Transaction Costs and Crowding

Ludwig B. Chincarini*

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Abstract

We use industry data to determine whether crowding of the investment space is caused by portfolio construction processes typical to the investment community. In particular, this paper examines the extent that transaction cost models cause crowding of the investment space, even when the investment models are completely unrelated to one another. We find that as transaction costs become more significant in the portfolio creation process as portfolios increase in size from $500 million to $5 billion, crowding actually declines for long-only portfolios and mainly declines, but sometimes increases for market neutral portfolios. This research sheds more light on how crowding develops through actions by players within the financial system.

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1 Introduction

Financial crises have various causes, but they are frequently caused or at least amplified by trade crowding in the investment space (Chincarini (1998), Chincarini (2012)). Crowding can take place through a variety of mechanisms. First, if investors follow similar trading models, it is likely that their resulting portfolios will be very similar. Investors may arrive at similar models either by coincidental or outright replication of each other's ideas. Thus, trading spaces can become crowded because too many investors are constructing portfolios using similar expected return models. The key feature that makes the space crowded is that too many assets are chasing a strategy compared to the availability of liquidity. For example, if a large proportion of manager's of a given type own a large percentage of shares of a stock relative to the stock's typical turnover, this may be a crowded space.

Crowding may also occur, ironically, when investors use similar techniques to construct their portfolios. Investors can have different models for generating their expected returns, but if they use the similar techniques for constructing their portfolios, this can also cause their portfolios to converge to one another. One component of building portfolios is to consider explicitly or implicitly the costs of trading securities. If portfolio managers all face similar transaction cost models, then their portfolios might be more similar than they would want.

Crowding is a problem for investors because it alters the risk and return dynamics of a trade (Cahan and Luo (2013), Ibbotson and Idzorek (2014), Menkveld (2014), and Pojarliev and Levich (2011)). Specifically, it makes the risk of a trade endogenous to the trade itself. Many hedge fund managers and quants argued that the quant crisis of 2007 was caused by crowding.[1] Some of them argued that it was crowding of the alpha models (Cerla (2007), Khadani and Lo (2007), and Rothman (2007)), while others argued that it was really about liquidity and that transaction cost models may have crowded the types of trades they made (Chincarini 2012). That is, with large portfolios, market impact costs might lead many quant funds to trade only a handful of very liquid securities subsequently causing extreme movements in these select stocks when managers decided to sell.

Crowding among quants happens for several reasons, but the transaction costs model was of primary importance, as it caused us to trade similar securities at each point in time.— Mark Carhart interview, Former Co-CIO of Quantitative Strategies at GSAM and Founder of Kepos Capital, October 11, 2011. (Chincarini (2012))

[1]For a first-hand account of the crisis, see chapter 8 of Chincarini (2012).

This paper studies the interactive role of transaction cost models, portfolio construction, and crowding. This paper makes several contributions to the literature on crowding. First, the paper helps to clarify the role that transaction cost models might play in causing the crowding of the investment space. Second, the paper introduces a simple method to approximate transaction costs of several varieties so as to use them in portfolio optimization construction. This approximation is very accurate and very simple to use, allowing practitioners to model a variety of complex transaction costs within a standard portfolio optimization framework. These issues of transaction costs and crowding are important ultimately for understanding the systemic risk from fund management practices and how portfolio formation may lead to fragile investment conditions. We find that crowding and transaction costs may not be related in the way that many portfolio managers believed them to be.

The paper is organized as follows: Section 2 we first define a measure of crowding that will be used in this paper; Section 3 describe our empirical framework for examining the crowding from portfolio construction and transaction costs; Section 4 describes the transaction cost models used in order to construct portfolios; Section 5 discusses the empirical results from the simulated portfolios; and Section 6 concludes the paper.

2 A Definition of Crowding

For the purposes of this paper we define crowding to be when investors own portfolios with similar holdings. Let the similarity between two portfolios be measured by s_{ij}, which is the dot product between the position weight vectors ($\mathbf{w}$) of each portfolio i and j divided by the product of the Euclidean norm of each vector. Thus,

$$s_{ij} = \frac{\mathbf{w}_i' \mathbf{w}_j}{|\mathbf{w}_i||\mathbf{w}_j|} \tag{1}$$

where the Euclidean norm is defined across N assets as

$$|\mathbf{w}_i| = \sqrt{\sum_{n=1}^{N} \mathbf{w}_{in}^2} \tag{2}$$

This measure will have a value between 0 and 1 for portfolios that can only be long securities (i.e. long-only portfolios). This measure will have a value between -1 and 1 for portfolios that can have negative weights.[2]

In our paper, we will study more than just two portfolios. Thus, for studying a group of M portfolios, we define the N-by-M portfolio holdings matrix as the matrix, H, which consists of columns of position weight vectors on N assets for each of M portfolios. The similarity matrix amongst all portfolios is computed as

$$S = \left(H'H\right) \circ \hat{\hat{H}} \tag{3}$$

where $\circ$ represents the Hadamard product or the element-by-element multiplication of the matrices, and

$$\hat{\hat{H}} = \begin{bmatrix} \frac{1}{\hat{h}_{11}} & \frac{1}{\hat{h}_{12}} & \cdots & \frac{1}{\hat{h}_{1M}} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{1}{\hat{h}_{M1}} & \frac{1}{\hat{h}_{M2}} & \cdots & \frac{1}{\hat{h}_{MM}} \end{bmatrix} \tag{4}$$

and $\hat{H} = |H|'|H|$, where $|H|$ contains the Euclidean norm of each manager's weight vector. The matrix S contains the similarities of each portfolio with every other portfolio. For example, element S_{12} represents the similarity of the portfolios of managers 1 and 2. For a specific set of portfolios, our measure of crowding is given by the average of the off-diagonal elements of this matrix.[3]

From the similarity matrix of M portfolios or portfolio managers, we measure the crowding, C, amongst the group of portfolios as the average similarity between portfolios.[4]

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[2]This measure is related to a more commonly used measure known as Pearson correlation. One can think of Pearson correlation as a de-meaned version of Cosine Similarity.

[3]The diagonal elements are the similarity of each portfolio with itself, which are irrelevant. Our measure of the similarity of portfolios to measure crowding is related to a more commonly known cosine similarity, which is a measure of the similarity between two vectors of an inner product space that measures the cosine of the angle between them. This measure is given as $\theta = \cos^{-1}\left(\frac{\mathbf{w_i'w_j}}{\|\mathbf{w'_i}\|\|\mathbf{w_j}\|}\right)$.

[4]Essentially, the numerator represents the summation of all the similarities between every portfolio manager and every other, including it's own. By subtracting m, we normalize this measure to be the average similarity in excess of a group of portfolio managers that are completely dissimilar to each other. In that case, the similarity matrix would be a diagonal of 1s.

$$C = \frac{\sum_{i=1}^{M}\sum_{j=1}^{M} S_{i,j} - M}{M^2 - M} \tag{5}$$

A simple example with a universe of three portfolios holding 3 stocks each might help to illustrate the concept of crowding. Suppose our matrix H of manager holdings is given as

$$H = \begin{bmatrix} 0.4 & 0.8 & 0.45 \\ 0.4 & 0.1 & 0.45 \\ 0.2 & 0.1 & 0.10 \end{bmatrix}. \tag{6}$$

This example includes the portfolios of 3 managers. Each manager has a portfolio whose holding sum to 1. The portfolio of manager 1 has 40% in stock 1, 40% in stock 2, and 20% in stock 3. Portfolio 2 has 80% in stock 1 and 10% in stocks 2 and 3. Portfolio 3 has 45% in stock 1 and 2 and 10% in stock 3.

One can see that manager 1 and manager 3 have very similar or "crowded" portfolios. Manager 2's portfolio is less related to the other two. Using our formula for computing the similarity matrix, we find that[5]

[5]The components of S are given by,

$$H'H = \begin{bmatrix} 0.36 & 0.38 & 0.38 \\ . & 0.66 & 0.415 \\ . & . & 0.415 \end{bmatrix}.$$

$$\hat{H} = \begin{bmatrix} 0.36 & 0.4874 & 0.3865 \\ . & 0.66 & 0.5234 \\ . & . & 0.4150 \end{bmatrix}.$$

and

$$\hat{\hat{H}} = \begin{bmatrix} 2.778 & 2.0515 & 2.5872 \\ . & 1.5152 & 1.9108 \\ . & . & 2.4096 \end{bmatrix}.$$

$$S = \begin{bmatrix} 1 & 0.7796 & 0.9831 \\ . & 1 & 0.7906 \\ . & . & 1 \end{bmatrix}. \tag{7}$$

The resulting similarity matrix corresponds with our intuition. That is, portfolios 1 and 2 have a similarity measure of 0.7796, which is high, but not as high as portfolios 1 and 3, which have a measure of 0.9831. For this universe of portfolios, the crowding measure is $C = 0.8454$. This indicates that there is a high level of similarity or crowding in the investment space from these 3 portfolio managers.

In addition to capturing the crowding of a group of portfolios or portfolio manager holdings, we also wish to study specifically how the portfolio construction process creates additional crowding in the investment space. One way to do this is to measure the crowding of portfolios *before* the portfolio construction process and *after* the portfolio construction process. Specifically, if we were able to observe the expected return models or alpha models of portfolio managers before they assigned weights to their portfolios, we could infer the amount of crowding that is added or removed from portfolio construction techniques.

Let's define S_α as the similarity matrix of portfolio managers from their alpha models. This is the similarity of their stock picking models, whether quantitative or qualitative managers. Define S_p as the similarity of portfolios after the manager has combined his alpha model with his optimization model to construct his final portfolio. Thus, S_p is the similarity matrix of actual portfolio holdings. Both measures are computed as described previously. For both S_α and S_p, the crowding measures are also computed and given by C_α and C_p. In our analysis, we will compute the ratio of these two as

$$\Omega = \frac{C_p}{C_\alpha}. \tag{8}$$

When this ratio is greater than one, it means that the portfolio construction process has caused portfolios to become more crowded than they were just from the different portfolio manager beliefs about the attractiveness of different stocks and vice versa. In other words, this metric represents how much more similar on average the portfolios are than the expected return models.[6]

In our simulation analysis, we will look at the crowding of portfolios, C, as well as the ratio of crowding before and after portfolio construction, Ω.

3 The Empirical Framework

Our strategy to analyze the real-world implications of crowding from the portfolio construction process is to simulate the construction of portfolios using random alpha signals combined with real-world risk models and realistic transaction cost models in order to examine the extent of crowding that occurs indirectly due to the transaction cost considerations of portfolio managers.[7] The data for our empirical study was obtained from several sources. We obtained all stock return from Factset. We obtained our risk model data from the three major risk model providers in the financial industry; Barra, Axioma, and Northfield.

In this section, we discuss the techniques of our simulation process, including the creation of our expected return or α models for stocks, the risk models, and the portfolio construction techniques.

3.1 Portfolio Construction

In order to examine the extent of crowding from the portfolio construction process due to transaction cost considerations, we created portfolios that were more common in the professional investment world. We considered two types of portfolio management techniques; a long-only portfolio and a market-neutral portfolio.[8]

The long-only portfolio manager maximizes net expected return (i.e. returns after transaction costs) while keeping a portfolio that has a volatility equal to the historical volatility of the S&P 500,

[6]We could have also taken the average of the absolute values in this similarity matrix. This would not be as representative of crowding itself, but would also be important for a measure of financial fragility. That is if 50% of managers are long a portfolio and 50% of managers are short a portfolio, our current measure would have a lower level of crowding than the absolute measure. However, this particularly extreme case might indicate a very fragile financial system when crowding is considered in this broader context. One could also consider measuring the absolute value of stock weights when computing the similarity matrix, however, this would not represent crowding as much as it would represent activity in similar stocks.

[7]The procedures used are very similar to those used by sophisticated portfolio managers. For example, Goldman Sachs quant equity group managed portfolios in a similar way. "Our approach to portfolio construction uses these individual company alphas in combination with other optimization criteria with the goal of maximizing each portfolios risk-adjusted expected return net of transaction costs. The inputs to our optimization process are return forecasts, transaction cost estimates, risk estimates, and of course, client objectives. Our risk model and risk forecasts are central to the optimization process." (Daniels (2009)).

[8]For more details on the optimization process, see Appendix B

is not levered, has a maximum stock weight of 10% in any one name, and whose sector composition matches that of the benchmark.[9] We also consider the same long only portfolio manager, however, rather than maximize alpha subject to a risk target, the portfolio manager minimized risk subject to an net expected return target.[10]

The market-neutral manager maximizes expected return subject to having no more than 5% volatility over the risk-free rate, a dollar-neutral portfolio (i.e. the weights of the longs sum to the weights of the shorts), a leverage of 2 (i.e. the weights of the long portfolio sum to 1 and the weights of the short portfolio sum to 1), that no stock can have a weight less than -10% or more than 10%, and that the stocks are sector neutral with respect to the long and short side of the portfolio.[11] We also considered a market neutral manager that minimized the volatility of the portfolio subject to a target alpha.[12]

For both types of portfolio construction, we also considered liquidity constraints, that is we constrained the manager to not purchase too much of a certain portfolio with respect to the average daily trading volume, but did not report these results in the paper.[13]

3.2 The Alpha Model

In order to focus on the amount of crowding that is caused from the portfolio construction process when considering transaction costs, we used random alpha models for the different portfolios. That is, each portfolio receives signals about the stock universe that are random. Thus, the degree of crowding from the alpha models, prior to portfolio construction, has an average value of zero.

[9] These portfolio parameters are quite reasonable. In fact, we surveyed several portfolio managers before creating our parameters. We also experimented with other maximum and minimum weights for the portfolio. The benchmark portfolio for the purposes of sector neutralization was the top 2000 companies selected by market capitalization each month and weighted by market capitalization.

[10] The target for the randomly generated models was chosen as half of the S&P 500 5-year historical volatility. There was no specific reason for these choices, except that they seemed to be reasonable. If a specific target could not be achieved, we searched for the next reasonable target.

[11] These portfolio parameters are quite reasonable. In fact, we surveyed several portfolio managers before creating our parameters. While it is true that different managers may use slightly different parameters, the main purpose of this paper is to describe the potential crowding effects that may occur when portfolio managers use reasonable parameters and similar risk models.

[12] The target alpha was the same as with the long only case.

[13] We did not include liquidity constraints the randomly generated alphas, because for large portfolios, liquidity constraints could not be kept at the 30% level of average daily trading volume (ADV). They had to be increased up to 70% for the portfolio optimizer to solve. For market neutral portfolios with 2x leverage it was even more difficult to satisfy these constraints. This has interesting implications for portfolio management. As a portfolio increases in size and one seriously considers liquidity constraints, the portfolio manager must either accept to trade over several days and accept an increasing position over time or the portfolio manager must increase the portfolio tolerance as a function of average daily trading volume. Both of these increase the problems with liquidity and crowding in an exit situation.

In order to construct the random alpha signals for each portfolio manager, we drew 100 random alpha signals for all stocks from a normal distribution, $\boldsymbol{\alpha} \sim N(0, \boldsymbol{\Sigma}_{\boldsymbol{\alpha}})$, where $\boldsymbol{\Sigma}$ is the historical variance-covariance of asset returns up to the time of portfolio selection with the off-diagonals set to 0.[14] That is, we used the historical volatility of each asset, but ignored the correlations.[15]

3.3 The Risk Models

Crucial to the portfolio management process is the use of a risk model for the securities. In order to understand how crowding occurs in the financial system from the portfolio construction process, we used the leading risk models in the industry to build portfolios.[16]

Most professional money managers use standard third-party risk models to manage their portfolios. The most well-known risk models are that of MSCI-Barra, Northfield, and Axioma.[17] In this paper, we use the the Barra US Equity Model (USE4) which has has been active since June 30, 1995.[18] We also use Northfield's U.S. Fundamental Equity Risk Model which has been active since January 30, 1990.[19] Finally, we use Axioma's Robust Risk Model for the U.S. which has been active since January 4, 1982.[20] Barra is believed to lead most providers with around a 50% market share.

All of these risk models are multi-factor models. That is, these factor models assume that asset returns can be modeled as a linear combination of common risk factors (Ross (1976), Chincarini and Kim (2006)). The three risk models differ by the factors chosen and other estimation techniques. We use the three prominent risk models in the industry to reconstruct the variance-covariance

[14]The reason for choosing 100 random draws rather than a larger number had to with the tradeoff between sufficiently large and the computation time required. To create the 100 random portfolios for twelve months of data took 20 days on a supercomputer that used twelve cores.

[15]Further research might wish to consider a random model which draws from a standard normal distribution, $\boldsymbol{\alpha} \sim N(0, 1)$, where signals for individual assets are independent of their historical volatility. Further research may also wish to consider a model that draws from a full variance-covariance matrix of asset returns rather than just the diagonals.

[16]In order to allow for comparisons across risk models, we match all data across risk model providers and the top 2000 stocks by market capitalization every month of the analysis. We matched the data by CUSIP identification.

[17]The majority of asset managers use either Barra, Northfield, or Axioma and thus are a very representative group (Fabozzi et. al (2007) and Fabozzi and Markowitz (2011)). Other providers include APT and R-squared. APT's Market Risk Model for the US has been active since January 2000. For more info, see http://www.sungard.com/campaigns/fs/alternativeinvestments/apt/solutions/apt _market_risk_models.aspx. R-Squared Customized Hybrid Risk Model (CHRM) has been active since June 29th, 2007. For more info, see http://www.rsquaredriskmanagement.com/Customised-Hybrid-Risk-and-Return-Models.

[18]For more info, see http://www.msci.com/products/portfolio_management_analytics/equity_models/barra _us_equity_model_use4.html. BARRA has another popular risk model , the Barra US Equity Model (USE3), which has been active since 1973.

[19]For more info, see http://www.northinfo.com/documents/8.pdf.

[20]For more info, see http://axioma.com/robust.htm.

matrix of asset returns that real-world portfolio managers would be using to build their optimal portfolios so that we can get an accurate estimation of the crowding that may or may not occur through the portfolio construction process.

4 Transaction Cost Models

Transaction costs are broken down into two categories. These include fixed costs (or those easily observed in the market place) and variable costs (those that are less observable and therefore require more modeling to estimate).

Fixed transaction costs will typically include a per share commission that the manager must pay to the broker to execute the trades. It is reasonable to estimate a cost of $0.005 per share for commissions. Additionally, there is the issue of the bid/offer spread. Since we know that market makers make money on trades through the spread, it is reasonable to assume that a manager will pay the offer on purchases and receive the bid for sales. These costs are assumed to be more or less constant costs that do not vary much with the size of the trade.

Variable transaction costs will typically include an estimate of the likely impact of the size of the trade on the price. There is a positive relationship between the size of a trade and market impact. However, the relationship is not always linear. As trades increase in size up to and beyond a certain threshold, the estimated market impact will increase at an increasing rate. For example, a trade that is two times the average daily trading volume (ADTV) is likely to have more than two times the market impact as a trade that is equal to the ADTV.

Transaction costs have the potential to contribute to crowding. While the fixed costs are not likely to vary with the size of the trade, the variable costs will vary and can affect the final positions. For large portfolios, the transaction costs for assets with low ADTV are likely to be high and the positive gross alphas will become smaller (and possibly even negative) after transaction costs. The final portfolios are less likely to include these names. For assets with higher ADTV, more of the positive gross alpha will translate into positive net alpha and we are more likely to see these names included in the final portfolios. When multiple managers use the same transaction cost model, there may be crowding in liquid assets.

4.1 Model 1

In order to study the impacts of the transaction costs or market impact models on crowding, we use two market impact models. The first model is a structural model estimated from U.S. equity data (Almgren, Thum, Haptmann and Li (2005)). The model for market impact on trading is given by:[21]

$$c_{it} = \frac{I}{2} + \operatorname{sgn}(n_{it})\eta\sigma_{it}\left|\frac{n_{it}}{V_{it}T}\right|^{3/5} \tag{9}$$

where $I = \gamma\sigma_{it}\frac{n_{it}}{V_{it}}\left(\frac{N_{it}}{V_{it}}\right)^{1/4}$, $\gamma = 0.314$, $\eta = 0.142$, σ_{it} is the daily volatility of stock return i at the beginning of month t, N_{it} is the total amount of shares outstanding in the security, V_{it} is the average daily trading volume of the stock (shares traded, not dollars traded), T is the time interval in which the trade takes place in number of days, for this paper we use $T = 1$, and n_{it} represents the number of shares of the security the portfolio is trading.[22]

4.2 Model 2

The second model is the Northfield model for transaction costs which has been available since March 2009 (see Northfield (2012)). This model of market impact is estimated every month by Northfield with dynamically generated parameters for each stock. The model is of the form,

$$c_{it} = B_{it}|n_{it}| + C_{it}|n_{it}|^{0.5} \tag{10}$$

where B_{it} and C_{it} are parameters estimated by Northfield, n_{it} is the number of shares to be purchased for security i in month t, and c_{it} is expressed in terms of percentage price movement.[23]

4.3 Spreads

For both market impact models, we add the percentage spread cost of trading, by adding a term equal to the bid-ask spread divided by 2 divided by the current stock price multiplied by 100. Thus,

[21]For the purposes of this paper, the preciseness of the transaction cost model is not crucial. Any model of the form, $c_{it} = \frac{s}{2} + \frac{n_{it}}{V_{it}}^{\phi}\psi$ will be sufficient, where s is the bid-ask spread and ϕ and ψ are parameters that need to be estimated.

[22]The parameter symbols have been changed from the original paper so as to be more consistent with symbols in this paper.

[23]Northfield prefers to use the symbol S_{it} to represent the shares traded.

the final transaction cost model is given by

$$tc_{it} = \left|\frac{100 s_{it}/2}{p_{it}}\right| + |c_{it}| \tag{11}$$

where s_{it} is the bid-ask spread of stock i at time t. These transaction costs, tc_{it} are in percentage points.

For example, for December 2013, take two stocks, AT&T (Ticker Symbol: T), a very liquid stock, and AGL Resources (Ticker Symbol: GAS), a less liquid stock. AT&T for this particular period had a market capitalization of $183 billion, a stock price of $35.16, and a 10-day average daily trading volume of 18,930,000 shares. The spread was 1 cent or a 0.0284% spread. The trading costs in percentage terms for a 1% position in a $500 million portfolio was 0.0232%. That is, a $5 million trade of AT&T representing 142,000 shares would cost the trader $1,160. This does not represent commissions, it is simply the market impact and spread costs. AGL Resources for this particular period had a market capitalization of $5.6 billion, a stock price of $47.23, and a 10-day average daily trading volume of 490,000 shares. The spread was 2 cents or a 0.0423% spread. The trading costs in percentage terms for a 1% position in a $500 million portfolio was 0.1621%. That is, a $5 million trade of AGL Resources representing 105,865 shares would cost the trader $8,105.

4.4 Approximation of Transaction Costs for Optimization Model

The transaction cost models used in this paper are difficult to use in a standard optimization framework. In the case of transaction cost model 1, it is not usable even in the leading software provider platforms for portfolio optimization, like Axioma, Northfield, and BARRA.[24] In this paper, I present an easy to execute and remarkably reliable approximation to transaction costs, which could prove useful for practitioners needing to deal with a variety of transactional cost models. In this paper, an approximation method is used for the transaction costs. First, the transaction costs are computed for each stock in the portfolio by varying the portfolio weight of each stock from zero to 0.10 (the maximum possible value for any stock in the portfolio) for each net asset level. Second, a regression is run on each stock of the following form:

[24] When I began working on this paper, I considered partnering with the research staff at Axioma and the research staff of other commercial portfolio optimization software providers to alleviate the work load. However, they informed me that their systems would have trouble incorporating certain transaction cost models, like transaction cost model 1. BARRA and Bloomberg also do not support such a functional form, although they can be tweeked to approximate the costs within a given range of trade size.

$$\widetilde{tc}_{it} = a_{it}w_{it} + b_{it}w_{it}^2 \tag{12}$$

where $\widetilde{tc}_{it}$ is a vector of net transaction costs from the transaction model corresponding to each stock's particular weight, a_{it} and b_{it} are parameters estimated from the linear regression.[25] That is, $\widetilde{tc}_{it}$ represents the percentage transaction cost of each stock multiplied by the stock's weight, w_{it}, representing the net transaction cost impact of each stock at each weight to the entire portfolio.

This approximation model works extremely well for all stocks.[26] For example, the maximum and minimum $\bar{R}^2$ for all stocks in December 2013 is 1 and 0.9998 respectively. The approximate cost model works very well at estimating the transaction costs of each stock. Figure 1 shows the actual transaction costs and approximate transaction costs for AT&T for December 2013. The approximation is excellent with $\hat{\alpha} = 0.0206$, $\hat{\beta} = 0.5096$, and $\bar{R}^2 = 0.9999$. Figure 2 shows the actual and approximate transaction costs for AGL Resources. The approximation is excellent with $\hat{\alpha} = 0.0697$, $\hat{\beta} = 0.11.1110$, and $\bar{R}^2 = 0.9999$.

[INSERT FIGURE 1 ABOUT HERE]

[INSERT FIGURE 2 ABOUT HERE]

5 Empirical Simulation

5.1 Methodology

Given these realistic portfolio construction techniques described in the previous sections, we constructed 100 optimal portfolios for every risk model and for every month in our sample period from a security universe of the largest 2000 publicly traded stocks in the United States.[27] In every month of the sample, each portfolio was constructed from 2000 random alpha signals. For each group of portfolios, four market scenarios were considered. In scenario 1, all of the portfolios were constructed without considering transaction costs. In scenario 2, each portfolio was assumed to have a total of $500 million in assets under management (AUM). In scenario 3, each portfolio was assumed

[25] There is a different a_{it} and b_{it} for every net asset level, since the transaction costs of each stock vary with assets under management.

[26] The Weierstrass approximation theorem states that every continuous function defined on a closed interval [a, b] can be uniformly approximated as closely as desired by a polynomial function. In the case of the transaction costs functions used in this paper, a quadratic function is sufficient for a very good approximation.

[27] This security universe was updated every month in our sample period.

to have a total of \$5 billion in AUM. In scenario 4, each portfolio was assumed to have a value of \$20 billion in AUM. Since the market impact (a primary driver of transaction costs) is driven by the size of the trades, adjusting the value of portfolios from \$500 million to \$20 billion in AUM allows for a comparison of how transaction costs affect crowding during the portfolio construction process. The transaction costs are estimated for every stock and the approximation parameters are re-estimated for every size scenario, since they naturally would change as the portfolio size changes. The portfolios are rebalanced once per month.[28] For each scenario, the optimal portfolio weights are stored for all 100 portfolios in every month. These weights are used to measure crowding in each of the four scenarios.

Our analysis of these simulated portfolios covered the period 2006 to 2013 for transaction cost model 1 and March 2009 to 2013 for transaction cost model 2.[29] The reason that we used a different time period for transaction cost model 2 is that it was created by Northfield in 2009 and did not exist prior to this date.

5.2 General Results

Tables 1 and 2 report the crowding measures by optimization framework (e.g. Long Only portfolio), by risk model 1, 2, or 3, with and without transaction costs, and by average size of the portfolio.[30] We also show in the tables, the maximum weight in any given portfolio, the minimum weight in any given portfolio, and the average number of stocks in the portfolios that are constructed. We split the analysis into two periods since one transaction cost model only existed from March 2009 until 2013. For both periods, 2006 to 2009, and 2009 to 2013, independent of risk model used, as the average portfolio size increased from \$500 million with no transaction costs to \$5 billion with transaction costs, the average crowding declined. This is true for both market neutral and long only portfolios. Crowding amongst managers only starts to increase when the average size of the portfolio moves from \$5 billion to \$20 billion. Even in this case, crowding is greater when

[28] Portfolio managers may not trade as often or as strictly as the simulation does, however, the point of the research is to examine how portfolio construction and transaction costs affect crowding and thus a controlled setting is required.

[29] We have data for a longer time period, but the simulations take an enormous amount of time to compute and thus we limited our sample from 2006 to 2013. For example, the 100 random alpha portfolios can take 20 days to complete one historical year of analysis when running on 12 processors in parallel.

[30] The empirical part testing of the crowding induced by risk models was extremely complicated. In order to dynamically simulate the portfolios, we had to create an entire program to do professional portfolio optimization. We used MATLAB 2014a and CPLEX from IBM through the MATLAB API to perform the empirical analysis. The random simulations took an enormous amount of time to run. For example, to obtain the results for one year of data took 20 days when running the program on 12 parallel processors.

transaction costs are not considered at all compared to when they are considered with portfolios of size \$20 billion (see columns 2, 8, and 14 of Tables 1 and 2). For example, from 2006 to 2009, for risk model 1 and long-only portfolios, crowding is 0.58 with \$500 million dollar portfolios and no transaction costs and 0.50 with portfolios of \$20 billion in size when considering transaction costs. For risk model 2 and risk model 3, the numbers are 0.60 and 0.43 and 0.59 and 0.46 respectively. The qualitative results are similar for the period 2009 to 2013 and also similar for both transaction cost model 1 and 2. Thus, the average crowding for long portfolios declines as portfolios get large, even though transaction costs are increasing.

In order to establish statistical significance, we also tested whether the average crowding with transaction costs was significantly different. In almost all cases, the average crowding when considering transaction costs was significantly lower than not considering transaction costs at the 99% confidence level. In Tables 1 and 2, this is indicated by *** (99% confidence level) and ** (95% confidence level). We also tested whether the average crowding from portfolios with \$20 billion and hence more transaction costs was significantly smaller than portfolios with \$500 million in assets, and found that it was for all of the long-only portfolios during the period 2006 to 2009, but only for risk model 1 and 3 during the period 2009 to 2013.

[INSERT TABLE 1 ABOUT HERE]

[INSERT TABLE 2 ABOUT HERE]

The crowding measure for market neutral portfolios was generally much smaller than for long-only portfolios. Thus, in order to examine the effect of transaction costs, we focused mainly on the measure of omega (Ω). Omega measures the crowding of the constructed portfolio to the crowding of the random alpha signals. Thus, a value greater than 1 indicates that the portfolio construction process led to more crowding than the alpha models amongst portfolios. The higher the value of omega, the more crowding that occurs from portfolio construction.

For market neutral portfolio managers, the evidence is similar. For portfolios of size \$500 million to \$5 billion, relative crowding (Ω) decreases when considering transaction costs. As the average portfolio size increases to \$20 billion, relative crowding is generally higher than portfolios of smaller size for the period 2006 to 2009, but once again lower for the period 2009 to 2013. For example, for the period 2006 to 2009, the average relative crowding is almost double for a portfolio of size \$20 billion than for portfolios that do not consider transaction costs using risk models 1 and 3 (see columns 3 and 15 of Table 1). However, for risk model 2, crowding declines. For the 2009 to 2013 period, relative crowding generally declines as the portfolio grows from $500 million without considering transaction costs to $20 billion. For risk models 2 and 3, the no transaction cost measure of relative crowding is 55 and 63 compared to 28 and 45 using transaction cost model 1. Only for risk model 1 is relative crowding greater for the $20 billion dollar average portfolio compared to the $500 million dollar average portfolio without transaction costs.

We also tested for the statistical significance of the differences in average crowding for market neutral portfolios. We found no statistical difference in the average crowding. Thus, although average crowding generally declines with transaction costs for market neutral portfolios, the lack of significance implies that transaction costs probably did not play a large role in crowding at these asset levels.

One may also notice the large negative pseudo-Sharpe ratios. When portfolio managers fail to consider transaction costs ex-ante, the ex-post Sharpe ratios are dramatically negative. For example, using risk model 1, the Sharpe ratios for a portfolio that doesn't consider transaction costs is -42.65 and -1.12 for long only and market neutral portfolios (see Table 2, column 4) versus -7.13 and 0.79 for portfolios that considered transaction costs ex-ante. This is exactly what would be expected. For the long only portfolios, the average Sharpe ratios are positive in the second period from 2009 to 2013, but not for the market neutral portfolios.

Figures 3 to 5 show the way crowding changes over time. For long-only portfolios, for most of the period between 2006 and 2013, the crowding from the largest portfolios considering transaction costs (red line) is lower than that of the smaller portfolios that do not even consider transaction costs (blue dotted line). However, in 2006 and in 2013, this was reversed (see Figure 3). This general pattern seems to be true for all risk models used (see Figure 4).

Figure 4 shows that for most of the time, the relative crowding for market neutral models without transaction costs is in-line with large portfolios that consider transaction costs, except for certain brief periods, where the relative crowding of large portfolios increases enormously. These particular periods are driving the average results discussed in the tables.

5.3 Implications

The crowding of investment in securities can lead to similar positions by similar investors that may eventually lead to a cascade when investors must rebalance their portfolios. Rebalancing cascades might occur when investors follow similar benchmarks (Chinco and Fos (2016)), when they follow similar investing strategies, or even as an unintended consequence of using similar methods to build portfolios. One of the ways that portfolios can become very similar is due to the portfolio construction process, like the use of similar transaction cost models to manage the direct drag (spreads) and the indirect drag (market impact) from trading securities. The simulations in this paper provide evidence that as equity portfolios grow in size from $500 million each to $5 billion each, crowding actually declines. Thus, for portfolio managers managing less than $5 billion and using similar transaction cost models and reasonable portfolio construction parameters, the unintended crowding from transaction cost models in the equity space may not be a worry. However, as the value of portfolios approaches $20 billion, crowding due to transaction cost parameters starts to increase and should be of concern. Of course, this will also depend on the number of managers in the space compared to the asset universe.

The results of the paper show that portfolio managers acting independently may lead to crowding. It also shows something else that is interesting and that can be understood by comparing the $500 million case with the $20 billion case. If portfolio managers construct portfolios using a larger asset base than their own portfolio value to estimate transaction costs, this may lead to portfolios constructed that have less crowding than if they only consider the actual size of their own portfolios. Considering a larger asset base for transaction costs in portfolio construction would make sense if they recognize many similar investors to themselves that might rebalance or trade at a similar time as them.

One of the explanations for lower crowding due to transaction costs as portfolios grow from $500 million to $5 billion is that as market impact costs become larger in a portfolio, it makes trading large amounts of particular stocks incredibly costly due to the non-linear nature of market impact costs. Thus, a portfolio optimizer will find it advantageous, ceteris paribus, for portfolios to trade very small amounts of many more stocks.[31] Of course, this is absent any particular alpha considerations. This logical behavior of the optimizer will result in less crowding amongst similarly optimized portfolios up to a certain portfolio size relative to the security universe.[32]

[31] A portfolio manager might split up the order over several days, but market impact is still present to a degree, since this is just a scaling of the magnitude of the impact. Even if a portfolio manager builds their positions over several days, they are essentially crowding the investment space regardless and may be in jeopardy when a shock arises that requires them to sell quickly.

[32] Of course, there is a complicated relationship between the optimization parameters, the alpha signals, and the constraints of the optimization problem.

The results also indicate the importance of considering transaction costs in portfolio optimization. These costs should be considered not only with respect to the size of one's own portfolio, but also with a consideration of the size of similar types of investors. Some portfolio managers do not even consider transaction costs in their portfolio construction, which can cause low ex-post returns as well as unintended crowding. Dan deBartolomeo, the CEO of Northfield, has said "There is a disconnect in the industry between portfolio construction and trading and many portfolio managers leave the issue of transaction costs to the trading team." Portfolio managers who do ignore transaction costs may create "crowding" due to transaction costs that will *only be realized ex-post* when it is too late.

The results of this paper indicate that crowding can occur from transaction cost considerations in equity portfolios as the size of the portfolios becomes very large relative to the size of the equity space — in this paper we considered the top 2000 U.S. equities by market capitalization. The results also indicate that considering more than one's own portfolio when accounting for transaction costs might actually reduce transaction costs across a group of similar managers. However, there are limits to this benefit. As portfolios and the group of portfolios grow in size beyond \$20 billion or beyond the capacity of their stock universe, crowding will be unavoidable. However, it is still better to consider transaction costs *ex-ante*, rather than realize them in spades *ex-post*.

6 Conclusion

The links between market participants is gaining more notice in the financial community. The behavior of market participants can lead to a change of equilibrium prices that depart from fundamentals and leave a trading space vulnerable for a collapse. One of the ways in which a trading space can become crowded is through portfolio managers copying each other's trade ideas or implementing similar trade ideas that lead to similar positions. The crowding of the investment space may lead to a mis-measurement of risk. Another way in which portfolio managers might find it difficult to trade positions is that their positions have become concentrated due to using similar transaction cost models. For example, suppose two portfolio managers wish to trade two stocks. Portfolio manager 1 likes stock A and portfolio manager 2 likes stock B. Ignoring diversification issues, portfolio manager 1 would like to buy 70% of A and 30% of B, while portfolio manager 2 would like to buy 30% of A and 70% of B. However, if stock B has a large transaction cost relative to A, then both managers might tilt more towards A. In fact, the result might be that portfolio manager 1 buys 75% of A and portfolio manager 2 buys 70% of A, which causes crowding and ironically may lead to *ex-post* trading costs being even larger than *ex-ante* trading costs. Thus, it may be the case that transaction costs impose some sort of crowding.

When portfolio managers using independent alpha models account for transaction costs, it actually leads to less crowding or an insignificant amount of additional crowding from individual portfolio sizes of $500 million to $5 billion, however crowding starts to increase beyond $20 billion for a U.S. stock universe of 2000 companies. For long only portfolio managers, we find that the average crowding from portfolios with no transaction costs compared to portfolios that consider transaction costs with an average size of $20 billion is 9% to 22% more. For market neutral portfolios, we find no statistical difference in the average crowding up to $20 billion in portfolio size. Portfolio managers during the quant crisis of 2007 mentioned transaction costs as a potential cause of crowding during the crisis (Chincarini (2012)). The simulation evidence in this paper indicates that it may have had less to do with transaction costs and more to do with other factors.

Our paper contributes to the understanding of systemic risk by understanding how the interaction of portfolio managers in preparing portfolios may lead to inadvertent crowding with a particular focus on transaction cost models. Our paper shows how crowding and transaction costs are related and introduces a simple and very useful method to incorporate transaction costs into the optimization framework.

There are many directions for further research in the area of crowding. A more detailed investigation of the tradeoff between trading liquidity and portfolio size might be interesting, including whether there are obvious limits to a portfolio's size given a trading strategy. One might also investigate how different parameters of the portfolio construction process influence the relationship between crowding and transaction costs. As portfolio size grows, it may also provide a linkage between what constitutes a short-term investor and a longer-term investor, since transaction cost constraints will force the honest and knowledgeable manager to be a longer-term investor.

There have been some studies relating crowding and momentum. It might be illuminating to study the links between momentum and transaction costs. As stocks perform better (momentum), they become a larger component of one's portfolio and those stocks will represent a larger potential sell to non-owners of that stock relative to other stocks, which might lead to higher transaction costs and might imply more crowding than a simple measure would detect. Related to this is the concept of how a portfolio manager's effective universe of securities declines as the portfolio size grows and how crowding depends on the size of the investment universe and the number of managers in that universe. There is also much needed theoretical work on crowding.

A Applied Optimization Details

Our optimizations involve three portfolio management techniques. This appendix describes the optimization problem set up. All of our optimizations were performed in MATLAB using MATLAB's optimization routines, in addition to user-adjusted optimization routines, and the CPLEX optimization tools from IBM.[33]

A.1 The Long Portfolio

Our approach is to maximize the expected return after transaction costs (or net alpha signal) of the portfolio subject to a variety of constraints, including that the portfolio volatility be equal to the 60-month historical volatility of the S&P 500[34], the weights of the portfolio sum to 1, the weights of any individual stock are between 0 and 10%, and that the portfolio has the same exposure to each sector as the benchmark universe of 5000 stocks.

$$\max_{\mathbf{w}} \mathbf{w}'\boldsymbol{\mu} - \widetilde{\mathbf{tc}} \tag{13}$$

$$\text{s.t.} \tag{14}$$

$$\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} = \sigma_{S\&P500} \tag{15}$$

$$\mathbf{w}'\boldsymbol{\iota} = 1 \tag{16}$$

$$0 \leq \mathbf{w} \leq 0.10 \tag{17}$$

$$\mathbf{S}\mathbf{w} = \mathbf{w}_s^{BM} \tag{18}$$

where $\mathbf{w}$ are the weights of the stocks in the portfolio, $\boldsymbol{\mu}$ is a vector of alpha signals for each stock, $\widetilde{\mathbf{tc}}$ is the net transaction costs, $\boldsymbol{\Sigma}$ is the variance-covariance matrix of stock returns, $\boldsymbol{\iota}$ is a vector of ones, $\mathbf{S}$ is an M-by-N matrix of zeros and ones representing the M sectors of the economy with a 1 if the security is in that sector and a 0 if not, and $\mathbf{w}_s^{BM}$ is an M-by-1 vector of sector weights for the benchmark universe.

We also consider the reverse optimization problem whereby the portfolio is contructed by minimizing the variance of the portfolio subject to achieving a target after transaction costs alpha equal to historical annualized volatility of the S&P 500 divided by $\sqrt{12}$.

[33] Some of the optimizations were not solvable in feasible time with versions of MATLAB older than 2014a.

[34] In cases where it is not feasible to achieve the S&P 500 historical volatility, the closest feasible volatility is used in the optimization.

A.2 The Market Neutral Portfolio

Since many quantitative portfolio managers construct market neutral portfolios, we also investigate crowding with the market neutral construction. The approach is to maximize the expected return after transaction costs or net alpha of the portfolio, while constraining the portfolio to have a target volatility equal to 5% over the risk-free rate, have leverage of 2 and be dollar-neutral (that is, sum of long weights sum to 1 and sum of short weights sum to 1), the long portfolio is sector neutral to the short portfolio, the weights of an individual stock cannot be less than -10% or greater than 10%[35], and beta neutral (that is, the weighted average beta of the long portfolio equals the weighted average beta of the short portfolio).

$$\max_{\mathbf{w}} \mathbf{w}'\boldsymbol{\mu} - \widetilde{\mathbf{tc}} \tag{19}$$

$$\text{s.t.} \tag{20}$$

$$\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} = 0.05 \tag{21}$$

$$\mathbf{w}_L'\boldsymbol{\iota} = 1 \ \forall w_i \geq 0 \tag{22}$$

$$\mathbf{w}_S'\boldsymbol{\iota} = -1 \ \forall w_i \leq 0 \tag{23}$$

$$-0.10 \leq \mathbf{w} \leq 0.10 \tag{24}$$

$$\mathbf{w}'\boldsymbol{\beta}|_{w_i \geq 0} = \mathbf{w}'\boldsymbol{\beta}|_{w_i \leq 0} \tag{25}$$

$$\mathbf{S}\mathbf{w}_L = -\mathbf{S}\mathbf{w}_S \tag{26}$$

where $\mathbf{w}$ are the weights of the stocks in the portfolio, $\boldsymbol{\mu}$ is a vector of alpha signals for each stock, $\boldsymbol{\Sigma}$ is the variance-covariance matrix of stock returns, $\boldsymbol{\iota}$ is a vector of ones, $\boldsymbol{\beta}$ is a vector of the CAPM beta for each stock estimated on 5-year historical return data, $\mathbf{S}$ is an M-by-N matrix of zeros and ones representing the M sectors of the economy with a 1 if the security is in that sector and a 0 if not, $\mathbf{w}_L$ and $\mathbf{w}_L$ represents the weights of the long and short portfolio respectively.

We also consider the reverse optimization problem whereby the portfolio is contructed by minimizing the variance of the portfolio subject to achieving a target after transaction costs alpha equal to historical annualized volatility of the S&P 500 divided by $\sqrt{12}$.

A.3 The Market Neutral Portfolio with Liquidity Constraints

We constructed market neutral portfolios that incorporated reasonable self-imposed liquidity constraints. The optimization approach was exactly the same as for the market neutral portfolio,

[35] We initially started with smaller weight restrictions of 0.03 and -0.03, but many of the optimizations could not be solved, thus we expanded the weight constraint.

however we added a liquidity purchase constraint that is a fraction of the average daily trading volume.

Liquidity constraints are relatively straightforward to add to the optimization problem. The constraint takes the form of a portfolio manager not wishing to trade more than some percentage of the average daily trading volume of the stock. That is, the constraint is $V_t w_{it} \leq cADTV_{it}$ or $w_i \leq \frac{c}{V_t} ADTV_{it}$, where c represents the constant indicating the threshold percentage that the portfolio manager wishes to trade in any given stock, V_t is the dollar value of the portfolio, and $ADTV_{it}$ is the average daily trading volume of stock i at time t in dollars. A typical value for this in the quantitative world is 15%.[36]

Since the liquidity constraint is essentially an upper bound weight constraint, it makes sense to adjust the existing upper bound weight constraint for each stock rather than adding a new series of constraints. Thus, the upper bound and lower bound weight constraint for every stock was adjusted using the following algorithm. If the liquidity constraint was higher than the existing stock constraint (i.e. 10%), then we didn't alter the stock's weight constraint. If smaller, we changed the upper and lower bound constraint to be equal to the liquidity constraint value for each stock. We did this for both the long and short side of the portfolio.

Unfortunately, when we added these constraints and increased the size of the portfolios, oftentimes there was no feasible solution. Also, since our main goal was to investigate transaction costs and their impact on portfolio construction, we removed the liquidity constraints for the randomly generated portfolios.

A.4 Market Neutral Construction

One of the challenges of the market neutral optimization was to set up the problem so that leverage can be limited. The method we employed for every one of the N stocks in our buy list, we created an additional set of weights called buy weights and an additional set of sell weights. Thus, for n stocks, we created weights, $w_1 ... w_N$, $w_1^b ... w_N^b$, and $w_1^s ... w_N^s$. We then constructed our entire optimization with these $3N$ weights. In preparing our inputs for the optimization, we formulated the following:

[36] The quantatitive manager might also have a total limit on the ultimate size of any position, for example, 3 times the ADTV. We did not consider this additional consideration for this study.

$$\boldsymbol{\mu} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_N \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{27}$$

where there are $2N$ zero values in the column. The variance-covariance matrix was also modified as follows:

$$\boldsymbol{\Sigma} = \begin{bmatrix} V(r_1) & C(r_1, r_2) & \cdots & C(r_1, r_N) & 0 & \cdots & 0 \\ C(r_2, r_1) & V(r_2) & \cdots & C(r_2, r_N) & 0 & \cdots & 0 \\ \vdots & & & & 0 & \cdots & 0 \\ C(r_N, r_1) & C(r_N, r_2) & \cdots & V(r_N) & 0 & \cdots & 0 \\ 0 & \cdots & 0 & & & & \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix} \tag{28}$$

Most importantly, we altered the constraints in such a way as to keep the main constraints on the final weights (i.e. $w_1 ... w_N$), while achieving our market-neutral leverage and dollar-neutral constraints. Thus,

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \cdots & 0 & -1 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 & 0 & -1 & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 & 0 & 0\cdots & 0 & \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 1 & 1\cdots & 1 & \end{bmatrix} \tag{29}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \tag{30}$$

These constraints created an optimization whereby $w_i = w_i^b - w_i^s$, $\sum_i^{n_b} w_i^b = 1$, and $\sum_i^{n_s} w_i^s = 1$. We also added constraints that $\mathbf{w}_B \geq 0$ and $\mathbf{w}_S \geq 0$. This ensured that our market neutral portfolio was dollar neutral and had a leverage limited to 2. One could modify this for other forms of leverage very easily. The weights we ultimately care about are the $\mathbf{w}$. Any additional constraints on these weights, such as upper and lower bounds or sector constraints can be added to the constraint matrix, A, simply by adding rows and placing zeros wherever the $\mathbf{w}^b$ and $\mathbf{w}^s$ occurred.

Although this solution enabled leverage and dollar-neutral constraints on our market neutral portfolio, it did not guarantee that we didn't have wasteful solutions such that we purchase and sell pieces of the same stock. In order to reduce this possibility, we introduced a penalty function into our objective function. That is,

$$\max_{\mathbf{w}} \mathbf{w}'\boldsymbol{\mu} - \boldsymbol{\Lambda}(\boldsymbol{\iota}'\mathbf{w_B} + \boldsymbol{\iota}'\mathbf{w_S}) \tag{31}$$

B Transaction Costs Construction

Due to the recursive nature of transaction costs, that is, the optimal weight of a stock depends on the transaction costs of that stock, but the transaction costs, due to market impact, depends on the optimal weight of the stock, we use the technique outlined in the paper of approximating transaction costs by a quadratic function and show how to modify the optimization problem to deal with this.

B.1 Long Only Portfolio

In order to incorporate our approximate transaction costs into the portfolio optimization problem, we must modify the quadratic optimization program slightly.[37] First, we must use a quadratic optimization routine that can accept quadratic constraints, in addition to linear constraints.[38] Second, we must modify the traditional portfolio optimization setup to work with transaction costs.

The mathematical expression of the quadratic optimization with quadratic constraints is given as,

$$\min_{\mathbf{x}} \frac{1}{2}\mathbf{x}'\mathbf{Q}\mathbf{x} + \mathbf{x}'\mathbf{c} \quad s.t. \quad \mathbf{A}'\mathbf{x} \leq \mathbf{b} \tag{32}$$

$$\mathbf{l}'\mathbf{x} + \mathbf{x}'\mathbf{Q}^*\mathbf{x} \leq \mathbf{r} \tag{33}$$

$$\mathbf{lb} \leq \mathbf{x} \leq \mathbf{ub} \tag{34}$$

[37] Some books discuss using binary constraints as a way of including transaction costs. Usually, this is because those writers have not considered the empirical implications. It is extremely difficult for the optimizer to solve such problems. In fact, even impossible.

[38] For example, CPLEX's cplexqcp.

where $\mathbf{x}$ is the vector of unknowns in the problem, $\mathbf{Q}$ is a symmetric positive semi-definite matrix supplying the coefficients on the quadratic terms of the optimization problem, $\mathbf{c}$ is a vector of coefficients related to the linear objective function, $\mathbf{A}$ is a matrix of coefficients for the equality and inequality constraints, $\mathbf{b}$ is a vector of constraint values, $\mathbf{l}$ is a vector, $\mathbf{Q}^*$ is a matrix, $\mathbf{lb}$ is a lower bound vector, and $\mathbf{ub}$ is an upper bound vector.

In the traditional mean-variance optimization problem, we substitute the following variables; $\mathbf{x} = \mathbf{w}$, the stock weights, $\mathbf{Q} = \mathbf{\Sigma}$, the variance-covariance matix of stock returns, $\mathbf{c} = \mathbf{0}$, $\mathbf{l} = \mathbf{0}$, $\mathbf{Q}^* = \mathbf{0}$, $\mathbf{A}$ is chosen typically to have a row of ones and a row of expected returns, and the lower and upper bounds are set as desired.

In order to create an optimal portfolio which minimizes the risk of the portfolio and achieves a desired after-transaction cost alpha, the parameters chosen were as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \multicolumn{4}{c}{\cdots\cdots\cdots\cdots} \end{bmatrix} \tag{35}$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ . \end{bmatrix} \tag{36}$$

$$\mathbf{Q} = 2\mathbf{\Sigma} \tag{37}$$

$$\mathbf{Q}^* = \begin{bmatrix} \hat{\beta}_1 & 0 & \dots & 0 \\ 0 & \hat{\beta}_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \hat{\beta}_N \end{bmatrix} \tag{38}$$

and $\mathbf{l} = -\tilde{\boldsymbol{\mu}}$, $\tilde{\boldsymbol{\mu}} = \boldsymbol{\mu} - \hat{\boldsymbol{\alpha}}$, $\mathbf{c} = \mathbf{0}$, and $r = -\mu^T$, where $\tilde{\boldsymbol{\mu}}$ is a vector of the expected returns of each stock minus the constant estimate in the transaction cost regression, $\boldsymbol{\mu}$ is the expected return of each stock, and μ^T is the after transaction costs expected return for the portfolio to match.

For the dual problem of maximizing the after transaction cost return, while achieving a target variance, the parameters chosen are as follows:

$$
\mathbf{A} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \cdots\cdots\cdots\cdots \end{bmatrix} \tag{39}
$$

$$
\mathbf{b} = \begin{bmatrix} 0 \\ . \end{bmatrix} \tag{40}
$$

$$
\mathbf{Q} = \begin{bmatrix} \hat{\beta}_1 & 0 & \dots & 0 \\ 0 & \hat{\beta}_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \hat{\beta}_N \end{bmatrix} \tag{41}
$$

$$
\mathbf{Q}^* = \mathbf{\Sigma} \tag{42}
$$

and $\mathbf{l} = \mathbf{0}$, $\mathbf{c} = -\tilde{\boldsymbol{\mu}}$, $\tilde{\boldsymbol{\mu}} = \boldsymbol{\mu} - \hat{\boldsymbol{\alpha}}$, and $r = \sigma^T$, where $\tilde{\boldsymbol{\mu}}$ is a vector of the expected returns of each stock minus the constant estimate in the transaction cost regression, $\boldsymbol{\mu}$is the expected return of each stock, and σ^T is the target volatility for the portfolio to match.

B.2 Market Neutral

The market neutral problem is slightly more complicated. As explained previously, we create phantom weights for the long and the short. In order to create an optimal portfolio which minimizes the risk of the portfolio and achieves a desired after-transaction cost alpha, the parameters chosen were as follows:

$$
\mathbf{A} = \begin{bmatrix} \quad \end{bmatrix} \tag{43}
$$

$$
\mathbf{b} = \begin{bmatrix} \quad \end{bmatrix} \tag{44}
$$

$$
\mathbf{Q} = 2\mathbf{\Sigma} \tag{45}
$$

where $\mathbf{\Sigma}$ is as in Equation (28).

$$
\mathbf{Q}^* = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Sigma}_2 \end{bmatrix} \tag{46}
$$

where

$$
\mathbf{\Sigma}_2 = \begin{bmatrix} \hat{\beta}_1 & 0 & \dots & 0 \\ 0 & \hat{\beta}_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \hat{\beta}_N \end{bmatrix} \tag{47}
$$

and $\mathbf{l} = -\tilde{\boldsymbol{\mu}}$, $\tilde{\boldsymbol{\mu}} = [\boldsymbol{\mu}, -\hat{\boldsymbol{\alpha}}, -\hat{\boldsymbol{\alpha}}]'$, $\mathbf{c} = \mathbf{0}$, and $r = -\mu^T$, where $\tilde{\boldsymbol{\mu}}$ is a $3 \times N$ matrix of the expected returns of each stock and the constant estimates in the transaction cost regression, $\boldsymbol{\mu}$ is the expected return of each stock, and μ^T is the after transaction costs expected return for the portfolio to match.

For the dual problem of maximizing the after transaction cost return, while achieving a target variance, the parameters chosen are as follows:

$$
\mathbf{A} = \begin{bmatrix} \dots \end{bmatrix} \tag{48}
$$

$$
\mathbf{b} = \begin{bmatrix} . \end{bmatrix} \tag{49}
$$

$$
\mathbf{Q} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Sigma}_2 \end{bmatrix} \tag{50}
$$

where

$$
\mathbf{\Sigma}_2 = \begin{bmatrix} \hat{\beta}_1 & 0 & \dots & 0 \\ 0 & \hat{\beta}_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \hat{\beta}_N \end{bmatrix} \tag{51}
$$

$$
\mathbf{Q}^* = 2\mathbf{\Sigma} \tag{52}
$$

where $\mathbf{\Sigma}$ is as in Equation (28) and $\mathbf{l} = \mathbf{0}$, $\mathbf{c} = -\tilde{\boldsymbol{\mu}}$, $\tilde{\boldsymbol{\mu}} = [\boldsymbol{\mu}, -\hat{\boldsymbol{\alpha}}, -\hat{\boldsymbol{\alpha}}]'$, and $r = -\sigma^T$, where $\tilde{\boldsymbol{\mu}}$ is a $3 \times N$ matrix of the expected returns of each stock and the constant estimates in the transaction cost regression, $\boldsymbol{\mu}$ is the expected return of each stock, and σ^T is the after transaction costs risk for the portfolio to match.

C Tables

Table 1: Summary of Crowding from Random Alpha Models and Transaction Costs from 2006 to February 2009

	Risk Model 1						Risk Model 2						Risk Model 3					
	C	Omega	SR	Max	Min	N	C	Omega	SR	Max	Min	N	C	Omega	SR	Max	Min	N
Alpha	-0.00																	
Long Only																		
MN NTC	-0.00	0.75	-3708.352	0.004	-0.004	645	-0.00	0.84	-2437.77	0.005	-0.005	611	0.00	0.50	-3296.92	0.006	-0.01	632
LONG NTC	0.58	-141.26	-140.911	0.076	0.000	63	0.60	-181.90	-175.48	0.072	0.000	75	0.59	-156.62	-184.22	0.079	0.00	64
Port. Size ($500M)																		
MN TC1	-0.00	0.27	-8.171	0.007	-0.006	567	0.00	-0.04	-7.84	0.006	-0.006	543	0.00	0.11	-7.49	0.009	-0.01	556
LONG TC1	0.49	-127.77	-0.512	0.079	0.000	67	0.45**	-123.77	-1.00	0.071	0.000	89	0.46**	-116.86	-0.84	0.080	0.00	71
Port. Size ($5B)																		
MN TC1	0.00	0.63	-15.027	0.007	-0.007	527	0.00	0.10	-13.88	0.010	-0.011	514	0.00	0.47	-13.98	0.009	-0.01	519
LONG TC1	0.42***	-91.04	-1.427	0.077	0.000	102	0.38***	-113.74	-1.59	0.072	0.000	138	0.38***	-111.11	-1.71	0.077	0.00	114
Port. Size ($20B)																		
MN TC1	0.00	1.42	-21.240	0.013	-0.013	157	0.00	0.09	-20.03	0.014	-0.014	456	0.00	1.13	-20.05	0.014	-0.01	460
LONG TC1	0.50	294.63	-2.152	0.072	0.000	157	0.43***	151.19	-2.26	0.064	0.000	217	0.46***	241.19	-2.33	0.072	0.00	176

Note: This table presents various crowding measures from the constructed portfolios using various portfolio optimization structures that minimize volatility using various risk models over the period 2010 to 2013. Risk Model 1,2, and 3 represent leading risk models used in the industry. The names are purposely omitted so as to not identify any particular risk model. All numbers in the Exhibit are averages of various variables constructed from monthly portfolios. The computations are based on 100 portfolios formed from random alpha signals. C represents our crowding measure as described in the paper, $C = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{m^2 - m}$. Ω measures the relative crowding between random signals and actual portfolios, $\Omega = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{\alpha:i,j} - m}$. A higher value means that risk model creates more crowding. *S.R.* is a pseudo-Sharpe ratio for each portfolio defined as the portfolio's annualized forward one-month return minus transaction costs divided by its ex-ante standard deviation. Max represents the median of the maximum weight of any portfolio over all months, Min represents the median of the minimum weight of any portfolio over all months, and N represents the average number of stocks across portfolios in any given month over all months. The optimizations represent optimizations which attempt to minimize the variance of the portfolio subject to a target alpha subject to various constraints as explained in the paper. Two transaction cost models are considered, model 1 (TC1) and model 2 (TC2). The transaction costs include spreads and market impact of the following form: $tc_{it} = \left|\frac{100 s_{it}/2}{p_{it}}\right| + |c_{it}|$, where s_{it} is the bid-ask spread of stock i at time t. For model 1, $c_{it} = \frac{I}{2} + \text{sgn}(n_{it})\eta\sigma_{it}\left|\frac{n_{it}}{V_{it}T}\right|^{3/5}$, where $I = \gamma\sigma_{it}\frac{n_{it}}{V_{it}}\left(\frac{N_{it}}{V_{it}}\right)^{1/4}$, $\gamma = 0.314$, $\eta = 0.142$, σ_{it} is the daily volatility of stock return i at the beginning of month t, N_{it} is the total amount of shares outstanding in the security, V_{it} is the average daily trading volume of the stock, T is the time interval in which the trade takes place in number of days, for this paper we use $T = 1$, and n_{it} represents the number of shares of the security the portfolio is trading. For model 2, $c_{it} = B_{it}|n_{it}| + C_{it}|n_{it}|^{0.5}$, where B_{it} and C_{it} are parameters estimated by Northfield, n_{it} is the number of shares to be purchased for security i in month t, and c_{it} is expressed in terms of percentage price movement. Model 2 exists only since March 2009. ***, ** indicates the a 99% and 95% significant difference respectively in the average crowding from this portfolio and a portfolio that doesn't consider transaction costs. MN is for the market-neutral portfolios and LONG is for the long portfolios.

Table 2: Summary of Crowding from Random Alpha Models and Transaction Costs from March 2009 to 2013

	Risk Model 1						Risk Model 2						Risk Model 3					
	C	Omega	SR	Max	Min	N	C	Omega	SR	Max	Min	N	C	Omega	SR	Max	Min	N
Alpha	0.00																	
Long Only																		
MN NTC	-0.00	27.11	-42.646	0.006	-0.006	747	-0.00	55.14	-60.02	0.008	-0.008	731	-0.00	62.98	-32.70	0.008	-0.01	736
LONG NTC	0.37	-19958.74	-1.123	0.090	0.000	44	0.41	-24910.52	-8.03	0.087	0.000	48	0.39	-21522.80	-4.13	0.092	0.00	46
Port. Size ($500M)																		
MN TC1	-0.00	26.23	-7.132	0.007	-0.008	663	-0.00	27.70	-6.66	0.009	-0.009	652	-0.00	57.83	-6.51	0.010	-0.01	652
LONG TC1	0.32	-17843.58	0.794	0.090	0.000	50	0.27***	-16434.66	0.32	0.085	0.000	66	0.31**	-16026.43	0.53	0.091	0.00	56
Port. Size ($5B)																		
MN TC1	-0.00	26.74	-14.669	0.009	-0.009	594	-0.00	19.87	-13.70	0.012	-0.012	588	-0.00	51.50	-13.61	0.012	-0.01	586
LONG TC1	0.29***	-15581.70	0.167	0.088	0.000	77	0.25***	-13758.30	-0.08	0.082	0.000	106	0.27***	-14462.44	-0.04	0.088	0.00	92
Port. Size ($20B)																		
MN TC1	-0.00	40.62	-22.275	0.013	-0.013	122	-0.00	27.97	-20.73	0.015	-0.016	503	-0.00	45.28	-21.27	0.015	-0.02	502
LONG TC1	0.38	-6862.70	-0.491	0.082	0.000	122	0.32***	-7217.42	-0.66	0.075	0.000	159	0.36	-6539.03	-0.66	0.082	0.00	144
Port. Size ($500M)																		
MN TC2	-0.00	29.83	-7.346	0.007	-0.007	655	-0.00	45.16	-6.44	0.009	-0.009	644	-0.00	60.42	-6.73	0.010	-0.01	644
LONG TC2	0.34	-17964.16	0.960	0.091	0.000	43	0.33***	-20577.48	0.51	0.088	0.000	47	0.33	-16635.14	0.80	0.093	0.00	44
Port. Size ($5B)																		
MN TC2	-0.00	28.76	-13.438	0.009	-0.009	591	-0.00	30.47	-11.91	0.011	-0.011	583	-0.00	58.93	-12.31	0.011	-0.01	581
LONG TC2	0.30**	-16088.35	0.558	0.091	0.000	45	0.27***	-16026.43	0.06	0.086	0.000	52	0.28***	-14002.91	0.30	0.092	0.00	47
Port. Size ($20B)																		
MN TC2	-0.00	28.39	-19.124	0.010	-0.010	513	-0.00	1.88	-17.08	0.013	-0.013	506	-0.00	56.05	-17.56	0.014	-0.01	506
LONG TC2	0.26***	-14485.43	0.157	0.091	0.000	49	0.22***	-12348.61	-0.25	0.086	0.000	59	0.24***	-13175.77	-0.12	0.091	0.00	53

Note: This table presents various crowding measures from the constructed portfolios using various portfolio optimization structures that minimize volatility using various risk models over the period 2010 to 2013. Risk Model 1,2, and 3 represent leading risk models used in the industry. The names are purposely omitted so as to not identify any particular risk model. All numbers in the Exhibit are averages of various variables constructed from monthly portfolios. The computations are based on 100 portfolios formed from random alpha signals. C represents our crowding measure as described in the paper, $C = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{m^2 - m}$. Ω measures the relative crowding between random signals and actual portfolios, $\Omega = \frac{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{p:i,j} - m}{\sum_{i=1}^{m}\sum_{j=1}^{m} S_{\alpha:i,j} - m}$. A higher value means that risk model creates more crowding. *S.R.* is a pseudo-Sharpe ratio for each portfolio defined as the portfolio's annualized forward one-month return minus transaction costs divided by its ex-ante standard deviation. Max represents the median of the maximum weight of any portfolio over all months, Min represents the median of the minimum weight of any portfolio over all months, and N represents the average number of stocks across portfolios in any given month over all months. The optimizations represent optimizations which attempt to minimize the variance of the portfolio subject to a target alpha subject to various constraints as explained in the paper. Two transaction cost models are considered, model 1 (TC1) and model 2 (TC2). The transaction costs include spreads and market impact of the following form: $tc_{it} = \left|\frac{100 s_{it}/2}{p_{it}}\right| + |c_{it}|$, where s_{it} is the bid-ask spread of stock i at time t. For model 1, $c_{it} = \frac{I}{2} + \text{sgn}(n_{it})\eta\sigma_{it}\left|\frac{n_{it}}{V_{it}T}\right|^{3/5}$, where $I = \gamma\sigma_{it}\frac{n_{it}}{V_{it}}\left(\frac{N_{it}}{V_{it}}\right)^{1/4}$, $\gamma = 0.314$, $\eta = 0.142$, σ_{it} is the daily volatility of stock return i at the beginning of month t, N_{it} is the total amount of shares outstanding in the security, V_{it} is the average daily trading volume of the stock, T is the time interval in which the trade takes place in number of days, for this paper we use $T = 1$, and n_{it} represents the number of shares of the security the portfolio is trading. For model 2, $c_{it} = B_{it}|n_{it}| + C_{it}|n_{it}|^{0.5}$, where B_{it} and C_{it} are parameters estimated by Northfield, n_{it} is the number of shares to be purchased for security i in month t, and c_{it} is expressed in terms of percentage price movement. Model 2 exists only since March 2009. ***, ** indicates the a 99% and 95% significant difference resprectively in the average crowding from this portfolio and a portfolio that doesn't consider transaction costs. MN is for the market-neutral portfolios and LONG is for the long portfolios.

D Figures

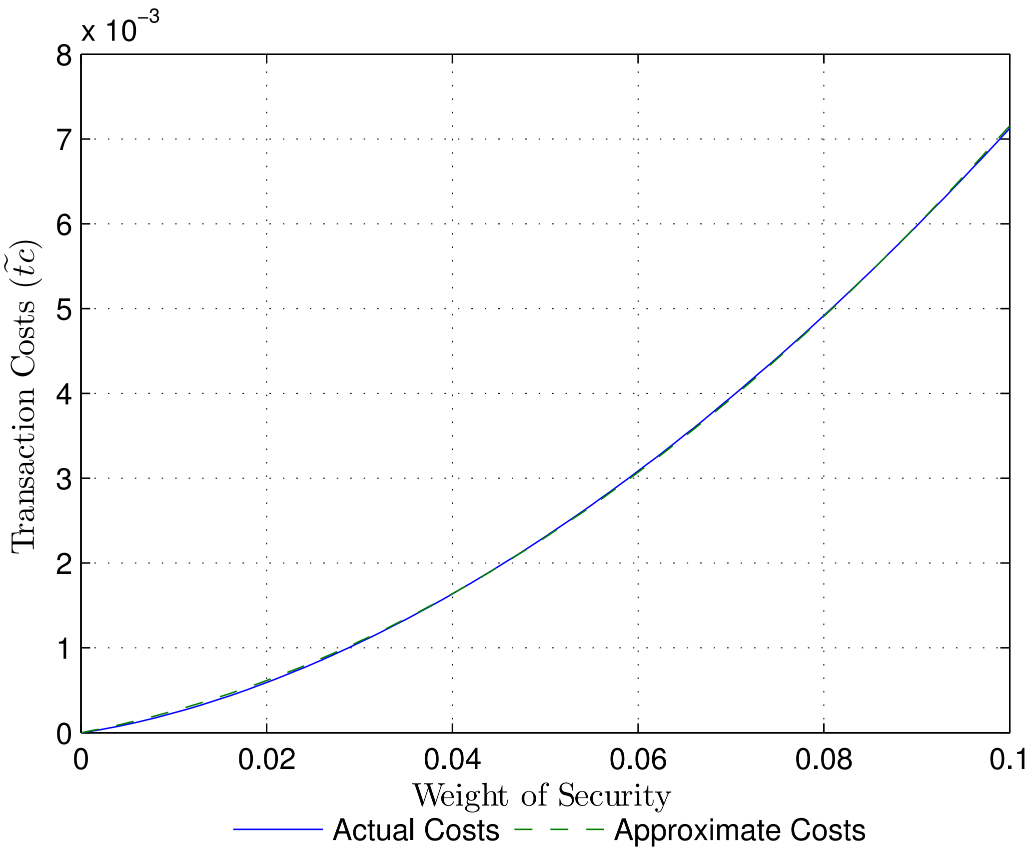

Figure 1: Actual Trading Costs with Approximate Trading Costs for AT&T. This figure shows the trading costs, $\widetilde{tc}$, using transaction cost model 1 for December 2013.

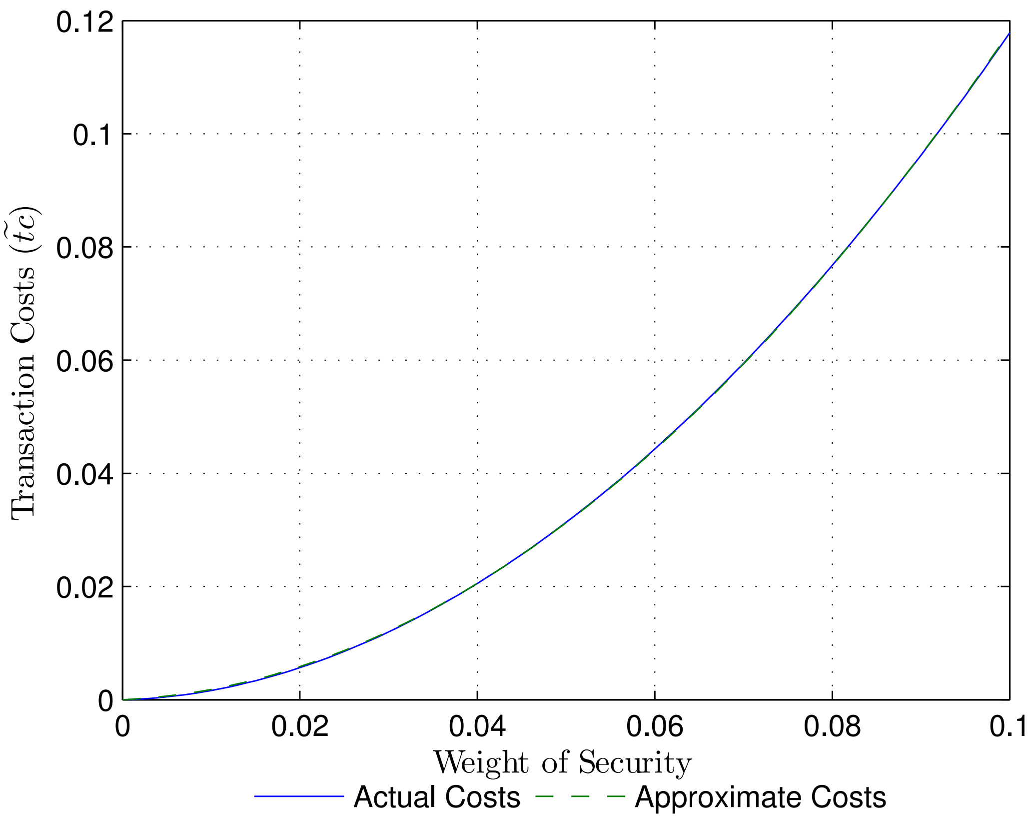

Figure 2: Actual Trading Costs with Approximate Trading Costs for AGL Resources. This figure shows the trading costs, $\widetilde{tc}$, using transaction cost model 1 for December 2013.

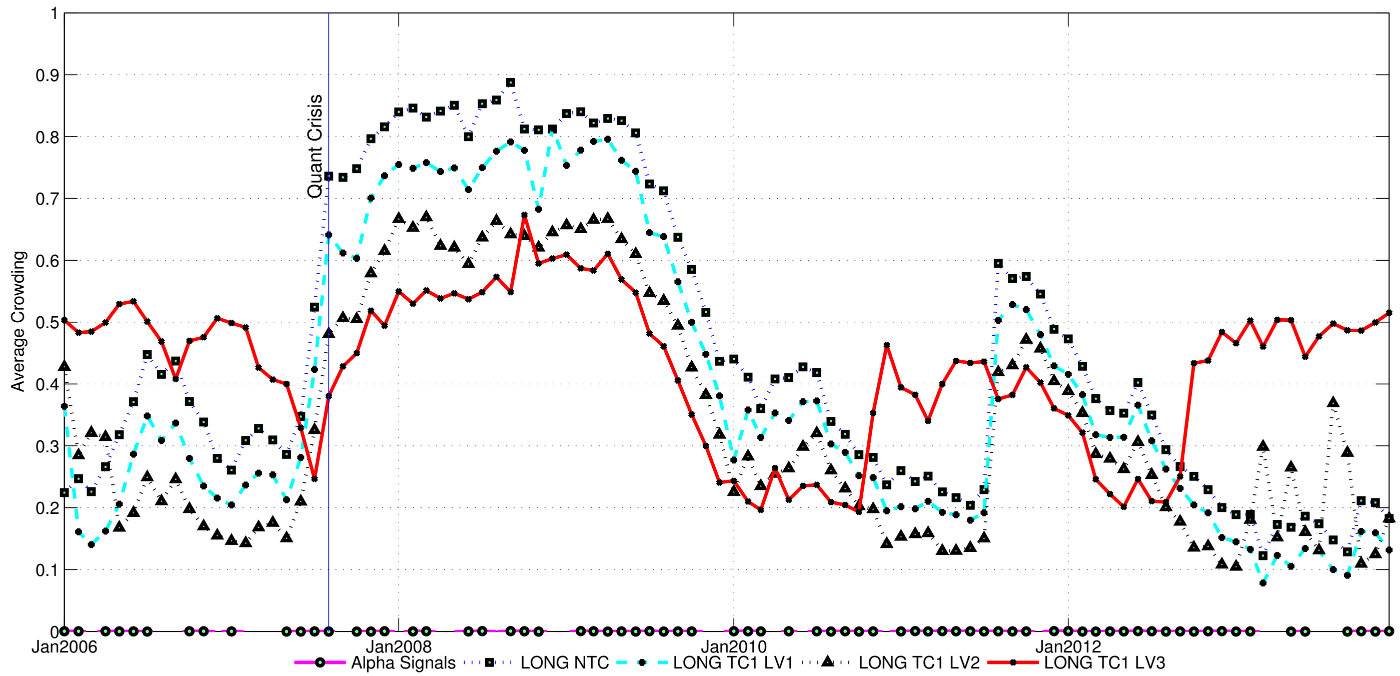

Figure 3: Crowding over Time for Random Alpha Models' Long Portfolios. This figure represents the crowding measures for long-only portfolios constructed every month from 2006 to 2013 using three industry risk models. At every point in time, actual data was used to construct 100 random alpha signals and consequently 100 optimized portfolios. The vertical line represent that starting month of the quantitative crisis of 2007. Two transaction cost models are considered, model 1 (TC1) and model 2 (TC2). These graphs only show the crowding numbers for portfolios created with model 1. The transaction costs include spreads and market impact of the following form: $tc_{it} = \left|\frac{100 s_{it}/2}{p_{it}}\right| + |c_{it}|$, where s_{it} is the bid-ask spread of stock i at time t. For model 1, $c_{it} = \frac{I}{2} + \mathrm{sgn}(n_{it})\eta\sigma_{it}\left|\frac{n_{it}}{V_{it}T}\right|^{3/5}$, where $I = \gamma\sigma_{it}\frac{n_{it}}{V_{it}}\left(\frac{N_{it}}{V_{it}}\right)^{1/4}$, $\gamma = 0.314$, $\eta = 0.142$, σ_{it} is the daily volatility of stock return i at the beginning of month t, N_{it} is the total amount of shares outstanding in the security, V_{it} is the average daily trading volume of the stock, T is the time interval in which the trade takes place in number of days, for this paper we use $T = 1$, and n_{it} represents the number of shares of the security the portfolio is trading. LONG NTC is the monthly crowding for long-only portfolios that do not consider transaction costs, LONG TC1 LV1, LV2, and LV3 are long portfolios that consider transaction costs with portfolios of size \$500M, \$5B, and \$20B respectively.

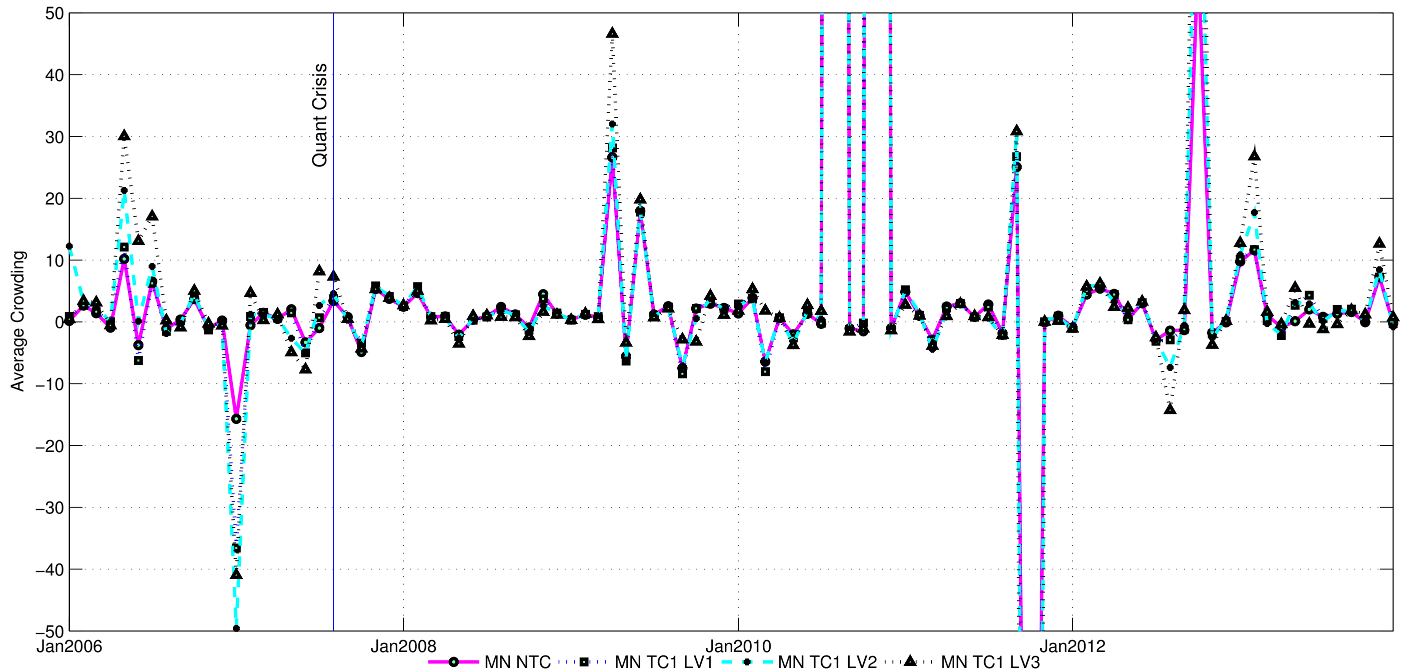

Figure 4: Crowding over Time for Random Alpha Models' Market Neutral Portfolios. This figure represents the crowding measures for long-only portfolios constructed every month from 2006 to 2013 using three industry risk models. At every point in time, actual data was used to construct 100 random alpha signals and consequently 100 optimized portfolios. The vertical line represent that starting month of the quantitative crisis of 2007. Two transaction cost models are considered, model 1 (TC1) and model 2 (TC2). These graphs only show the crowding numbers for portfolios created with model 1. The transaction costs include spreads and market impact of the following form: $tc_{it} = \left|\frac{100 s_{it}/2}{p_{it}}\right| + |c_{it}|$, where s_{it} is the bid-ask spread of stock i at time t. For model 1, $c_{it} = \frac{I}{2} + \text{sgn}(n_{it})\eta\sigma_{it}\left|\frac{n_{it}}{V_{it}T}\right|^{3/5}$, where $I = \gamma\sigma_{it}\frac{n_{it}}{V_{it}}\left(\frac{N_{it}}{V_{it}}\right)^{1/4}$, $\gamma = 0.314$, $\eta = 0.142$, σ_{it} is the daily volatility of stock return i at the beginning of month t, N_{it} is the total amount of shares outstanding in the security, V_{it} is the average daily trading volume of the stock, T is the time interval in which the trade takes place in number of days, for this paper we use $T = 1$, and n_{it} represents the number of shares of the security the portfolio is trading. LONG NTC is the monthly crowding for long-only portfolios that do not consider transaction costs, LONG TC1 LV1, LV2, and LV3 are long portfolios that consider transaction costs with portfolios of size \$500M, \$5B, and \$20B respectively.

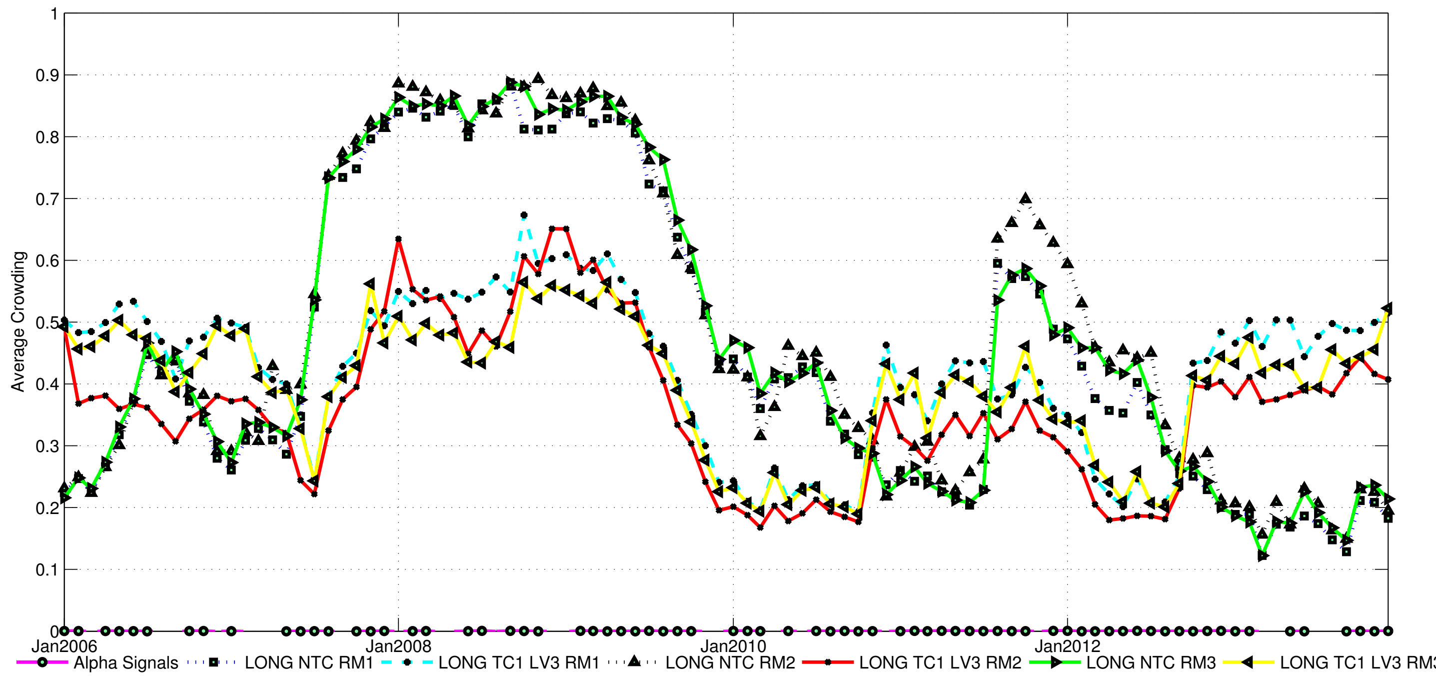

Figure 5: Crowding over Time for Random Alpha Models' Long and Market Neutral Portfolios using All Risk Models. This figure represents the crowding measures for long-only portfolios constructed every month from 2006 to 2013 using three industry risk models. At every point in time, actual data was used to construct 100 random alpha signals and consequently 100 optimized portfolios. The vertical line represent that starting month of the quantitative crisis of 2007. Two transaction cost models are considered, model 1 (TC1) and model 2 (TC2). These graphs only show the crowding numbers for portfolios created with model 1. The transaction costs include spreads and market impact of the following form: $tc_{it} = \left|\frac{100 s_{it}/2}{p_{it}}\right| + |c_{it}|$, where s_{it} is the bid-ask spread of stock i at time t. For model 1, $c_{it} = \frac{I}{2} + \mathrm{sgn}(n_{it})\eta\sigma_{it}\left|\frac{n_{it}}{V_{it}T}\right|^{3/5}$, where $I = \gamma\sigma_{it}\frac{n_{it}}{V_{it}}\left(\frac{N_{it}}{V_{it}}\right)^{1/4}$, $\gamma = 0.314$, $\eta = 0.142$, σ_{it} is the daily volatility of stock return i at the beginning of month t, N_{it} is the total amount of shares outstanding in the security, V_{it} is the average daily trading volume of the stock, T is the time interval in which the trade takes place in number of days, for this paper we use $T = 1$, and n_{it} represents the number of shares of the security the portfolio is trading. LONG NTC RM1, RM2, and RM3 represent the monthly crowding for long-only portfolios that do not consider transaction costs using risk model 1, risk model 2 and risk model 3 respectively. LONG TC1 LV3 RM1, RM2, and RM3 are long portfolios that consider transaction costs with portfolios of size \$20B using risk model 1, risk model 2 and risk model 3 respectively.

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