Market Depth and Order Size

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Abstract

In this paper we measure market depth by investigating the relation between net order flow and price changes. Two aspects are our main focus. Is the relation linear? Is the relation different for positive and negative net order flow? Answers to these questions are important for the design of market liquidity studies and for optimal trading.

We use intraday data on German index futures. Our analysis based on a neural network model provides us with two main results. First, the relation between net order flow and price changes is strongly non-linear. Large orders lead to relatively small price changes whereas small orders lead to relatively large price changes. We provide an example which shows that the optimal trading strategy of informed investors depends crucially on whether the price impact is linear or not. Second, we find that buyer initiated trades lead to a smaller price change than seller initiated trades of the same size. This finding contradicts the assumption of a symmetric price impact of buy and seller orders which is commonly used in theoretical models.

Overall, the results of our paper suggest that the assumption of a linear and symmetric impact of orders on prices is highly questionable. Thus, market depth cannot be described sufficiently by a single number. Therefore, empirical studies comparing liquidity of markets should be based on the whole price function instead of a simple ratio. A promising avenue of further theoretical research might be to allow the price impact per unit to depend on the trade volume. This should lead to quite different trading strategies as in traditional models.

JEL Classification: C45, G10

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1. Introduction

„Investors want three things from the markets: liquidity, liquidity, and liquidity.“ [Handa and Schwartz (1996)] In general, a market for an asset is liquid if the asset can be converted into cash easily [Hasbrouck and Schwartz (1988)]. This allows investors to adjust their asset portfolios quickly and at a low cost.

An intuitive measure of market liquidity is to divide total trading volume within an interval by the absolute price change.[1] According to Bernstein (1987), such a liquidity ratio is the most popular liquidity measure used in the marketplace.[2] It quantifies the trade volume required to move the price by one unit and is intended to characterize the depth of a market.[3]

The liquidity ratio as a measure of market depth suffers from several limitations.[4] First, a market should be termed deep if it is able to absorb order imbalances without large price changes. Buyer initiated trades should lead to only a small price increase and seller initiated trades to a small price decrease. Consequently, a measure of market depth has to be based on signed trade volume (buys minus sells), but not on aggregate trading volume (buys plus sells) during an interval. The liquidity ratio, however, is based on aggregate trading volume. A second major limitation of the liquidity ratio is its inability to distinguish between different sources of price changes. Price changes can result from new public information about the fundamental value of an asset or

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* We are indebted to Dirk Schiereck for helpful comments.

1 This measure is used, for example, by Cooper, Groth, and Avery (1985).

2 Alternative measures of market liquidity are discussed, for example, in Baker (1996).

3 See, for example, Hasbrouck and Schwartz (1988) for the dimensions depth, breath, and resiliency of market liquidity.

from order imbalances. In the former case, there may be quote adjustments even if trading does not actually take place. Price changes due to quote adjustments do not provide an insight into the depth of a market. However, the liquidity ratio relates total price change to volume, but should exclusively be based on price changes due to order imbalances. The final limitation is particularly important to this paper. The liquidity ratio is meaningful only when the relation between trading volume and absolute price change is linear. Only in this case the relation between trading volume and price change can be characterized by a single number, a ratio. However, such linearity is questioned by empirical findings of Marsh and Rock (1986), Gallant, Rossi, and Tauchen (1992, 1993), and Plexus Group (1996). For all the reasons mentioned, the liquidity ratio is not an adequate measure of market depth.

In this paper, we measure market depth in a new way that overcomes the limitations of the liquidity ratio. First, our study is based on net order flow instead of trade volume. Second, we distinguish between price changes due to order flow and price changes due to other reasons such as the release of public information. Third, our approach allows for non-linearity between net order flow and price changes, i.e. it is valid even if the relation between order flow and price change is non-linear.

The literature dealing with market depth is typically concerned with comparing the liquidity of different markets [see, for example, Dubofsky and Groth (1984), Cooper, Groth, and Avery (1985), Marsh and Rock (1986), Pirrong (1996), and Plexus Group (1996)]. Although our approach could be used for such a comparison, we do not contribute to this line of research. Instead, we focus on more basic, yet unanswered questions. How can the depth of a market be characterized ? Does market depth change with order size ? Is market depth identical for buy and sell orders of the same size ?

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[4] See, for example, Marsh and Rock (1986), Bernstein (1987), Grossman and Miller (1988).

Answers to these questions are important for the design of studies which compare the liquidity of different markets. Can market liquidity be characterized by a simple liquidity ratio or do we need more complex measures? If market depth depends on the order size, it is not obvious which market is deeper. It might be the case that one market is deeper than another for small trades, but not for large trades. In this case, an investor (and a researcher as well) has to judge market depth with respect to a given trade volume. In addition, one market may be deeper than another market for buy orders, but not for sell orders. If this is case, then it is optimal to buy a dually listed asset in one and sell it in another market. Answers to our fundamental questions are even important for investors who are restricted to a single market. Suppose that the price change increases at a rate less than proportional to the net order flow. In this case it may pay for investors to trade large blocks instead of splitting their orders. Thus, the optimal order size of an investor depends on the structure of market depth.

The remainder of the paper is organized as follows. Section 2 describes our approach to measure market depth and states the hypotheses to be tested. As a tool for measuring market depth, we use neural networks. They are introduced in Section 3. In Section 4 a description of the data is given. The empirical results are provided in Section 5. Implications of our results are illustrated by an example in Section 6. Section 7 concludes.

2. Measuring Market Depth

A market is deep if it absorbs large quantities without large price effects.[5] Price changes may result from information and from noise trading. The part of the information which is publicly released leads to quote adjustments but not to trading. The other part of information is known only to some traders. These traders take advantage of this information by trading. Price changes due to quote adjustments give

[5] See, for example, Kyle (1985).

no information on market liquidity, whereas price changes associated with trading do. In such a setting, price changes can be attribute either to the release of public information release (without trading) or to trading. This is formalized in model (1)

$$(1) \qquad \Delta p_t = \mu + \lambda [B_t - S_t] + \varepsilon_t ,$$

where Δp_t is the price change between t-1 and t. The constant μ and the stochastic, zero mean disturbance term ε_t capture the price effect of new public information which is not connected with trading. $[B_t - S_t]$ is the net order flow [buyer initiated trade volume minus seller initiated trade volume] over the time interval. The depth of a market is measured by the parameter λ. One would expect λ to be positive.

For example, the study of Ferguson, Mann, and Schneck (1995) applies model (1) to measure market depth. This approach has two advantages in comparison with the simple liquidity ratio. First, it distinguishes between price changes due to order flow and price changes due to other events. Second, it is based on net order flow instead of trading volume. However, one limitation of the liquidity ratio remains in model (1). It assumes a linear relation between net order flow and price change. The assumption of a linear relation between net order flow and price change is questioned by empirical results. Marsh and Rock (1986) and Plexus Group (1996) analyze price changes for different order sizes. They report that the relation between trade size and price change is non-linear. Gallant, Rossi, and Tauchen (1992, 1993) find a non-linear relation between (aggregate) trading volume and absolute price change. Hiemstra and Jones (1994) report a non-linear lead-lag relation between (aggregate) trading volume and price change. Based on these findings, we generalize approach (1). We allow the market liquidity parameter λ to be a (yet unspecified) function of the net order flow:

$$(2) \qquad \Delta p_t = \mu + \lambda_{(B-S)} [B_t - S_t] + \varepsilon_t$$

We estimate the function $\lambda_{(B-S)}$ from our data using a neural network. This allows us to test the following hypotheses: (i) Market depth does not depend on order size. (ii) Market depth is identical for net demand and net supply of the same size. (iii) Net demand leads to a price increase. (iv) Net supply leads to a price decrease. The results of the tests give us the information necessary to answer the fundamental questions stated in Section 1.

3. Neural Network Models

As a starting point, it is instructive to look at the hypotheses (i) to (iv) in the context of a linear model. Let X_t be the net order flow

(3) $$X_t \equiv [B_t - S_t].$$

Then our model (1) can be rewritten as

(4) $$\Delta p_t = \alpha_0 + \alpha_1 X_t + \varepsilon_t \ ,$$

where $\alpha_0 = \mu$ and $\alpha_1 = \lambda$. The model allows for a joint test of the hypotheses (iii) and (iv). To test the symmetry hypotheses (ii) we allow for different slope coefficients for positive and negative order flow. Therefore, we define X_t^+ as

(5) $$X_t^+ \equiv \begin{cases} [B_t - S_t], & \text{when } [B_t - S_t] > 0 \\ 0, & \text{when } [B_t - S_t] \leq 0 \end{cases},$$

i.e. X_t^+ equals the net order flow whenever it is positive and takes a value of zero otherwise. With these definitions the following model of the relationship between price change and net order flow is specified:

(6) $$\Delta p_t = \alpha_0 + \alpha_1 X_t + \alpha_2 X_t^+ + \varepsilon_t \ .$$

In the framework of model (6), the hypotheses (ii) to (iv) can be formulated as the following parameter restrictions. If $\alpha_2 = 0$, then market depth is identical for net demand and net supply of the same size (hypothesis ii). If $\alpha_1 + \alpha_2 > 0$, then net demand leads to a price increase (hypothesis iii). If $\alpha_1 > 0$, then net supply leads to a price decrease (hypothesis iv). A crucial limitation of model (6) is the assumption of a (piecewise) linear relationship between net order flow and price change. Consequently, model (6) does not allow for testing the linearity of the relationship. What is needed instead is a more flexible functional form, which encompasses the linear model. Furthermore, this enriched model should still allow us to formulate and test the hypotheses (i) to (iv) statistically.

Neural networks are ideally suited for this purpose. They can approximate virtually any (measurable) function up to an arbitrary degree of accuracy, as was shown in Hornik, Stinchcombe, and White (1989). While neural networks allow for the same functional flexibility as nonparametric kernel estimators, they are still based on parameters. This facilitates the statistical analysis of the resulting regression function, as parameter inference can be applied.[6]

The term neural network is not uniquely defined. It comprises a variety of different network types and models. We restrict our attention to single layer perceptron networks. A linear regression model can easily be nested in such a network. For our application this leads to the following specification:

(7) $$\Delta p_t = \alpha_0 + \alpha_1 X + \alpha_2 X^+ + \sum_{h=1}^{H} \beta_h \cdot g(\gamma_{1h} X + \gamma_{2h} X^+) + \varepsilon_t \ .$$

[6] Statistical inference in neural network models was developed by White (1989a,b).

Model (7) encompasses model (6) and adds a non-linear network part. The market depth function $\lambda_{(B-S)}$ resulting from model (7) is given as $\alpha_1 + \frac{\alpha_2 X^+ + \sum_{h=1}^{H} \beta_h \cdot g(\gamma_{1h} X + \gamma_{2h} X^+)}{X}$.[7] The network part of (7) consists of H hidden units, where $\beta_1,\ldots,\beta_H,\gamma_{11},\ldots,\gamma_{1H}$, and $\gamma_{21},\ldots,\gamma_{2H}$ are unknown parameters and g is a non-linear transfer function. The number H of hidden units is as yet unspecified. In general, the more complex the relationship between order flow and price change, then the more hidden units will be needed to approximate it adequately. Most commonly, the transfer function g is chosen to be either the logistic or the hyperbolic tangent function. The latter is used here. As $\tanh(-x) = -\tanh(x)$ and $\tanh(0) = 0$, this choice simplifies the analysis of the model. In the framework of model (7) the hypotheses (i) to (iv) can be examined as follows:

(i) If $\beta_1 = \beta_2 = \ldots = \beta_H = 0$, then price changes depend linearly on order flow.[8]

(ii) If $\alpha_2 = 0$ and $\gamma_{2h} = 0, \forall\, h = 1,\ldots,H$, then market depth is identical for net demand and net supply of the same size.[9]

(iii) If $\alpha_1 + \alpha_2 > 0$ and $\beta_h \cdot (\gamma_{h1} + \gamma_{h2}) > 0, \forall\, h = 1,\ldots,H$, then net demand leads to a price increase, whose magnitude strictly increases with net demand.[10]

(iv) If $\alpha_1 > 0$ and $\beta_h \cdot \gamma_{h1} > 0, \forall\, h = 1,\ldots,H$, then net supply leads to a price decrease, whose magnitude strictly increases with net supply.

We test the linear model against the alternative of a non-linear neural network (hypothesis i) using tests suggested by White (1989c) and Teräsvirta, Lin and Granger

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7 The relation between $B-S$, X, and X^+ is given in equations (3) and (5).
8 The linear model (5) allows for different slope coefficients for positive and negative order flow.
9 This follows from the symmetry of the hyperbolic tangent function around the origin.
10 This follows from the fact that the hyperbolic tangent is a strictly increasing function.

(1993). The first method was compared to other linearity tests in the study of Lee, White, and Granger (1993) and was found to be very powerful against a variety of non-linear alternatives. The second test performed even better than the first one in the simulation study of Teräsvirta, Lin, and Granger (1993).

Once the number of hidden units is specified by the above tests, all the β-parameters included in the model are different from zero. Then hypotheses (ii) to (iv) can be tested by standard Wald-tests as proposed by White (1989b).

4. Data

Our empirical study is based on data on German Stock Index futures (DAX futures) from September 1993 to September 1994. DAX futures are screen traded on the fully computerized German Futures and Options Exchange (DTB) between 9.30 a.m. and 4.00 p.m.. Liquidity is provided by traders and voluntary market makers who place limit orders into the centralized electronic orderbook. This orderbook is open to all market participants. All orders (market and limit orders) are submitted electronically to the market via a trading terminal where orders are automatically matched, based on price and time priority.[11]

Our data set consists of all time stamped best bid quotes, best ask quotes, transaction prices, and transaction quantities for DAX futures contracts nearest to deliver. There is no information in the data set on whether a trade is initiated by a buyer or a seller. However, this information is needed to estimate equation (7). Therefore, we classify trades as buyer or seller initiated using an algorithm similar to Lee and Ready (1991). A trade is classified as buyer-initiated if the transaction price is equal to or higher than the current best ask price. If the transaction price is equal to or lower than the current best bid price, the trade is classified as seller-initiated. Prearranged trades can take

[11] See for example Bühler and Kempf (1995) for a more detailed description of DAX futures.

place inside the quote. These trades are eliminated from our sample since they do not provide information concerning market depth. The classification procedure leads to a time series which includes transaction prices, transaction sizes and information on whether the trade is buyer or seller initiated.

In our study 5-minute intervals are used.[12] We calculate for each interval the net order flow, $B-S$, as the difference between the buyer and seller initiated trading volume. Trading volume is measured as the number of futures contracts traded. The logarithmic price change during each interval, Δp, is calculated from the last transaction price of the previous interval and the current interval. To avoid possible biases at the beginning of a trading session, observations within the first 15 minutes after the opening of the DTB are excluded. This procedure leaves us with 18969 observations. Descriptive statistics concerning price changes and net order flow are provided in Table 1.

Table 1: Descriptive Statistics of Logarithmic Price Changes and Net Order Flow.

	Logarithmic Price Changes Δp in Percentage Points	**Net Order Flow (B - S)**
Mean	-0.00025	0.28
Standarddeviation	0.101	97.57
Minimum	-0.748	-1811
1%-Quantile	-0.272	-262
25%-Quantile	-0.050	-46
50%-Quantile	-0.0	1
75%-Quantile	0.052	47
99%-Quantile	0.257	266
Maximum	1.195	1816

[12] We tested the sensitivity of our results with respect to the length of the time interval. In addition to 5-minute intervals, we used 15-minute, 30-minute, and 60-minute intervals. Our results are insensitive to interval variation.

For both, price changes and net order flow, the mean values are very small and not significantly different from zero. As can be seen from the quantiles, price changes and net order flow are almost symmetric around zero. There are a few extremely large absolute values of net order flow. One does not obtain reliable and economically meaningful results from a neural network model for regions with hardly any data points. Thus, we restrict the investigation to a net order flow in the range from -300 to 300. This excludes about one percent of all values.[13] 18729 observations remain in our sample.

5. Results

We first estimate the relation between price changes and net order flow based on the linear approach (5). The results are provided in Table 2.

Table 2: Regression Results for the Linear Model (5).

	Estimated Parameters	Standard Errors	p-value
α_0	-0.000201	0.000518	0.699
α_1	0.000781**	0.000008	0.000
$\overline{R}^2 = 0.473$	Significantly different from zero at a 1% level: **, at a 5% level: * Estimation is carried out by Ordinary Least Squares. Standard errors are obtained by the heteroscedasticity consistent estimator of White (1980).		

The slope coefficient is significantly positive as expected. With an estimated value of about 0.0008 a net supply (demand) of 100 units will on average cause a price decrease (increase) of 0.08 percentage points. If one assumes a DAX futures level of 2100 index points, which is about the average index level for the time period under study, the corresponding decrease in the futures price is around 1.7 index points. An adjusted R-squared value of roughly 0.5 indicates that one half of the variation in the price changes can be attributed to net order flow, the other half to quote adjustments.

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[13] We also performed the analysis without excluding large net order flows. The results for the [-

We proceed by estimating the piecewise linear model (6) which allows to test the symmetry of the price function (hypothesis ii). The results are shown in Table 3.

Table 3: Regression Results for the Piecewise Linear Model (6).

	Estimated Parameters	Standard Errors	p-value
α_0	0.001810*	0.000760	0.017
α_1	0.000812**	0.000014	0.000
α_2	-0.000063**	0.000023	0.006
$\overline{R}^2 = 0.474$	Significantly different from zero at a 1% level: **, at a 5% level: * Estimation is carried out by Ordinary Least Squares. Standard errors are obtained by the heteroscedasticity consistent estimator of White (1980).		

The negative parameter α_2 indicates that market depth differs for buyer initiated and seller initiated trades. The futures market is deeper for market buy orders than for market sell orders. This difference results from the behavior of traders supplying liquidity to the market via limit orders. This finding may be based on the preference of investors to place limit sell orders in the futures market instead of in the underlying stock market. The preference could be based on short selling restrictions which exist in the stock market but not in the futures market. For limit buy orders such a preference should not exist.

The results provided thus far are based on the linear models. Since earlier studies question the assumption of linearity, we proceed by examining whether a non-linear relation between price changes and net order flow exists in our data. For this purpose, we test for neglected non-linearity using the neural network tests of White (1989c) and Teräsvirta, Lin, and Granger (1993). Both tests strongly reject the null hypothesis of a linear model against the alternative of some neglected hidden units. The corresponding test statistics are shown in Table 4. Thus our hypothesis (i) is rejected

300,+300] range are essentially the same.

and a non-linear model such as (7) describes the relationship between net order flow and price changes more adequately than model (6).

These test results still leave us with the question of how many hidden units should enter into the neural network. The number of hidden units, H, in equation (7) is chosen based on the Schwarz Information Criterion (SIC)[14]. The SIC takes its minimal value for a network with one hidden unit. Since the choice of an appropriate number of hidden units is important for the power of our subsequent tests, we validate the network by another criterion. The network model (7) with one hidden unit is estimated, and tests for additional non-linearity in the data were carried out. The tests of White (1989c) and Teräsvirta, Lin, and Granger (1993) do not detect any further non-linear structure, thus no additional hidden unit is necessary.

Once the number of hidden units is specified we can test hypotheses (ii) to (iv) formulated in Section 2. Table 4 summarizes the corresponding results: The relationship between net order flow and price change is non-linear (hypothesis i), non-symmetric (hypothesis ii)[15], strictly increasing with net demand (hypothesis iii), and strictly decreasing with net supply (hypothesis iv).

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14 See Schwarz (1978).

15 This holds only for a significance level of 5 percent, so the evidence on this point is weaker than for the other hypotheses.

Table 4: Test Results for the Hypotheses (i) to (iv).

Hypothesis (H_0)	Test-Statistic	Distribution under H_0	p-value
(i): $\beta_1 = \beta_2 = \ldots = \beta_H = 0$	(Teräsvirta et. al.), (White)[16] 123.10, 273.47	$\chi^2(2)$, $\chi^2(2)$	0.000, 0.000
(ii): $\alpha_2 = 0 \wedge \gamma_{12} = 0$	(Wald) 7.83	$\chi^2(2)$	0.020
(iii): $(\alpha_1 + \alpha_2) > 0$ $\wedge \beta_1 \cdot (\gamma_{11} + \gamma_{12}) > 0$	(Wald) 268.01	$\chi^2(2)$	0.000
(iv): $\alpha_1 > 0$ $\wedge \beta_1 \cdot \gamma_{11} > 0$	(Wald) 215.36	$\chi^2(2)$	0.000

Up to this point, we have no information on how the linear and non-linear parts of model (7) contribute to the overall results. Such information is provided by the estimated parameters shown in Table 5. The slope coefficient α_1 takes a smaller value than the one estimated in the linear models. However, the positive coefficients β_1 and γ_{11} indicate an additional positive, but non-linear price impact of net order flow. As α_2 remains significantly negative of almost the same size as in the linear approach, the asymmetry of model (6) is maintained. The non-linear part of the model has no impact on the asymmetry as γ_{12} is not significantly different from zero. Thus, the differences in the functional form for positive and negative net order flows can be attributed solely to the linear part.

Table 5: Regression Results for the Neural Network Model (7).

	Estimated Parameters	Standard Errors	p-value
α_0	0.001831	0.002186	0.402
α_1	0.000579**	0.000042	0.00
α_2	-0.000064*	0.000027	0.018
β_1	0.031251**	0.007608	0.000

[16] The test was performed using the first two principal components of the output of originally ten hidden transfer functions. These principal components account for more than 95% of the total variation.

γ_{11}	0.019301**	0.003640	0.000
γ_{12}	-0.000237	0.007680	0.975
$\overline{R}^2 = 0.483$	Significantly different from zero at a 1% level: **, at a 5% level: * Estimation is carried out by Non-linear Least Squares. Standard errors are obtained by the heteroscedasticity consistent estimator of White (1980).		

Figure 1 depicts the relationship between net order flow and price changes together with the fitted regression line. The non-linearity of the relation is apparent. A small net order flow leads to a relatively large absolute price change. A large net order flow leads to a relatively small absolute price change.

Figure 1: Relation between Net Order Flow and Logarithmic Price Change.

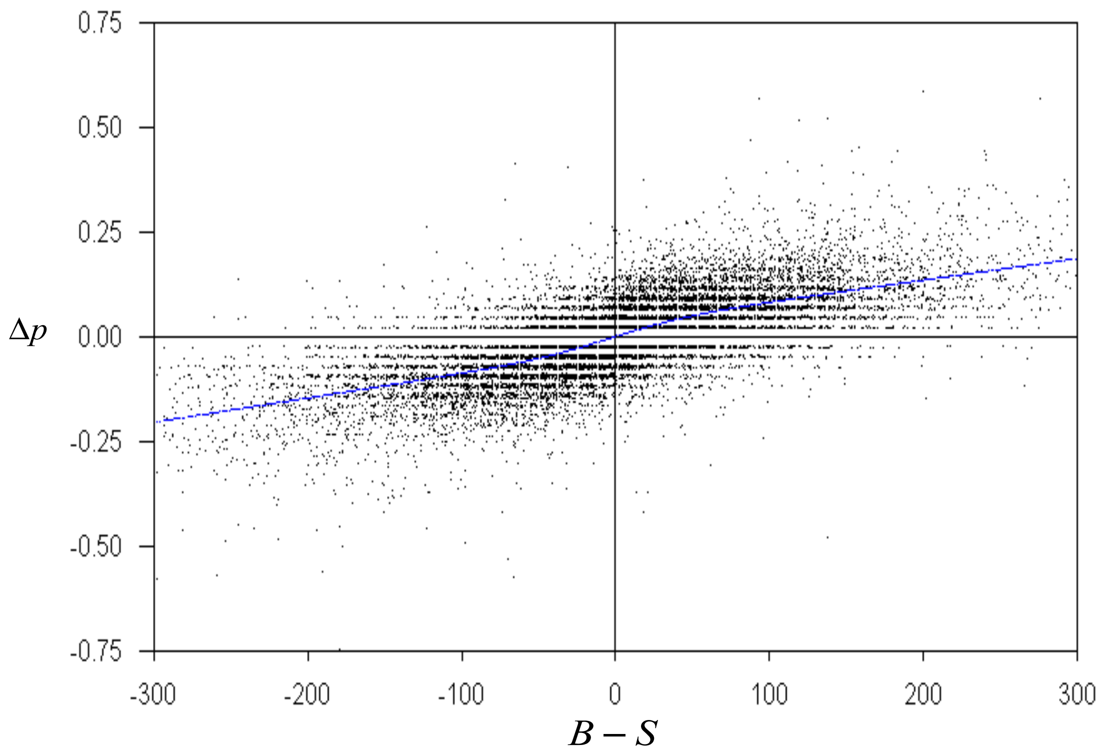
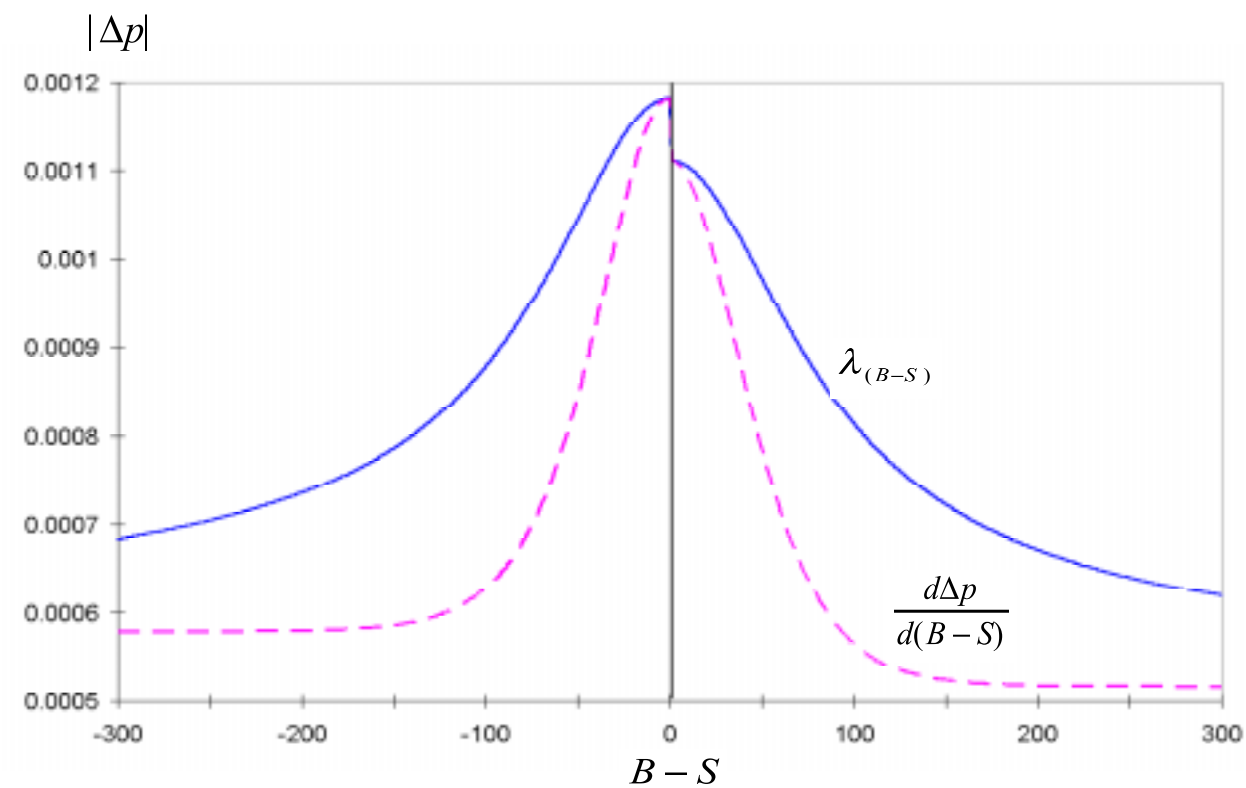


Figure 2 provides a further look at the non-linearity between net order flow and price changes. The dashed line shows the effect of a marginal change of net order flow on the absolute price change for different levels of order flow. It is derived from the estimated network model (7). It is only for large positive or negative values of $(B-S)$, that the price change increases with net order flow almost linearly. This result is consistent with Marsh and Rock (1986), who find an almost linear relationship between price change and trade size only for large trades. For absolute values of net order flow less then about 150 contracts the non-linear part of the model dominates. The price impact of an additional unit of net order flow can be twice as high for small orders than for large orders.

The solid line in Figure 2 shows the function $\lambda_{(B-S)}$. It gives the average absolute price change caused by one unit of net order flow for different order sizes. The average price impact decreases monotonically with order flow. This means that the price impact per trade unit is smaller, the larger the order size.

Figure 2: Effect of a Marginal Unit of Extra Net Order Flow on Price Changes and $\lambda_{(B-S)}$ Implied by the Estimated Model (7).



From our results, one important implication for market liquidity studies can be derived. Measures of market liquidity, which reduce market depth to a single number, may lead to erroneous conclusions. Instead, a meaningful comparison of market depth should be based on the whole function $\lambda_{(B-S)}$.

6. Example

In this section we give a simple example how the non-linearity of the price function may affect the trading strategy of an informed investor even if she is restricted to a single market. Suppose that an investor knows that prices change only in response to her trades. Furthermore, assume that she wants to buy m futures contracts and that the futures price is currently P_0. Finally assume that the price changes with net order flow according to a function $\Delta P(X)$ where $\Delta P \equiv P_t - P_{t-1}$.[17] The question is: What is her optimal market order size ? Is it better to split the demand into several orders or is it better to place one large order.

In the market setting of DAX futures (continuous trading), a large market order will be matched with several limit orders in the order book. The limit orders can differ with respect to their quantities and their limit prices. A large market order hitting the order book is not executed at a single price, but at the different limits. Given this market setting, we will analyze two different trading strategies of an informed investor: First, the investor submits one large order to buy m contracts. Second, she submits n orders consecutively where the size of each order is k contracts. Since the investor wants to buy a total amount of m contracts, $m = n \cdot k$ holds.

We first study the strategy of submitting one large order. In this case, the investor has to spend a total amount of money which equals $m \cdot P_0 + \sum_{i=1}^{m} \Delta P(i)$. The first term of the sum gives the amount the investor would have to pay if the market was perfectly liquid. The second term is due to the price impact of her trade. It reflects the cost of illiquidity when one large order for m contracts is placed.

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17 The function $\Delta P(X)$ should not be confused with the price function $\Delta p(X)$ shown in Figure 1, which measures logarithm price changes. However, both functions can be deduced from eachother easily.

Next, we analyze the strategy of order splitting. The investor places n orders with a size of k contracts each. We assume that the traders who supply market liquidity via limit orders recognize the market order as information induced, but do not anticipate further information based orders. If this is the case, then they will adjust their limit orders according to latest transaction price. Under these assumptions, the total amount of money to spend is $m \cdot P_0 + \sum_{j=1}^{n} \left[(j-1) \cdot \Delta P(k)k + \sum_{i=1}^{k} \Delta P(i) \right]$. The cost of illiquidity in this case is the sum over the bracket. The price change up to the end of trading round $j-1$ is given by $(j-1) \cdot \Delta P(k)k$. The price impact of trading round j is given by $\sum_{i=1}^{k} \Delta P(i)$.

The cost of illiquidity in the two strategies determine which strategy is better. If $\sum_{i=1}^{m} \Delta P(i) < \sum_{j=1}^{n} \left[(j-1) \cdot \Delta P(k)k + \sum_{i=1}^{k} \Delta P(i) \right]$ holds, then the investor should place one large order. If the inequality has the opposite direction, order splitting is better. Only if both strategies lead to identical costs of illiquidity, the order size does not matter.

It can be easily shown that the optimal trading strategy depends heavily on the form of the price function $\Delta P(X)$. It is optimal for the investor to place one large order whenever $\Delta P(X)$ is strictly concave. The investor should split his demand whenever $\Delta P(X)$ is strictly convex. The order size does not matter whenever the price function $\Delta P(X)$ is linear.

In our empirical study we found that the price function $\Delta p(X)$, which is based on logarithmic price changes, is strictly concave. Unfortunately, this does not assure, in general, that the price function $\Delta P(X)$ is also concave. Therefore, it is necessary to derive its properties from the estimated function $\Delta p(X)$. It can be shown that the price function $\Delta P(X)$ is concave up to an net order flow of 106 futures contracts.

This has the following implication for the DAX futures market. As long as an investor wants to buy up to 106 contracts, she should place one large order instead of splitting.

Our example aims to highlight trading implications of non-linear market depth. The results obtained were based on two specific assumptions. First, a large order is executed at several limit prices, but not at a single market clearing price. Second, the investors supplying liquidity recognize the trade as completely information based and do not expect a sequence of buy orders to follow. However, even if these assumptions are modified, similar examples can be constructed where the properties of the market depth function have a significant impact on the optimal trading strategy.

7. Conclusion

In this paper we measure market depth by investigating the relation between net order flow and price changes. Two aspects are our main focus. Is the relation linear ? Is the relation different for positive and negative net order flow? Answers to these questions are of particular importance for the design of market liquidity studies and for optimal trading.

We use intraday data on German index futures. Our analysis based on a neural network model provides us with two main results. First, the relation between net order flow and price changes is strongly non-linear. Large orders lead to relatively small price changes whereas small orders lead to relatively large price changes. We provide an example which shows that the optimal trading strategy of informed investors depends crucially on whether the price impact is linear or not. Second, we find that buyer initiated trades lead to a smaller price change than seller initiated trades of the same size. This finding contradicts the assumption of a symmetric price impact of buy and seller orders which is commonly used in theoretical models.

Overall, the results of our paper suggest that the assumption of a linear and symmetric impact of orders on prices is highly questionable. Thus, market depth cannot be described sufficiently by a single number. Therefore, empirical studies comparing liquidity of markets should be based on the whole price function instead of a simple ratio. A promising avenue of further theoretical research might be to allow the price impact per unit depend on the trade volume. This should lead to quite different trading strategies as in traditional models.

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