The old ways are (sometimes) the best: the performance of simple mean-variance portfolio optimization in various markets

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We study the performance of mean-variance optimized (MVO) equity portfolios for retail investors, in various markets in the U.S. and around the world. Actively managed equity mutual funds have relatively high fees and tend to underperform their benchmark. Index funds such as ETFs still charge appreciable fees, and only deliver the performance of the benchmark. We find that an MVO is relatively easy to manage by a retail investor, and that they tend to outperform their benchmark or, at worst, equal its performance, even after adjusting for risk. Moreover, we show that the performance of these funds is not particularly sensitive to the frequency at which they are rebalanced so that, in the limit, an investor might have to rebalance her portfolio only once per year. This last finding translates into very low trading costs, even for a retail investors. Thus, we conclude that MVOs offer an easy, cheap alternative for a retail investors to invest in the world's equity markets.

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1. Introduction

Since its introduction in 1952 by Harry Markowitz, the mean-variance criteria has become the most widely known form of portfolio selection. Due to the simplicity of its underlying theory, as well as its ease of computation, it is taught in every business school, both at undergraduate as well as graduate levels, and there is a large strand of the literature devoted to its analysis and improvement. However, modern investors rarely entrust their portfolio selection to this venerable methodology, preferring instead to invest either in actively managed portfolio, or index funds. In order to demonstrate the value of Mean Variance Optimization (MVO), we apply this methodology in its simplest form in various markets around the world and find that, at worst, the resulting portfolios obtain the same level of performance as their respective index benchmark and, at best, beat these benchmarks with long-term results that are statistically as well as economically significant.

Over the last few decades there has been a strong and constant trend in the decline of the value of equities directly held by households, and the surge of household investments in mutual funds and other managed investment vehicles. As of year-end of 2013, there are 8,974 open-ended mutual funds in the U.S. with combined assets of more than $ 15 trillion, which collectively own 29% of U.S. firms' equity, as well as 1,332 ETFs with assets of more than $ 1.6 trillion. Overall, the share of household financial assets held by investment companies (including mutual funds, ETFs, closed-end funds and UITs) has gone from 2% in 1980 to 22% in 2013[1], with similar trends observable in other countries, as well as in the market for pension funds.

With the average fee charged by U.S. actively managed mutual funds being close to 1% of assets under management, these funds collectively receive revenues of close to $ 43 billion from their investors. And yet, not only is there ample of evidence that active management underperforms its

[1] "2014 Investment Company Fact Book", Investment Company Institute.

benchmark (Jensen (1968), Malkiel (1995), Fama and French (2010), and others), the very mathematics of active portfolio management imply that, net of fees, these funds must inevitably trail their relevant passive benchmarks, at least on average. Although there is evidence that a small sample of fund managers may be able to outperform its benchmark (see, for example, Avramov and Wermers (2006), Kosowski et al (2006), Cuthbertson et al (2010)), the debate is still ongoing in the academic world, and every methodology that purports to identify these 'winners' is complicated and ultimately unreliable.

The case is similar in other countries where, again, actively managed mutual funds tend to underperform their respective benchmarks (see, for example, Białkowski & Otten (2011), Chan & Yamada (1997), Gallagher & Jarnecic (2004), Aggrawal (2007), and others), and investor fees can be even higher than those in the U.S. market.

Thus, an investor can choose between an expensive, underperforming actively managed fund, or a somewhat cheaper fund (mutual fund, ETF or closed-end fund) that tracks the market index.

We explore a third possibility: that a retail investor might be able to construct her own mean-variance portfolio using simple analytic tools and publicly available information, and maintain that portfolio by rebalancing at a frequency that maximizes risk-adjusted performance, while reducing trading costs. To test this premise, we obtain stock price data from 22 markets (3 U.S. indexes and 19 foreign ones), and conduct a back-test of MVO portfolio optimization over a period of 10 years. We find that, on average, this methodology is superior to indexing, which implies it also outperforms the locally available actively managed funds. In fact, from 2005 to 2014 there is only one year in which our MVO portfolios trail the market index, and that is in 2008, during the worst period of the recent financial crisis. On average, the MVO portfolios outperform their benchmarks by 5.8% per year over the 10 year period, when rebalancing at

monthly intervals. However, even if portfolio rebalancing is carried out once a year, the average outperformance is still 3%. If, in addition, we consider the difference in trading costs between a fund that is rebalanced annually versus an index fund that is rebalanced more frequently, then we can see that we are easily striking down the main arguments against retail investors managing their own funds: performance and costs. The final argument, risk, is also weakened by the fact that, with a few exceptions, our country-index MVO portfolios tend to have the same standard deviation of returns than that of their respective index.

We explore some potential drawbacks to the application of MVO optimization by retail investors. The main one being that in some cases the resulting portfolios can contain very few assets, which runs counter to the goal of proper diversification. While this does not necessarily imply that these portfolios are far riskier than their benchmarks, we add analyses of portfolios generated adding constraints to the MVO problem. Although performance is somewhat reduced by these constrained solutions, our results remain qualitatively unchanged.

The rest of this paper is organized as follows. Section 2 presents the MVO methodology and the dataset used. Section 3 shows our main results on the performance of MVOs. Section 4 presents results for special cases, and Section 5 concludes.

2. Methodology and Data

The Mean-Variance Optimization (MVO) methodology is based in the maximization of the Sharpe ratio, which is the expected return of the portfolio divided by its variance. In other words, MVO attempts to obtain the highest possible return at the lowest possible risk. There are a number of variations on this methodology that claim to achieve higher levels of risk-adjusted performance, such as the use of Value-at-Risk (VaR) and other indicators of potential loss as optimization restrictions (CVaR, Drawdown, etc.), as well as maximizing risk-reward ratios other than Sharpe, such as the Sortino and Omega ratios (see, for example, Rockafellar & Uryasev (2000), Chekhlov, Uryasev, & Zabarankin (2004), Konno, & Yamazaki (1991) and others). We pursue the standard Markowitz approach to mean-variance optimization due to its simplicity and therefore its potential appeal to a large number of investors.

Let w be a vector of portfolio weights, r a vector of expected asset returns, and Ω an estimator of the variance-covariance matrix of these assets, then a Mean-Variance Optimized portfolio is the one formed by solving for w so that

$$max \frac{wr}{w\Omega w}$$

$$s.t. \ \sum w_i = 1 \ ; w \geq 0$$

Notice that in this case we are not allowing short sales (portfolio weights must all be positive or zero). This assumptions is consistent with retail investor habits.

In order to estimate expected returns and the covariance matrix we use historical stock returns data. Specifically, we use 5 years of monthly returns.

Finally, we test four portfolio rebalancing frequencies. We rebalance portfolios at annual, semiannual, quarterly and monthly intervals. For each rebalancing frequency and period, we use 5 years of past stock data and eliminate stocks from the respective index that do not have a full

set of monthly returns for that period. We then obtain the MVO portfolio weights, and proceed to simulate holding that portfolio until the next rebalancing date, when we repeat the process.

For each index from which stocks are sampled to be included in the optimized portfolio, this methodology generates 10 portfolios when we apply yearly rebalancing, 20 for semiannual, and so on, so that our back test always spans the same 10 year period.

We obtain daily stock and index price from Bloomberg. Data is obtained for 32 countries. However, for some countries, such as Cyprus, Egypt and Morocco, the dataset do not contain the minimum amount of data (15 years) to be included in this study. For others, like Panama, though the time series extends far enough, their markets contain too few stocks that actively trade, which invalidates the methodology and precludes any meaningful computation. Thus, we are left with 22 countries/indexes from which we compute optimized portfolios.

Table I shows descriptive statistics of these markets. The first 3 on the table are U.S. indexes, while the remaining 19 are foreign indexes. We tabulate the number of stocks listed in each index at the end of 2014, a measure of liquidity of each index obtained as the percentage of days in which all listed stocks show prices (i.e.: trade) during the last year of our sample, the mean price of all listed stocks in the local currency, and a measure of past index volatility calculated as the variance of index return over the past year and the past 5 years.

[TABLE I AROUND HERE]

By virtue of being included in a representative index, these stocks should have high levels of liquidity, relative to other stocks in the same market not included in the index (when the index does not include all market listed stocks). This is true in almost all cases, where our liquidity

measure is above 90%, and most of the time close to 100%. The notable exception is Indonesia, with a liquidity measure of only 73.66%. At the same time, we later show that Indonesia is one of the MVO portfolios which most improve performance when the rebalancing frequency is increased. Thus, we could argue that this performance improvement can be dubious, as trading costs might rise more than in other markets.

One year volatility varies between indexes from a low of 1.75% for Indonesia to a maximum of 7.77% for Greece. However, all one year volatilities show a reduction when compared to the five year volatility measure. This is logical, as the five year measure includes data starting in 2010, when the effects of the recent financial crisis were even more pronouncedly felt than they are now.

Ultimately, we find no relationship between MVO portfolio performance and any of the market variables presented above.

3. Performance of MVO portfolios

We employ the methodology and data described in the previous chapter to generate MVO portfolios from the stocks listed in each index in Table I, and then held until the next rebalancing period. In this chapter we present our main results as regards to the performance of these portfolios.

Table II contains our main performance measures. For each MVO portfolio we compute the annualized mean monthly return, standard deviation and Sharpe ratio, and we compare them with the same statistics for their respective index. Panel A of the table shows the results obtained when rebalancing portfolios at annual frequency, while panels B, C and D contain the performance statistics for portfolios rebalanced at semiannual, quarterly and monthly frequencies, respectively.

[TABLE II about here]

The first important observation is that most MVO portfolios obtain a higher level of return than their respective index, and in many cases do so without noticeable increases in their volatility. In fact, some MVO portfolios obtain Sharpe ratios that more than double that of their index, as is the case for the Dow Jones, the S&P 100, China and Portugal. One remarkable example, given the ongoing situation in that country, is that of the Greek portfolio, which attains a positive return, while its index averages negative returns, and even does so at a lower risk than that of its index. Additionally, even in cases when the portfolios trails the index in terms of performance, for the most part that difference is small. On such case is that of the Poland portfolio, which invariably trails its index by at least 50% in terms of annualized monthly returns. While this

difference appears large, we see later that it is not, in fact, statistically significant. Moreover, we can also ascertain that the outperformance of the MVO portfolios is very stable. As we can see in the time series plots in Figure I, the equal-weighted mean[2] annual MVO excess return is positive in almost every year, with the exception of 2008, the worst year of the recent financial crisis[3].

Second, since increasing the rebalancing frequency means that the information used to generate the portfolios is updated more frequently, we would expect that the performance of high rebalancing frequency portfolios would be higher than that of those rebalanced at lower frequencies. However, there does not seem to be any discernible pattern in the relationship between a portfolios rebalancing frequency and its risk or return. This is important since, while we do not perform a strict analysis of trading costs in this study, lower rebalancing frequency mechanically translates into lower execution costs. Thus, if the MVO portfolios can maintain their performance at relatively low rebalancing frequencies, then low trading costs can be added to their virtues.

Finally, the Market Efficiency Hypothesis would dictate that, if there is indeed an excess return to be had from these MVO portfolios, then investors should invest using this technique until market prices return to equilibrium and it is no longer possible to obtain an excess risk-adjusted return. Following the tenets of the MEH, we would expect to see a negative relationship between the level of competitiveness of the market (as proxied by size) and the outperformance of MVOs. Since the U.S. is the largest and most competitive of the markets in our sample, we would also assume that the MVOs based on the U.S. indexes should be the ones with the lowest performance. However, the MVOs based on two of the three U.S. indexes, the Dow Jones and the S&P 100, fare amongst the best performing of the group. Moreover, while the third MVO,

[2] We purposefully avoid a value-weighted mean, as it would be completely dominated by the U.S. indexes.

[3] A small dip is also observed for 2014, but we find that this purported underperformance is not statistically significant.

based on the stocks listed in the NASDAQ100 index, trails its index's performance, this difference is not statistically significant (see below).

In Table III we summarize the results of statistical tests performed on the portfolios' returns data. Specifically, we obtain a market model alpha by regressing the returns of the MVO on those of its benchmark index, and we also do a t-test of the difference between the monthly returns of the MVO and those of the benchmark. The table includes both, the point estimate of the test, as well as its respective t-statistic.

[TABLE III AROUND HERE]

As we can see in Panels A to D, independent of the rebalancing frequency the results remain very similar. There are between 6 and 7 MVOs with positive and significant factor model alphas and, although some alphas are negative, indicating underperformance, none of these are statistically significant. Moreover, these alphas are economically significant, showing outperformance of the respective MVO with respect to its benchmark of between 1 and 2% per month, over a period of 10 years.

The t-test results are very similar to those of the regression alphas, with the notable exception that we do observe a single negative and significant value, the difference between the return of the Polish MVO and its index. However, this negative t-test is only significant for yearly rebalancing, and it disappears at other frequencies.

In general, we see that MVOs tend to outperform their benchmark, and without adding much risk when compared to investing in an index fund. Moreover, their performance does not seem much affected by the investor's choice of rebalancing frequency, which allows a would-be MVO

investor to drastically reduce the trading costs associated with managing a portfolio by herself. Finally, within certain bounds on minimums, these results do not appear to be sensitive to market variables such as size and liquidity, although the latter should be studied further. As long as the market is large enough and there is enough liquidity for the MVO algorithm to work at all, its results deliver a much needed option to investors looking for an easy and low-cost way to access equity markets.

4. Portfolio Structure and Further Tests

Detractors of the MVO methodology of asset management point to the fact that the resulting portfolios lack diversification.

In Tables IV and V we study the structure of the MVO portfolios in our sample. Table IV shows the average number of stocks in each portfolio. With few exceptions, most portfolios contain between 3 and 4 stocks, while some average as low as 2. It is interesting to see that the market or index from which these stocks are picked does not seem to influence the size of the resulting portfolio, as we see that, for example, the large U.S. indexes produce portfolios with roughly the same number of stocks than those generated from far smaller market indexes. We also note that neither does the frequency of portfolio rebalancing seem to be related to the size of the resulting portfolio. Thus we conclude that this is a characteristic inherited from the MVO methodology itself.

[TABLE IV AROUND HERE]

Even with a relatively small number of assets, we might still be able to claim a certain acceptable level of diversification if investment capital was spread somewhat evenly amongst them. Table V shows the average portfolio weights in our sample. While the overall mean size of portfolio weights (column labeled ‘All’) is acceptable, ranging from 17% to 52%, the story is quite different if we analyze the highest and lowest allocation in each portfolio separately (‘Max’ and ‘Min’, respectively). Save for a few exceptions, the maximum allocation exceeds 70% of the capital invested (with 100% observed in various cases, indicating portfolios with a single asset), whereas the minimum does not exceed around 2% on average, and is often less than 1%.

[TABLE V AROUND HERE]

Taken together, these results confirm the claims that, in general, MVO portfolios may achieve high levels of risk-adjusted returns, but do so at the cost of almost insignificantly diversifying the investor's capital, and thus expose her to unnecessary levels of risk.

While various solutions have been proposed to this well-known issue with the MVO methodology (see, for example, Green & Hollifield (1992)), we test the simplest one which is to impose a single restriction on the maximum acceptable size for portfolio weights. Specifically, we impose a 20% maximum weight restriction[4]. This means that, mechanically, no portfolio can have less than 5 stocks and, even then, capital should be more evenly distributed. The main concern in adding this restriction is that any outperformance the MVO portfolios may have had when computed without the weight restriction might evaporate.

Table VI summarizes the performance of our restricted MVO portfolios. For brevity, we have included only the data for MVOs rebalanced at yearly intervals, and Panel A reports the same data as in Table 2, while Panel B reports the same statistics as are depicted in Table 3. Comparing the performance of the restricted MVOs to that of the unrestricted sample, we see that, indeed, performance has decreased somewhat. However, the number of MVOs that outperform their benchmark remains virtually unchanged, and the overall advantage of MVOs over other forms of stock funds remains qualitatively the same.

Ultimately, for the sake of increasing diversification we can add restrictions like the one tested in this chapter to the MVO methodology. What we discover is that the price of these restrictions is

[4] We opt for that level for the restriction in order to ensure that our sample remains intact. Imposing stronger restrictions, in the form of lower maximum weights, would inevitably reduce the size of our sample of MVOs as, for some countries with small markets, the algorithm fails to find a suitable solution.

not large enough to eliminate the appeal for investors of applying this methodology in the stock markets of the world.

5. Conclusions

We set out to test the viability of the venerable mean-variance portfolio methodology introduced in Markowitz (1950) as a tool that modern retail investors could use to improve the performance of their investments, over and above that offered by the average actively managed or index equity fund. To do so, we backtest the performance of portfolios formed by allocating capital to the stocks listed in 22 indexes of 19 different countries and, while varying the frequency at which these portfolios are rebalanced, simulating an investment made continuously over 10 years.

Since the average equity mutual fund tends to underperform its benchmark index, and since our mean-variance optimized portfolios tend to outperform the same index, we conclude that, in terms of risk-adjusted performance, MVO portfolios offer a better alternative to both types of funds currently available in the markets.

We tackle the main concern with MVO portfolios, their low levels of diversification, by adding a simple restriction on the size of resulting portfolio weights to the optimization problem. The results show an adequate increase in the level of diversification of the portfolios, without a noticeable change in our previous conclusions regarding portfolio performance.

Additionally, our results do not seem to be sensitive to the frequency with which the portfolios are rebalanced. That is, a monthly rebalanced portfolio does not necessarily offer better performance than a yearly rebalanced one. Thus, we can claim that, even if a retail investor faces higher transaction costs than an institutional investor, she can make up for this added cost by the lower turnover required to maintain a fund who's portfolio weights need only be updated once a year.

Taken together, we believe there is sufficient evidence to support the use of mean-variance optimization as a valid, value-adding tool for retail investors, that is, those investors who are unable to do exhaustive security and market analysis because they are not professionally trained and/or lack the access to some of the proprietary data and models used by professional money managers.

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Figure I: Excess Return of Mean-Variance Optimized Portfolios

MVO portfolios' yearly excess returns are calculated as the return of the MVO portfolio minus that of its benchmark index, and then averaged. The time series of these excess returns is plotted in each pane, for different MVO rebalancing frequency.

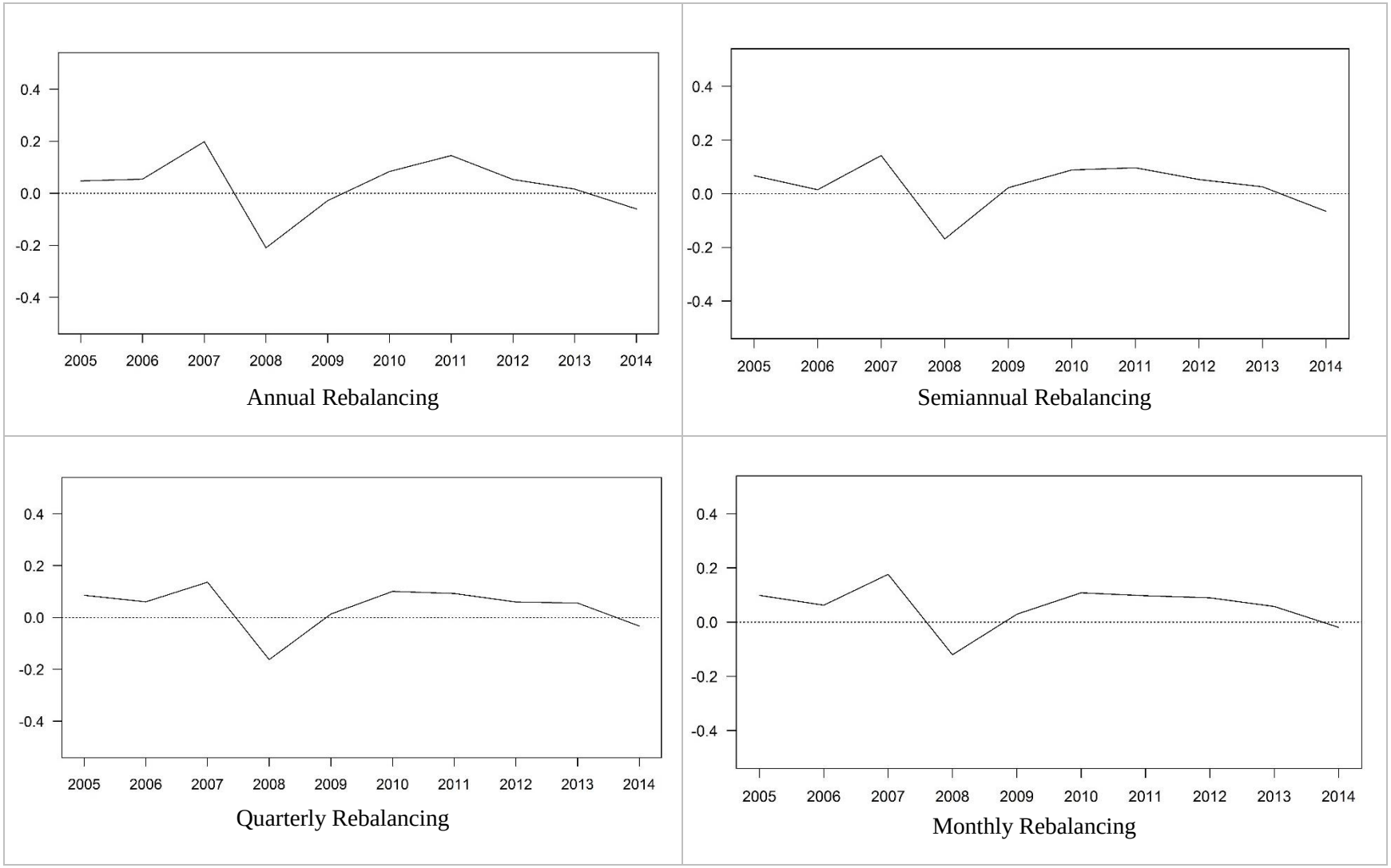

Table I: Descriptive Statistics of Country Indexes

Descriptive statistics are presented for each country / index which provide the sample of stocks eligible to be included in the mean-variance optimized portfolios. The data shown includes the number of stocks listed in the index as of the end of 2014 ('Num Stocks'), a measure of market liquidity ('Liquidity') obtained as the average of all days in the last year of the sample in which each index stock traded (i.e.: there is price information), the mean price of stocks in the last day of 2014 in each local currency (confirm!!!), and the volatility of the index expressed as the standard deviation of monthly returns using the past year ('Index Vol 1') and the past 5 years ('Index Vol 5') of data.

Country_Index	Num Stocks	Liquidity	Mean Price	Index Vol 1	Index Vol 5
SP100	101	97.79	92.5	2.19	3.65
NASDAQ100	107	98.37	102.76	2.92	4.3
DOW JONES	30	100	88.39	2.44	3.46
BRAZIL	68	98.43	20.81	6.58	5.37
CHILE	93	98.92	2685.95	3.45	3.8
CHINA	50	100	39.55	4.21	5.26
PHILIPPINES	30	99.92	294.16	2.28	4.58
GREECE	60	99.79	6.7	7.77	10.3
INDIA	50	98	907.62	3.58	4.97
INDONESIA	508	73.66	2101.95	1.75	4.46
ISRAEL	102	96.37	9889.17	2.22	3.63
JAPAN	225	99.89	2119.03	4.03	5.29
MALAYSIA	30	99.92	11.01	1.67	2.61
MEXICO	35	100	86.89	3.2	3.3
PERU	25	92.16	6.62	4.43	6.75
POLAND	80	96.45	28.77	3.77	5.12
PORTUGAL	18	94.44	3.7	6.02	5.48
SINGAPORE	30	99.95	9.57	2.36	3.8
SOUTH AFRICA	42	97.62	20830.14	2.31	3.8
SRI LANKA	287	97.57	247.05	3.22	5.91
THAILAND	50	97.33	86.04	3.2	5.06
TURKEY	100	99.14	20.06	6.13	6.81

Table II: Performance of Mean-Variance Optimized Portfolios

Portfolios of stocks listed in each index/country are rebalanced at different frequencies: yearly, semiannually, quarterly and monthly. Each time a portfolio is rebalanced, portfolio weights are determined by mean-variance optimization using the previous 5 years of monthly stock returns. Independent of rebalancing frequency, the full period time series of monthly returns are obtained for each portfolio and then used to calculate portfolio performance measures. The measures displayed on this table include the annualized mean monthly return ('Ret'), the annualized standard deviation of monthly returns ('SD'), and the portfolio's Sharpe Ratio ('Sharpe'). Both Ret and SD are expressed as percentages. , For comparison, the same measures are calculated for each index.

Panel A: Yearly Rebalancing

	Portfolio			*Index*		
Country	Ret	SD	Sharpe	Ret	SD	Sharpe
DOW JONES	35.54	16.65	2.13	13.49	15.00	0.90
NASDAQ100	24.57	28.84	0.85	24.04	16.64	1.44
SP100	32.28	17.37	1.86	13.73	15.54	0.88
BRAZIL	13.67	25.77	0.53	6.51	19.45	0.33
CHILE	5.15	17.43	0.30	9.95	14.92	0.67
CHINA	24.77	24.62	1.01	10.09	20.83	0.48
PHILIPPINES	51.22	26.22	1.95	25.48	17.69	1.44
GREECE	0.50	24.47	0.02	-8.26	37.53	-0.22
INDIA	18.75	26.19	0.72	16.22	21.95	0.74
INDONESIA	26.07	30.94	0.84	25.55	19.38	1.32
ISRAEL	46.12	40.42	1.14	16.33	15.26	1.07
JAPAN	9.34	24.19	0.39	12.89	20.22	0.64
MALAYSIA	10.05	13.65	0.74	16.21	10.96	1.48
MEXICO	3.53	38.84	0.09	13.73	16.24	0.85
PERU	18.13	25.22	0.72	17.33	30.38	0.57
POLAND	-2.26	22.26	-0.10	15.80	21.46	0.74
PORTUGAL	24.04	23.29	1.03	0.68	18.39	0.04
SINGAPORE	14.13	13.54	1.04	12.39	18.09	0.68
SOUTH AFRICA	28.18	18.55	1.52	16.25	16.31	1.00
SRI LANKA	32.16	25.95	1.02	31.11	25.09	1.24
THAILAND	32.69	25.95	1.26	22.58	19.97	1.13
TURKEY	16.40	28.00	0.59	20.17	26.26	0.77

Panel B: Semiannual Rebalancing

	Portfolio			*Index*		
Country	Ret	SD	Sharpe	Ret	SD	Sharpe
DOW JONES	27.44	19.04	1.44	13.49	15.00	0.90
NASDAQ100	16.16	33.99	0.48	24.04	16.64	1.44
SP100	21.48	18.81	1.14	13.73	15.54	0.88
BRAZIL	16.87	29.93	0.56	6.51	19.45	0.33
CHILE	10.85	16.76	0.65	9.95	14.92	0.67
CHINA	28.55	23.29	1.23	10.09	20.83	0.48
PHILIPPINES	53.37	26.49	2.01	25.48	17.69	1.44
GREECE	0.06	22.78	0.00	-8.26	37.53	-0.22
INDIA	23.54	20.90	1.13	16.22	21.95	0.74
INDONESIA	20.61	28.57	0.72	25.55	19.38	1.32
ISRAEL	48.32	41.74	1.16	16.33	15.26	1.07
JAPAN	10.52	25.90	0.41	12.89	20.22	0.64
MALAYSIA	17.61	13.32	1.32	16.21	10.96	1.48
MEXICO	11.29	40.03	0.28	13.73	16.24	0.85
PERU	18.35	23.60	0.78	17.33	30.38	0.57
POLAND	8.49	22.09	0.38	15.80	21.46	0.74
PORTUGAL	19.20	23.60	0.81	0.68	18.39	0.04
SINGAPORE	15.57	14.38	1.08	12.39	18.09	0.68
SOUTH AFRICA	33.58	17.73	1.89	16.25	16.31	1.00
SRI LANKA	12.97	44.02	0.29	31.11	25.09	1.24
THAILAND	35.94	26.85	1.34	22.58	19.97	1.13
TURKEY	11.60	28.98	0.40	20.17	26.26	0.77

Panel C: Quarterly Rebalancing

	Portfolio			*Index*		
Country	Ret	SD	Sharpe	Ret	SD	Sharpe
DOW JONES	33.57	18.64	1.80	13.49	15.00	0.90
NASDAQ100	20.40	32.00	0.64	24.04	16.64	1.44
SP100	25.27	18.01	1.40	13.73	15.54	0.88
BRAZIL	12.21	29.88	0.41	6.51	19.45	0.33
CHILE	4.17	15.13	0.28	9.95	14.92	0.67
CHINA	34.38	24.05	1.43	10.09	20.83	0.48
PHILIPPINES	52.33	27.89	1.88	25.48	17.69	1.44
GREECE	7.70	22.09	0.35	-8.26	37.53	-0.22
INDIA	27.67	20.54	1.35	16.22	21.95	0.74
INDONESIA	27.25	29.02	0.94	25.55	19.38	1.32
ISRAEL	48.58	37.84	1.28	16.33	15.26	1.07
JAPAN	8.05	24.85	0.32	12.89	20.22	0.64
MALAYSIA	18.02	13.44	1.34	16.21	10.96	1.48
MEXICO	13.63	38.11	0.36	13.73	16.24	0.85
PERU	21.01	24.12	0.87	17.33	30.38	0.57
POLAND	6.53	23.58	0.28	15.80	21.46	0.74
PORTUGAL	19.36	23.64	0.82	0.68	18.39	0.04
SINGAPORE	12.81	13.66	0.94	12.39	18.09	0.68
SOUTH AFRICA	33.52	17.92	1.87	16.25	16.31	1.00
SRI LANKA	15.94	42.56	0.37	31.11	25.09	1.24
THAILAND	35.05	28.32	1.24	22.58	19.97	1.13
TURKEY	9.19	26.91	0.34	20.17	26.26	0.77

Panel D: Monthly Rebalancing

	Portfolio			*Index*		
Country	Ret	SD	Sharpe	Ret	SD	Sharpe
DOW JONES	34.74	18.16	1.91	13.49	15.00	0.90
NASDAQ100	28.03	30.59	0.92	24.04	16.64	1.44
SP100	25.92	17.19	1.51	13.73	15.54	0.88
BRAZIL	14.67	29.93	0.49	6.51	19.45	0.33
CHILE	4.60	15.83	0.29	9.95	14.92	0.67
CHINA	36.09	24.18	1.49	10.09	20.83	0.48
PHILIPPINES	50.76	28.52	1.78	25.48	17.69	1.44
GREECE	9.77	22.17	0.44	-8.26	37.53	-0.22
INDIA	24.31	21.10	1.15	16.22	21.95	0.74
INDONESIA	35.27	28.29	1.25	25.55	19.38	1.32
ISRAEL	46.78	38.28	1.22	16.33	15.26	1.07
JAPAN	12.23	24.41	0.50	12.89	20.22	0.64
MALAYSIA	20.80	14.16	1.47	16.21	10.96	1.48
MEXICO	13.78	33.79	0.41	13.73	16.24	0.85
PERU	20.17	24.50	0.82	17.33	30.38	0.57
POLAND	11.78	22.17	0.53	15.80	21.46	0.74
PORTUGAL	21.66	24.20	0.90	0.68	18.39	0.04
SINGAPORE	14.68	14.12	1.04	12.39	18.09	0.68
SOUTH AFRICA	30.73	18.15	1.69	16.25	16.31	1.00
SRI LANKA	12.05	41.89	0.29	31.11	25.05	1.24
THAILAND	34.64	29.19	1.19	22.58	19.97	1.13
TURKEY	11.59	28.40	0.41	20.17	26.26	0.77

Table III: Statistical Tests of Performance of Mean-Variance Optimized Portfolios

Monthly returns are obtained for all mean-variance optimized portfolios formed from stocks in each index/country. These returns are then used to estimate to the statistical significance of the portfolio's performance. The first test is the estimation of a market-model alpha, where the benchmark market returns used are those of each index. The second is a t-test of the difference between the monthly returns of the mean-variance portfolios and those of their respective benchmark index. For each test we display the point estimate, as well as the corresponding t-statistic. Significance is denoted by ***, ** and * for the 1%, 5% and 10% levels, respectively.

Panel A: Yearly Rebalancing

	Alpha		$T-Test$	
Country	Estimate	T-stat	Estimate	T-stat
DOW JONES	1.82***	3.65	1.51***	2.96
NASDAQ100	0.25	0.25	0.04	0.04
SP100	1.51***	3.16	1.28***	2.69
BRAZIL	0.64	0.84	0.55	0.71
CHILE	-0.30	-0.73	-0.37	-0.92
CHINA	1.17*	1.82	1.06	1.65
PHILIPPINES	1.86**	2.26	1.60**	2.00
GREECE	0.34	0.48	0.76	0.70
INDIA	0.66	0.77	0.18	0.20
INDONESIA	0.19	0.19	-0.04	-0.04
ISRAEL	1.75	1.23	1.94	1.43
JAPAN	-0.31	-0.67	-0.27	-0.60
MALAYSIA	0.06	0.13	-0.46	-0.96
MEXICO	-1.63	-1.63	-0.79	-0.73
PERU	0.69	0.93	0.06	0.06
POLAND	-0.97	-1.43	-1.42*	-1.97
PORTUGAL	1.78**	2.25	1.76**	2.08
SINGAPORE	0.64	1.61	0.13	0.25
SOUTH AFRICA	1.77**	2.51	0.83	1.02
SRI LANKA	0.31	0.35	0.07	0.08
THAILAND	0.75	1.08	0.67	1.03
TURKEY	0.09	0.13	-0.27	-0.35

Panel B: Semiannual Rebalancing

	Alpha		$T-Test$	
Country	Estimate	T-stat	Estimate	T-stat
DOW JONES	1.34**	2.13	0.98	1.55
NASDAQ100	-0.54	-0.45	-0.55	-0.50
SP100	0.81	1.44	0.56	1.00
BRAZIL	0.81	0.91	0.78	0.89
CHILE	0.30	0.60	0.07	0.13
CHINA	1.49**	2.35	1.31**	2.03
PHILIPPINES	2.35**	2.35	1.72**	2.02
GREECE	0.26	0.37	0.72	0.64
INDIA	1.06	1.65	0.52	0.72
INDONESIA	-0.12	-0.13	-0.34	-0.40
ISRAEL	1.99	1.32	2.07	1.44
JAPAN	-0.27	-0.51	-0.18	-0.35
MALAYSIA	0.61	1.30	0.10	0.22
MEXICO	-1.07	-1.04	-0.18	-0.16
PERU	0.76	1.09	0.07	0.08
POLAND	-0.15	-0.23	-0.55	-0.81
PORTUGAL	1.44*	1.83	1.42*	1.71
SINGAPORE	0.64*	1.72	0.24	0.52
SOUTH AFRICA	2.02***	3.08	1.18	1.57
SRI LANKA	-1.12	-0.77	-1.26	-0.91
THAILAND	0.84	1.23	0.88	1.35
TURKEY	-0.26	-0.32	-0.62	-0.77

Panel C: Quarterly Rebalancing

	Alpha		*T − Test*	
Country	Estimate	T-stat	Estimate	T-stat
DOW JONES	1.75***	2.85	1.38**	2.23
NASDAQ100	-0.10	-0.09	-0.25	-0.24
SP100	1.12**	2.05	0.82	1.49
BRAZIL	0.50	0.54	0.44	0.48
CHILE	-0.01	-0.02	-0.45	-0.76
CHINA	1.83***	2.86	1.69***	2.63
PHILIPPINES	1.98**	2.08	1.66*	1.87
GREECE	0.86	1.24	1.34	1.15
INDIA	1.27**	2.17	0.80	1.23
INDONESIA	0.18	0.20	0.11	0.14
ISRAEL	2.20	1.60	2.09	1.59
JAPAN	-0.39	-0.73	-0.37	-0.71
MALAYSIA	0.63	1.34	0.13	0.28
MEXICO	-0.72	-0.70	-0.01	-0.01
PERU	0.95	1.31	0.26	0.29
POLAND	-0.41	-0.64	-0.70	-1.07
PORTUGAL	1.45*	1.85	1.43*	1.73
SINGAPORE	0.58	1.37	0.03	0.06
SOUTH AFRICA	1.98***	3.01	1.18	1.59
SRI LANKA	-0.61	-0.42	-1.04	-0.74
THAILAND	0.81	1.04	0.82	1.11
TURKEY	-0.36	-0.49	-0.81	-1.04

Panel D: Monthly Rebalancing

	Alpha		*T − Test*	
Country	Estimate	T-stat	Estimate	T-stat
DOW JONES	1.85***	3.08	1.46**	2.37
NASDAQ100	0.44	0.42	0.27	0.27
SP100	1.18**	2.30	0.86**	1.66
BRAZIL	0.70	0.74	0.62	0.66
CHILE	-0.01	-0.02	-0.42	-0.70
CHINA	1.94***	3.01	1.80***	2.78
PHILIPPINES	1.92*	1.94	1.57*	1.70
GREECE	1.01	1.46	1.50	1.28
INDIA	1.03*	1.69	0.57	0.86
INDONESIA	0.72	0.85	0.64	0.80
ISRAEL	2.13	1.52	1.98	1.48
JAPAN	-0.06	-0.12	-0.05	-0.10
MALAYSIA	0.81	1.60	0.33	0.67
MEXICO	-0.49	-0.52	0.00	0.00
PERU	0.89	1.20	0.20	0.22
POLAND	0.05	0.08	-0.30	-0.47
PORTUGAL	1.61**	2.01	1.59*	1.90
SINGAPORE	0.63	1.59	0.17	0.34
SOUTH AFRICA	1.77***	2.67	1.00	1.35
SRI LANKA	-0.99	-0.70	-1.33	-0.99
THAILAND	0.80	0.96	0.80	1.01
TURKEY	-0.14	-0.17	-0.62	-0.72

Table IV: Number of Assets in Portfolios

The number individual of stocks contained in each mean-variance optimized portfolio is averaged over the period of the analysis. Overall means are presented ('All'), as well as means for each portfolio rebalancing frequency ('Annual', 'Semiannual', 'Quarterly' and 'Monthly').

Country	All	Annual	Semiannual	Quarterly	Monthly
DOW JONES	2.00	1.90	1.90	2.10	2.10
NASDAQ100	4.00	4.10	4.00	3.90	3.90
SP100	3.10	3.30	3.00	3.10	3.20
BRAZIL	3.10	3.50	3.00	3.00	3.00
CHILE	5.00	5.70	4.80	4.80	4.90
CHINA	3.40	3.40	3.50	3.40	3.40
PHILIPPINES	3.20	3.30	3.20	3.20	3.00
GREECE	2.30	2.40	2.20	2.30	2.30
INDIA	3.20	3.00	3.40	3.30	3.30
INDONESIA	4.80	4.40	5.00	4.90	4.80
ISRAEL	3.80	3.70	3.80	3.80	3.90
JAPAN	3.70	4.00	3.50	3.50	3.70
MALAYSIA	2.20	2.20	2.20	2.20	2.10
MEXICO	3.50	3.70	3.50	3.50	3.40
PERU	3.00	2.80	3.30	3.10	3.00
POLAND	3.90	4.00	4.00	3.80	3.90
PORTUGAL	2.40	2.70	2.20	2.20	2.30
SINGAPORE	4.30	4.60	4.30	4.20	4.30
SOUTH AFRICA	2.50	2.70	2.60	2.40	2.40
SRI LANKA	7.50	7.40	7.50	7.50	7.60
THAILAND	3.90	3.80	4.20	3.80	3.90
TURKEY	5.00	4.90	4.80	5.00	5.10

Table V: Portfolio Weights of Mean-Variance Portfolios

Portfolio weights are averaged across time for each country and each portfolio rebalancing frequency. Means are shown for the weights of all assets in each portfolio ('All'), as well as for the assets with the highest ('Max') and lowest ('Min') weights in each portfolio.

	Annual			*Semiannual*			*Quarterly*			*Monthly*		
Country	All	Max	Min	All	Max	Min	All	Max	Min	All	Max	Min
DOW JONES	52.63	100.00	8.20	52.63	100.00	3.56	48.19	100.00	1.03	47.62	100.00	0.48
NASDAQ100	24.39	74.12	0.58	25.32	100.00	0.20	25.64	100.00	0.17	25.42	100.00	0.12
SP100	30.30	94.49	1.08	33.33	92.55	0.89	32.00	93.72	0.36	31.33	100.00	0.13
BRAZIL	28.57	100.00	1.44	33.33	100.00	0.40	33.06	100.00	0.40	32.97	100.00	0.12
CHILE	17.54	69.64	0.17	20.83	83.98	0.20	20.94	83.98	0.20	20.30	86.88	0.12
CHINA	29.41	95.19	0.93	28.57	95.90	0.32	29.20	100.00	0.32	29.41	100.00	0.32
PHILIPPINES	30.30	85.13	0.34	30.77	100.00	0.92	31.50	100.00	0.68	32.79	100.00	0.16
GREECE	41.67	100.00	0.31	44.44	100.00	0.38	43.01	100.00	0.38	42.70	100.00	0.28
INDIA	33.33	94.20	0.72	29.85	100.00	0.68	30.53	100.00	0.68	29.93	100.00	0.17
INDONESIA	22.73	55.02	0.48	20.20	68.71	0.50	20.30	68.71	0.50	20.94	68.74	0.22
ISRAEL	27.03	88.37	0.31	26.32	97.12	1.32	26.32	97.12	0.29	25.59	97.12	0.15
JAPAN	25.00	86.41	0.67	28.57	100.00	0.73	28.37	100.00	0.29	26.79	100.00	0.10
MALAYSIA	45.45	100.00	2.47	45.45	100.00	0.48	45.98	100.00	0.33	47.24	100.00	0.21
MEXICO	27.03	83.33	0.45	28.57	95.32	0.84	28.37	96.45	0.82	29.06	100.00	0.33
PERU	35.71	88.57	1.99	30.30	91.08	0.35	31.75	100.00	0.23	32.79	100.00	0.13
POLAND	25.00	85.80	1.36	25.32	85.67	2.12	26.14	93.27	1.01	25.75	93.27	0.12
PORTUGAL	37.04	100.00	0.20	44.44	100.00	0.51	44.44	100.00	0.51	43.01	100.00	0.26
SINGAPORE	21.74	89.89	0.50	23.26	88.21	0.29	23.67	88.21	0.29	23.12	100.00	0.14
SOUTH AFRICA	37.04	100.00	0.91	38.46	100.00	0.25	41.67	100.00	0.25	40.96	100.00	0.14
SRI LANKA	13.51	72.40	0.26	13.42	95.49	0.14	13.42	95.49	0.12	13.17	95.49	0.10
THAILAND	26.32	62.59	4.09	24.10	98.00	0.20	25.97	98.00	0.20	25.70	100.00	0.15
TURKEY	20.41	65.49	0.44	20.83	75.41	0.52	19.80	75.41	0.22	19.70	75.41	0.16

Table VI: Performance of Restricted Mean-Variance Optimized Portfolios

Portfolios of stocks listed in each index/country are rebalanced every year. Each time a portfolio is rebalanced, portfolio weights are determined by mean-variance optimization using the previous 5 years of monthly stock returns. Portfolio weights are restricted in the maximization problem to a maximum value of 20%. The full period time series of monthly returns is obtained for each portfolio and then used to calculate portfolio performance measures and perform statistical tests of this performance. Panel A of this table shows the annualized mean monthly return ('Ret'), the annualized standard deviation of monthly returns ('SD'), and the portfolio's Sharpe Ratio ('Sharpe'). Both Ret and SD are expressed as percentages. For comparison, the same measures are calculated for each index. Panel B contains the market-model alpha, where the benchmark market returns used are those of each index, as well as a t-test of the difference between the monthly returns of the mean-variance portfolios and those of their respective benchmark index. For each test we display the point estimate, as well as the corresponding t-statistic. Significance is denoted by ***, ** and * for the 1%, 5% and 10% levels, respectively.

Panel A: Performance Statistics

	Portfolio			*Index*		
Country	Ret	SD	Sharpe	Ret	SD	Sharpe
DOW JONES	17.33	13.12	1.32	13.49	15.00	0.90
NASDAQ100	33.13	25.14	1.32	24.04	16.64	1.44
SP100	20.99	17.08	1.23	13.73	15.54	0.88
BRAZIL	14.50	15.83	0.92	6.51	19.45	0.33
CHILE	7.20	16.11	0.45	9.95	14.92	0.67
CHINA	16.31	19.93	0.82	10.09	20.83	0.48
PHILIPPINES	34.19	20.49	1.67	25.48	17.69	1.44
GREECE	6.65	22.82	0.29	-8.26	37.53	-0.22
INDIA	24.72	18.56	1.33	16.22	21.95	0.74
INDONESIA	21.92	27.21	0.81	25.55	19.38	1.32
ISRAEL	35.98	23.06	1.56	16.33	15.26	1.07
JAPAN	9.04	21.64	0.42	12.89	20.22	0.64
MALAYSIA	16.12	9.55	1.69	16.21	10.96	1.48
MEXICO	12.31	21.40	0.58	13.73	16.24	0.85
PERU	19.86	23.37	0.85	17.33	30.38	0.57
POLAND	-2.68	19.27	-0.14	15.80	21.46	0.74
PORTUGAL	6.40	18.70	0.34	0.68	18.39	0.04
SINGAPORE	14.93	13.75	1.09	12.39	18.09	0.68
SOUTH AFRICA	27.19	14.80	1.84	16.25	16.31	1.00
SRI LANKA	27.74	32.95	0.84	31.11	25.09	1.24
THAILAND	33.09	25.12	1.32	22.58	19.97	1.13
TURKEY	13.59	28.74	0.47	20.17	26.26	0.77

Panel B: Statistical Tests

	Alpha		*T − Test*	
Country	Estimate	T-stat	Estimate	T-stat
DOW JONES	0.59**	1.98	0.28	0.85
NASDAQ100	0.61	0.81	0.60	0.85
SP100	0.63*	1.66	0.52	1.42
BRAZIL	0.83**	1.99	0.61	1.19
CHILE	-0.14	-0.42	-0.21	-0.65
CHINA	0.65	1.43	0.46	0.96
PHILIPPINES	0.63	1.40	0.57	1.37
GREECE	0.86	1.46	1.25	1.29
INDIA	0.98**	2.42	0.60	1.28
INDONESIA	-0.37	-0.53	-0.25	-0.38
ISRAEL	1.28*	1.95	1.33**	2.11
JAPAN	-0.23	-0.60	-0.29	-0.76
MALAYSIA	0.50*	1.77	-0.01	-0.02
MEXICO	-0.21	-0.46	-0.11	-0.24
PERU	0.68	1.32	0.18	0.27
POLAND	-0.97*	-1.80	-1.46**	-2.37
PORTUGAL	0.47*	1.78	0.46*	1.76
SINGAPORE	0.56*	1.83	0.19	0.48
SOUTH AFRICA	1.06***	3.40	0.76**	2.30
SRI LANKA	-0.16	-0.18	-0.22	-0.27
THAILAND	0.63	1.14	0.70	1.32
TURKEY	-0.17	-0.22	-0.48	-0.63