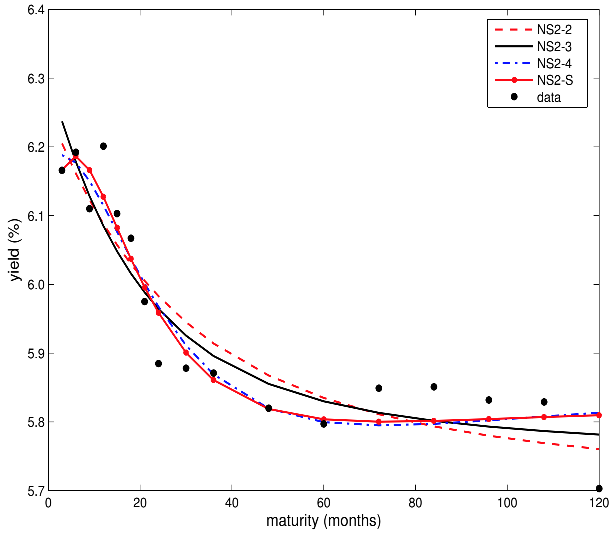
[a] June 30, 1989



[b] November 30, 1995



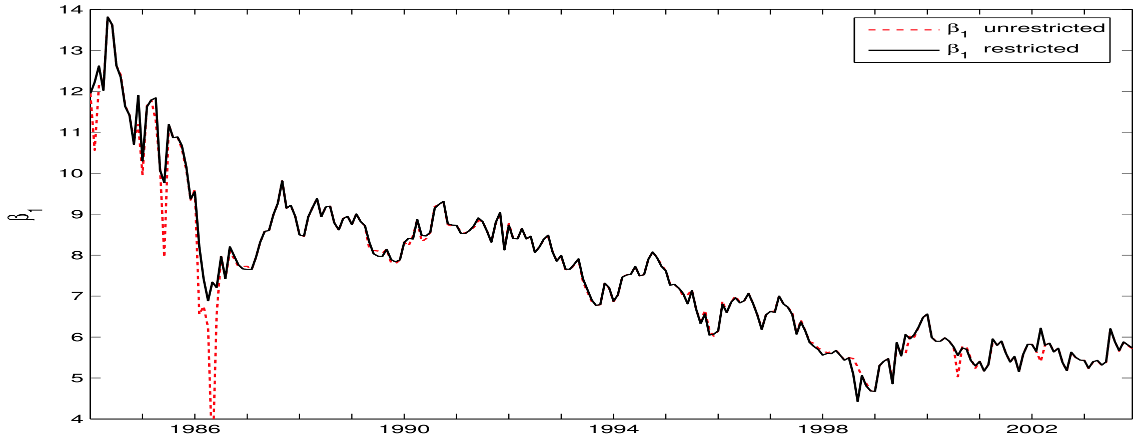
[c] August 31, 1998



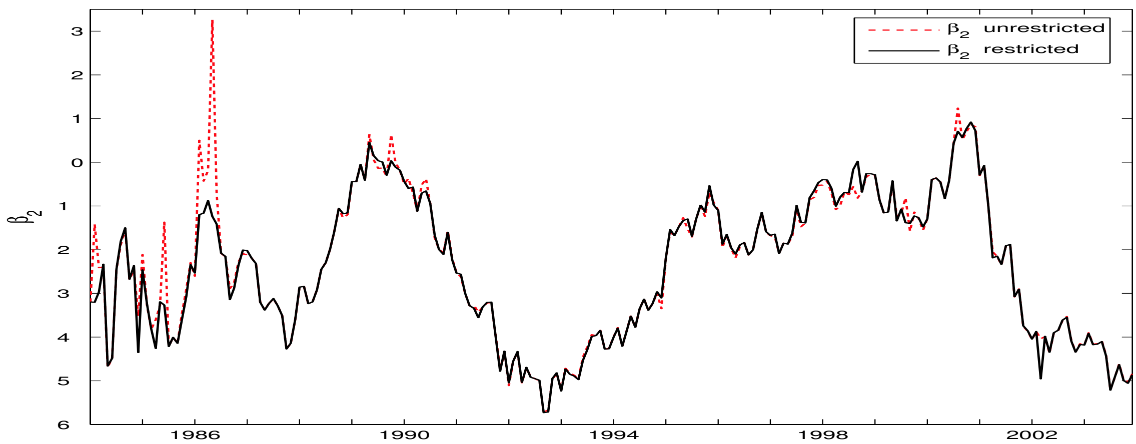
[d] September 29, 2000

Notes: The graph depicts the actual yield curve (black dots) and the fitted yield curve for a subset of models. Shown are four months from the full sample 1984:1 - 2003:12 (240 observations): [a] June 30, 1989, [b] November 30, 1995, [c] August 31, 1998 and [d] September 29, 2000. The fitted curve is shown for the two-factor, three-factor and four-factor model with fixed λ_t, the two-factor model where λ_t is estimated alongside β_1 and β_2, the Svensson model and the Adjusted Svensson model.

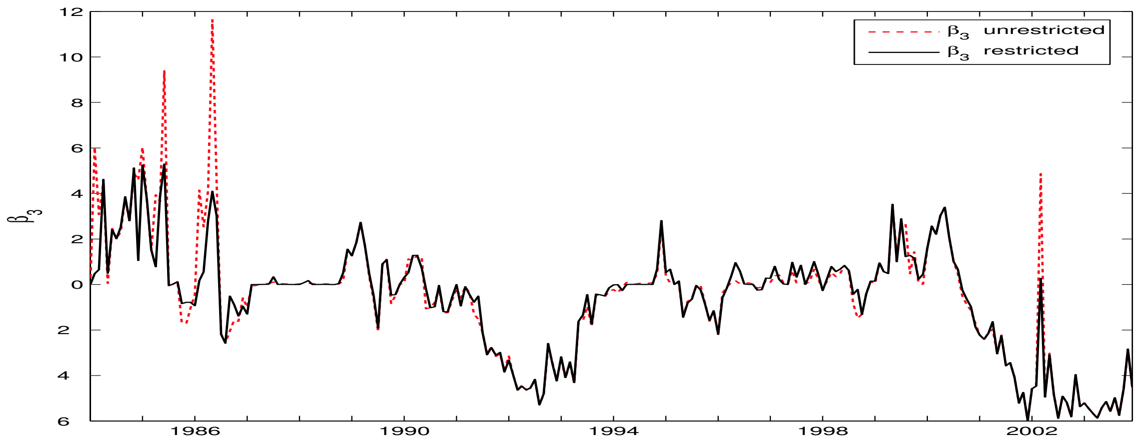
Figure 6: Nelson-Siegel factors with and without restrictions on λ_t



[a] β_1



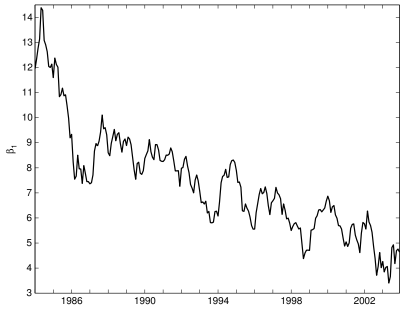
[b] β_2



[c] β_3

Notes: The graph shows time-series plots of the estimated Nelson-Siegel factors for the three-factor base model where λ_t is estimated alongside the factors ($\beta_{1,t}$ $\beta_{2,t}$ $\beta_{3,t}$). Shown are the estimate of the first factor in Panel [a], the second factor in Panel [b] and the third and last factor in Panel [c]. The solid line is the factor estimate when λ_t is restricted to the domain $[6.69, 33.46]$. The dotted line represented each factor when estimated without the restriction on λ_t. The sample period is 1984:1 - 2003:12 (240 observations).

Figure 7: Time-series of Nelson-Siegel factors with a fixed value for λ_t



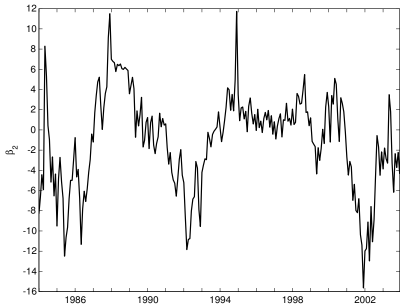
[a] NS2-2, β_1

[b] NS2-3, β_1

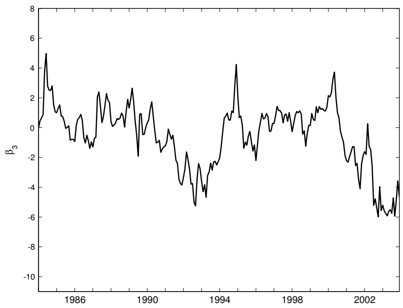
[c] NS2-4, β_1

[d] NS2-2, β_2

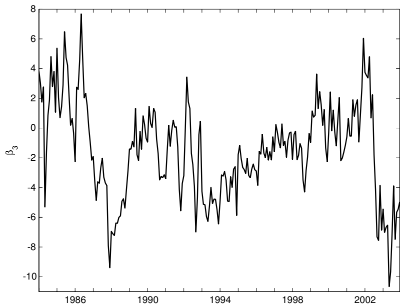
[e] NS2-3, β_2



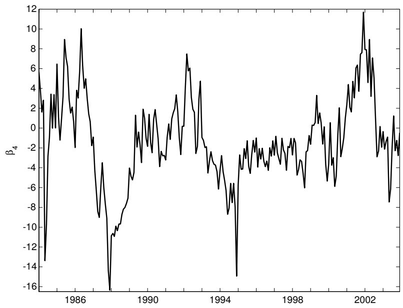
[f] NS2-4, β_2



[g] NS2-3, β_3



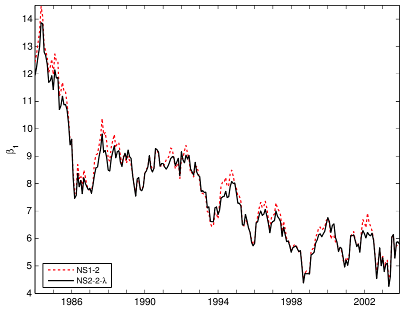
[h] NS2-4, β_3



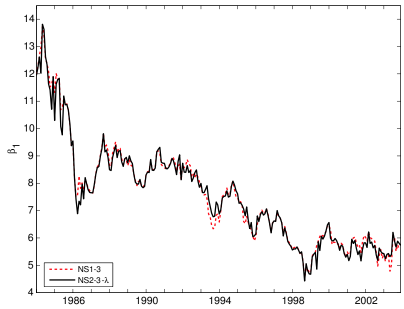
[i] NS2-4, β_4

Notes: The graph shows time-series plots of the estimated Nelson-Siegel factors for the two-factor model (first column), the three-factor model (second column) and the four-factor model (fourth column). The factors are estimated using OLS given a fixed λ_t which is set to 16.42. The sample period is 1984:1 - 2003:12 (240 observations).

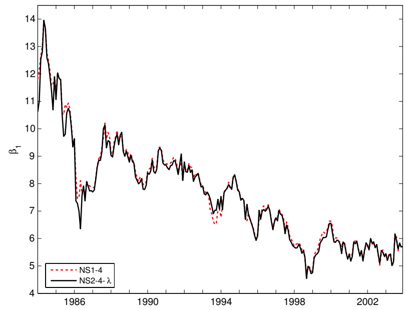
Figure 8: Time-series of Nelson-Siegel factors with estimated λ



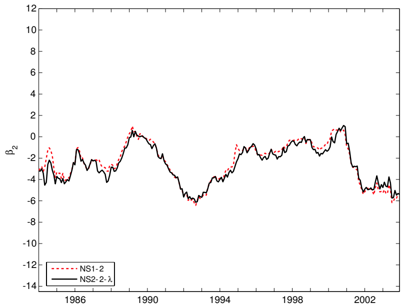
[a] β_1



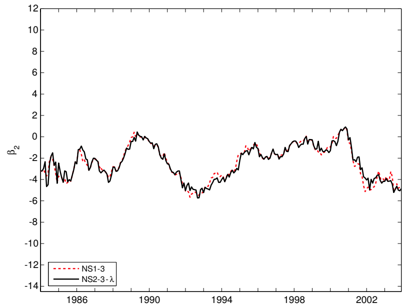
[b] β_1



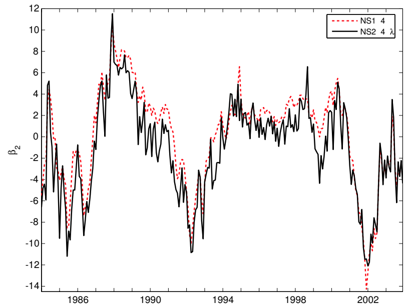
[c] β_1



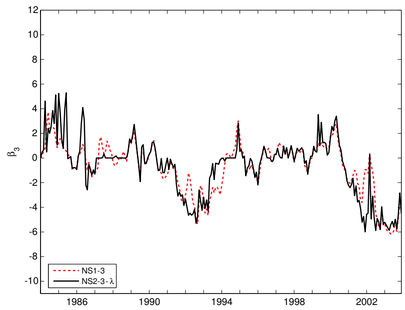
[d] β_2



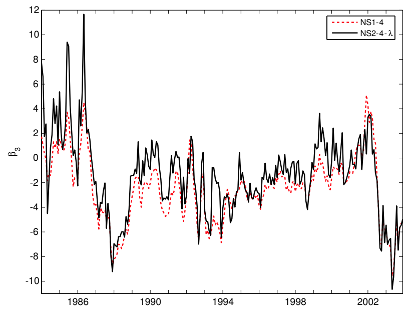
[e] β_2



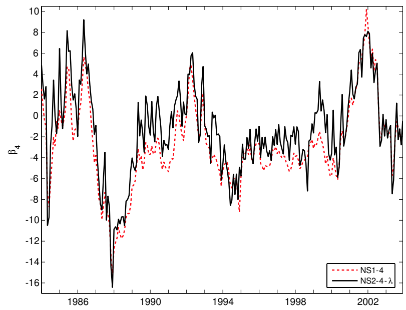
[f] β_2



[g] β_3



[h] β_3

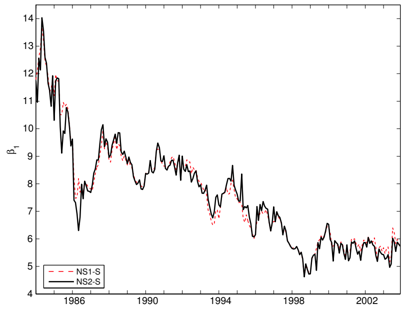


[i] β_4

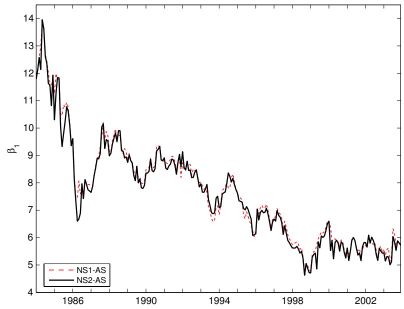
Notes: The graph shows time-series plots of the estimated Nelson-Siegel factors for the two-factor model (first column), the three-factor model (second column) and the four-factor model (fourth column) where λ_t is estimated alongside (β_1 β_2 β_3 β_4). Shown are the factors estimates from the two-step NLS (solid lines) and the one-step Kalman Filter (dotted lines) estimation methods. The sample period is 1984:1 - 2003:12 (240 observations).

Figure 9: Time-series of Nelson-Siegel factors with estimated λs

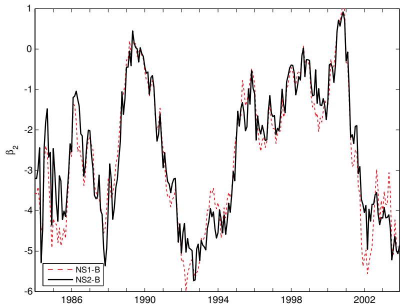
[a] β_1



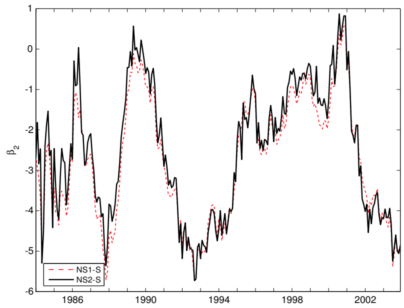
[b] β_1



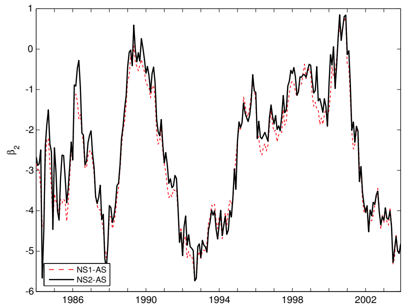
[c] β_1



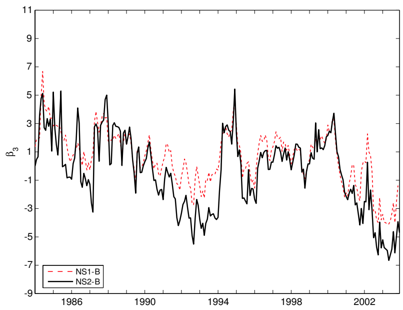
[d] β_2



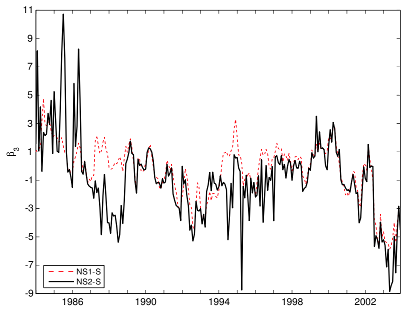
[e] β_2



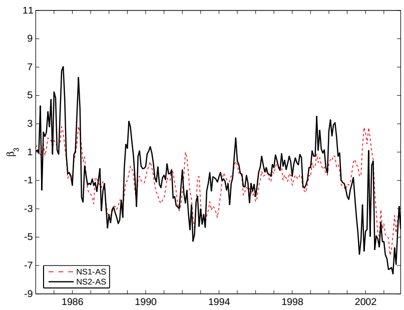
[f] β_2



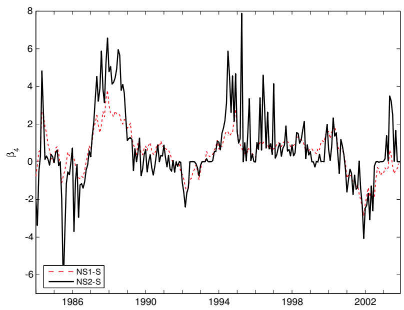
[g] β_3



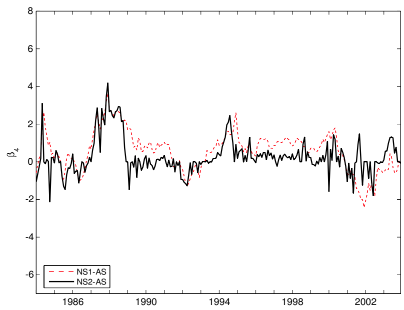
[h] β_3



[i] β_3



[j] β_4



[k] β_4

Notes: The graph shows time-series plots of the estimated Nelson-Siegel factors for the three-factor Bliss model (first column), the four-factor model Svensson extension (second column) and the four-factor model Adjusted Svensson model (fourth column) where $\lambda_{1,t}$ and $\lambda_{2,t}$ are estimated alongside (β_1 β_2 β_3 β_4). Shown are the factors resulting from the two-step NLS and the one-step Kalman Filter estimation methods. The sample period is 1984:1 - 2003:12 (240 observations).