Advances in High Frequency Strategies

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Notice:

The research contained in this presentation is the result of a continuing collaboration with

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SECTION I
The great divide

Is speed the real issue?

- Faster traders are nothing new:
  - Nathan Rothschild is said to have used racing pigeons to trade in advance on the news of Napoleon’s defeat at Waterloo.
  - Beginning in 1850s, only a limited number of investors had access to telegraphy.
  - The telephone (1875), radio (1915), and more recently screen trading (1986) offered speed advantages to some participants over others.
  - Leinweber [2009] relates many instances in which technological breakthroughs have been used to most investors’ disadvantage. ***So … what is new this time?***

A change in paradigm

- High Frequency Trading (HFT) is not Low Frequency Trading (LFT) on steroids.
- HFT have been mischaracterized as ‘*cheetah-traders*’.
- Rather than speed, the true great divide is a “change in the trading paradigm”.
- HFT are strategic traders. In some instances, they:
  - act upon the information revealed by LFT’s actions.
  - engage in sequential games.
  - behave like predators.
- Speed is an advantage, but there is more to it…

What is the new paradigm? (1/3)

- Time is a measuring system used to sequence observations.
- Since the dawn of time, humans have based their time measurements in chronology: Years, months, days, hours, minutes, seconds, and since recently milliseconds, microseconds ...
- This is a rather arbitrary time system, due to the key role played by the Sun in agricultural societies.

What is the new paradigm? (2/3)

- Machines operate on an internal clock that is not *chrono* based, but *event* based: The cycle.
- A machine will complete a cycle at various chrono rates, depending on the amount of information involved in a particular instruction.
- As it happens, HFT relies on machines, thus measuring time in terms of events.
- Thinking in volume-time is challenging for us humans. But for a ‘silicon trader’, it is the natural way to process information and engage in sequential, strategic trading.

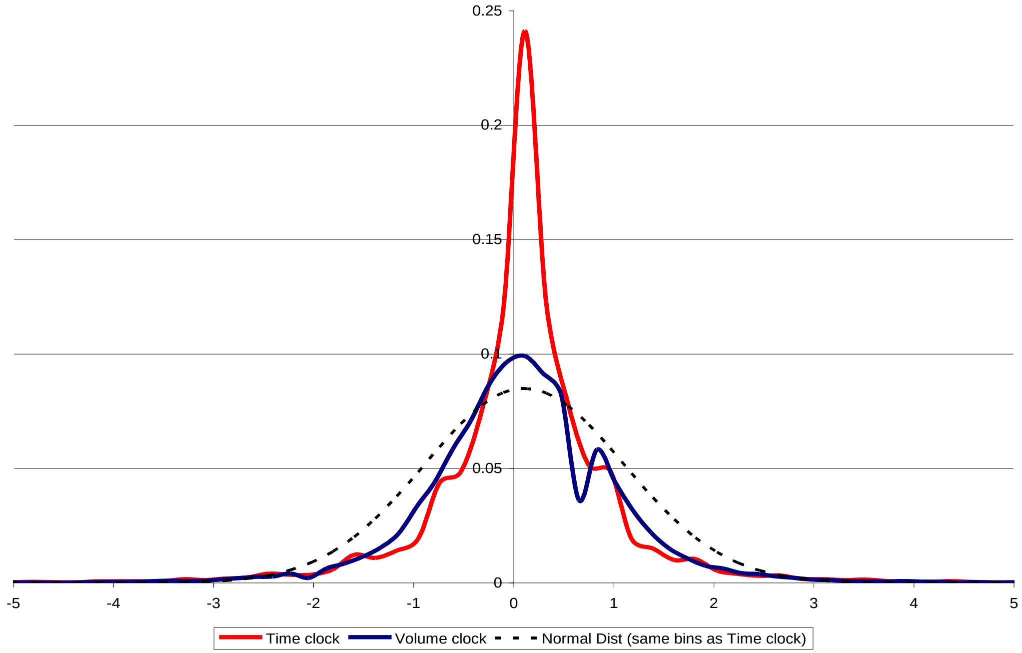
What is the new paradigm? (3/3)

- The new paradigm is "***event-based time***". The simplest example is dividing the session in equal volume buckets. This transformation removes most intra-session seasonal effects.
- For example, HF market makers may target to turn their portfolio every fixed number of contracts traded (volume bucket), regardless of the chrono time.
- In fact, working in volume time presents significant statistical advantages.

Volume time vs. Chrono time

Stats (50)	Chrono time	Volume time
Mean	0.0000	0.0000
StDev	1.0000	1.0000
Skew	-0.0788	-0.2451
Kurt	31.7060	15.8957
Min	-21.8589	-20.6117
Max	19.3092	13.8079
L-B*	34.4551	22.7802
White*	0.0971	0.0548
J-B*	34.3359	6.9392

Stats (100)	Chrono time	Volume time
Mean	0.0000	0.0000
StDev	1.0000	1.0000
Skew	-0.1606	-0.4808
Kurt	44.6755	23.8651
Min	-28.3796	-29.2058
Max	24.6700	15.5882
L-B*	115.3207	36.1189
White*	0.0873	0.0370
J-B*	72.3729	18.1782

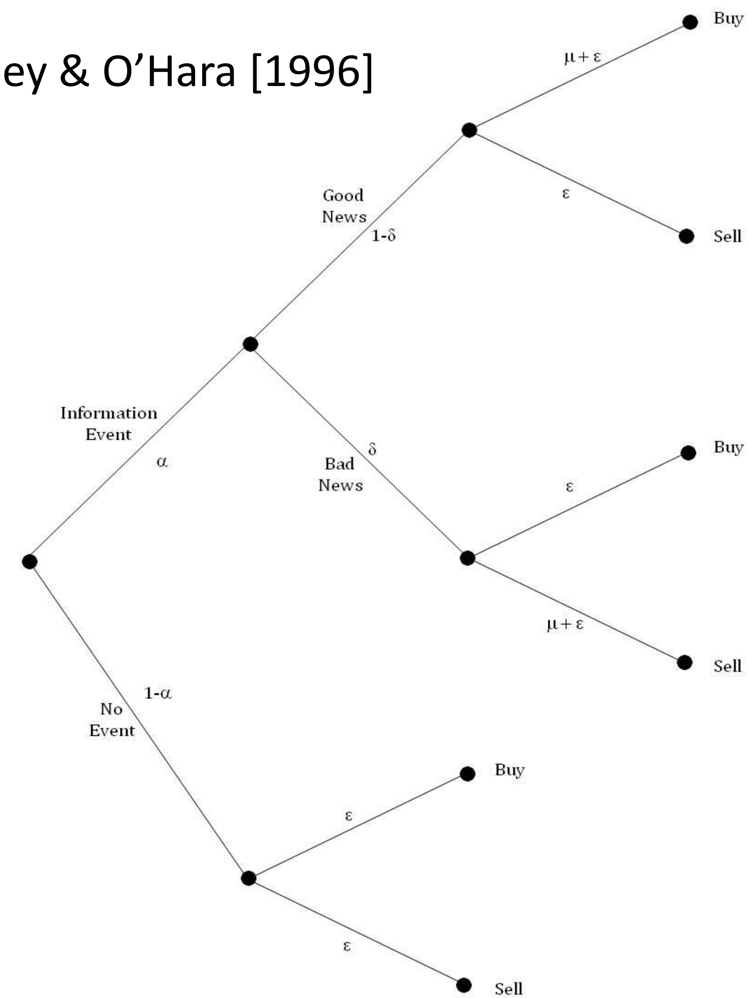


Sampling by Volume time allows for a partial recovery of Normality, IID

SECTION II
More than speed

Example of Sequential Trading model

Easley & O'Hara [1996]



$$E[S_i \mid t] = P_n(t)S_i^* + P_b(t)\underline{S}_i + P_g(t)\overline{S}_i$$

$$B(t) = E[S_i \mid t] - \frac{\mu P_b(t)}{\varepsilon + \mu P_b(t)}\left[E[S_i \mid t] - \underline{S}_i\right]$$

$$A(t) = E[S_i \mid t] + \frac{\mu P_g(t)}{\varepsilon + \mu P_g(t)}\left[\overline{S}_i - E[S_i \mid t]\right]$$

$$\Sigma(t) = \frac{\mu P_g(t)}{\varepsilon + \mu P_g(t)}\left[\overline{S}_i - E[S_i \mid t]\right] + \frac{\mu P_b(t)}{\varepsilon + \mu P_b(t)}\left[E[S_i \mid t] - \underline{S}_i\right]$$

$$If\ \delta = \frac{1}{2} \Rightarrow \Sigma = \frac{\alpha\mu}{\alpha\mu + 2\varepsilon}\left[\overline{S_i} - \underline{S_i}\right]$$

$$PIN = \frac{\alpha\mu}{\alpha\mu + 2\varepsilon}$$

Little known species you should be aware of

- **Predatory algorithms** are a special kind of informed traders. Rather than possessing exogenous information yet to be incorporated in the market price, they know that their endogenous actions are likely to trigger a microstructure mechanism, with foreseeable outcome. Examples include:
  - Quote stuffing: Overwhelming an exchange with messages, with the sole intention of slowing down competing algorithms.
  - Quote dangling: Sending quotes that force a squeezed trader to chase a price against her interests.
  - Pack hunting: Predators hunting independently become aware of each others activities, and form a pack in order to maximize the chances of triggering a cascading effect.

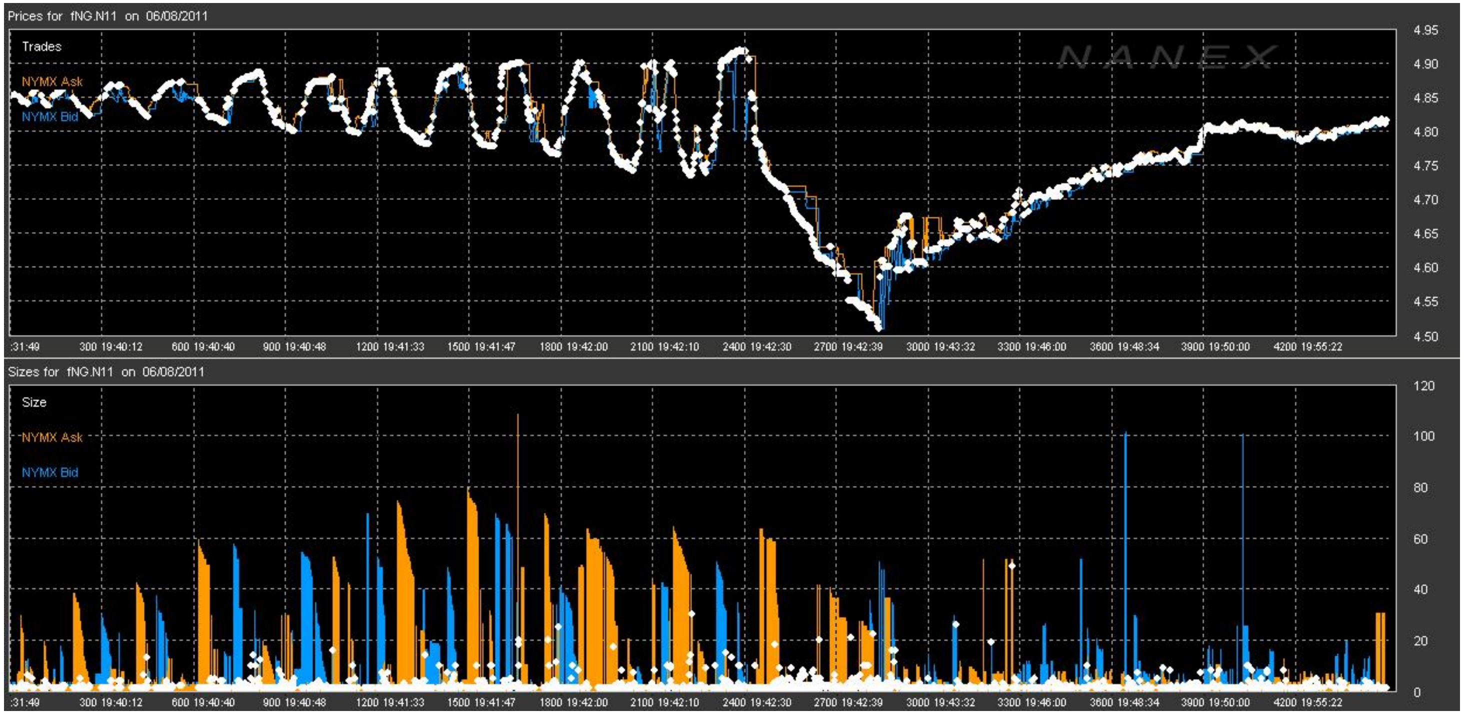
Slow chess may be harder than you think (1/2)

- O’Hara [2011] presents evidence of their disruptive activities.

- A quote dangler forcing a desperate trader to chase a price up. As soon as the trader gives up, the dangler quotes back at the original level, and waits for the next victim.

Slow chess may be harder than you think (2/2)

- NANEX [2011] shows what appears to be pack hunters forcing a stop loss.

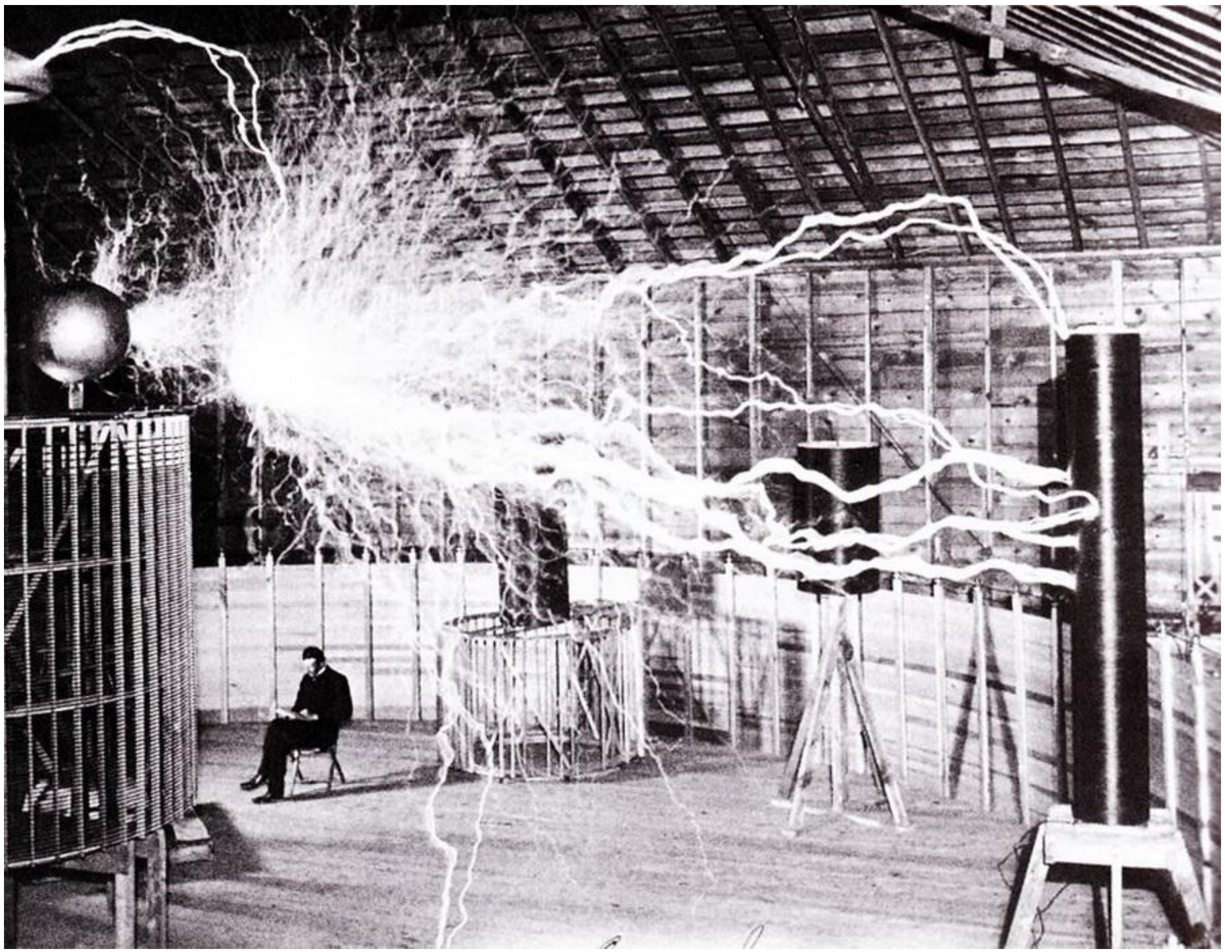


- Speed makes HFTs more effective, but slowing them down won't change their basic behavior: ***Strategic sequential trading***.

BIG data

- Perhaps more critically than speed is that HFT bases its decisions in a large amount of information, known as "big data":
  - Level 3 quotes (book depth).
  - Estimation of an order's position in the queue.
  - Estimation of other player's liquidity needs, asymmetric information.
  - NLP, etc.
- Regulators have acknowledged that they are not well-equipped for the task at hand (NYT [2010]). It took the SEC+CFTC nearly 5 months to analyze the events of May 6th 2010 ('flash crash').
- As a result, regulators have approached the CIFT at LBNL for advise.

SECTION III
Estimating PIN in a High Frequency world



Tesla tests the ‘High Frequency AC Oscillator’, giving twelve million volts

How can PIN be estimated – High Frequency (1/2)

1. Trades or bars are grouped in equal volume buckets (e.g., 40,000 contracts).
2. For each trade or bar, Z% is classified as buy ***and*** (1-Z)% as sell (denoted "**bulk classification**"). **Caution: Not all the volume of a single trade or bar is classified as buy *or* sell** (some researchers are confused by this).

$$V_\tau^B = \sum_{i=t(\tau-1)+1}^{t(\tau)} V_i \cdot Z\left(\frac{P_i - P_{i-1}}{\sigma_{\Delta P}}\right)$$

$$V_\tau^S = \sum_{i=t(\tau-1)+1}^{t(\tau)} V_i \cdot \left[1 - Z\left(\frac{P_i - P_{i-1}}{\sigma_{\Delta P}}\right)\right] = V - V_\tau^B$$

where $t(\tau)$ is the index of the last time bar included in the τ volume bucket, Z is the CDF of the standard normal distribution and $\sigma_{\Delta P}$ is the estimate of the standard derivation of price changes between time bars.

This procedure leads to better results than the *tick rule*, etc.

How can PIN be estimated – High Frequency (2/2)

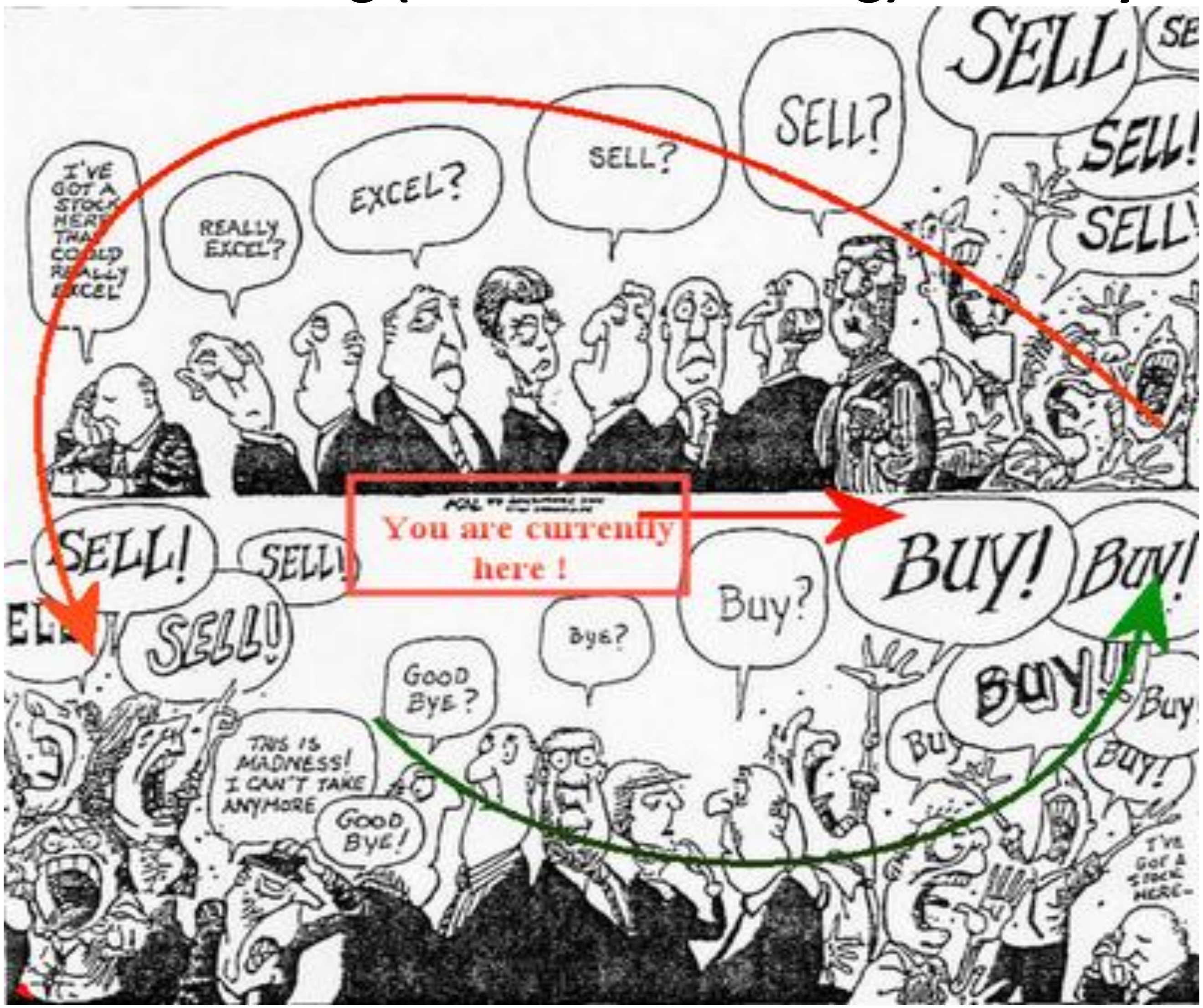
3. We know from Easley, Engle, O'Hara and Wu (2008) that the expected arrival rate of informed trades is $\mathrm{E}[V_\tau^S - V_\tau^B] = \alpha\mu(2\delta - 1)$, and $\mathrm{E}[|V_\tau^S - V_\tau^B|] \approx \alpha\mu$. The expected arrival rate of total trade is

$$\frac{1}{n}\sum_{\tau=1}^{n}\left(V_\tau^B + V_\tau^S\right) = V = \underbrace{\alpha(1-\delta)(\varepsilon+\mu+\varepsilon)}_{\text{volume from up event}} + \underbrace{\alpha\delta(\mu+\varepsilon+\varepsilon)}_{\text{volume from down event}} + \underbrace{(1-\alpha)(\varepsilon+\varepsilon)}_{\text{volume from no event}} = \alpha\mu + 2\varepsilon$$

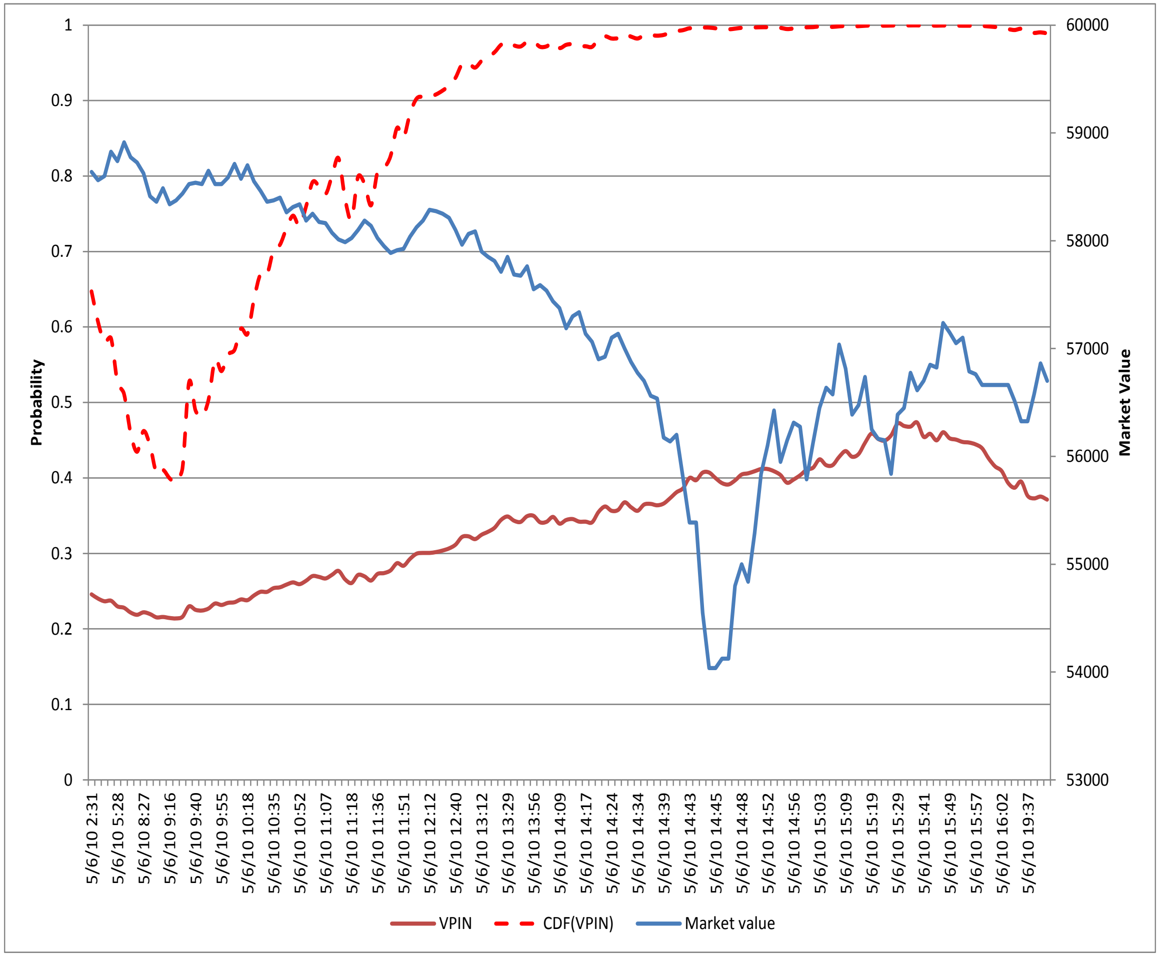
4. Note that with our convention about volume buckets, $V^B{}_\tau + V^S{}_\tau$ is constant, and is equal to $\frac{V}{n}$ for all buckets $\tau = 1, \ldots, n$.
5. From the values computed above, we can derive the *Volume-Synchronized Probability of Informed Trading* (VPIN) as

$$VPIN = \frac{\alpha\mu}{\alpha\mu + 2\varepsilon} = \frac{\alpha\mu}{V} \approx \frac{\sum_{\tau=1}^{n}|V_\tau^S - V_\tau^B|}{nV}$$

SECTION IV
Forecasting (and understanding) Volatility



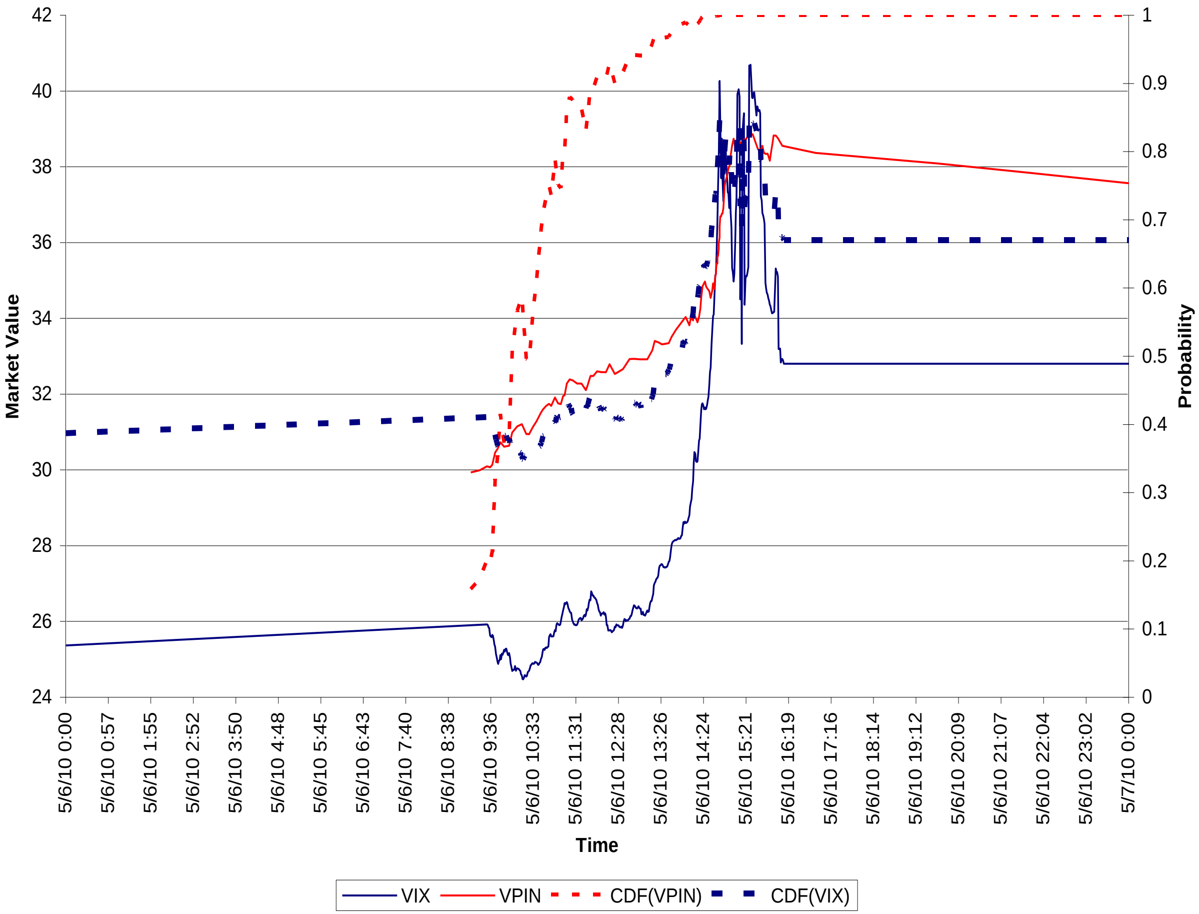
E-mini S&P500 futures on May 6th 2010



By 11:56am, the realized value of the VPIN metric was in the 10% tail of the distribution (it exceeded a 90% CDF(VPIN) critical value). By 1:08pm, the realized value of VPIN was in the 5% tail of the distribution (over a 95% CDF(VPIN)). At 2:32pm the crash begins according to the CFTC-SEC Report time line. [Link to video](#).

Note: The May 6th 2010 'Flash Crash' is just one of hundreds of [liquidity events](#) explained by VPIN!

VIX on May 6th 2010



VIX had a level of 25.92 at 9:30am, and reached a session high of 40.69 at 15:28pm.

VIX didn't reach historically high levels that day (VIX had a level of 89.53 On 10/24/2008). Rather than predicting the crash, it was impacted by it.

Toxicity-induced volatility

- There is a type of volatility that is not priced by VIX: Toxicity-induced volatility.
- Some characteristics:
  - Microstructural: This type volatility arises as a result of a failure in the liquidity provision process. Although Macro news may initiate the flow imbalance, it is MM’s underestimation of VPIN that generates the toxic inventory that ultimately forces them out of the market.
  - Endogenous: Unlike macro-volatility, this type of volatility can be predicted, as liquidity providers come under stress gradually. There is a lapse between the rise in VPIN and the liquidity crash, sometimes of hours!
  - Short-term: The liquidity failure is typically short-lived. A price jump will attract position takers, which will operate as tactical liquidity providers.

Forecasting Toxicity-induced volatility (1/3)

- An event *e* occurs every time that $CDF[VPIN(\tau)] \geq CDF^*$ while $CDF[VPIN(\tau-1)] < CDF^*$. We can index those events as $e = 1, \dots, E$, and record the volume bucket at which $CDF[VPIN(\tau)]$ crossed the threshold CDF^* as $\tau(e)$

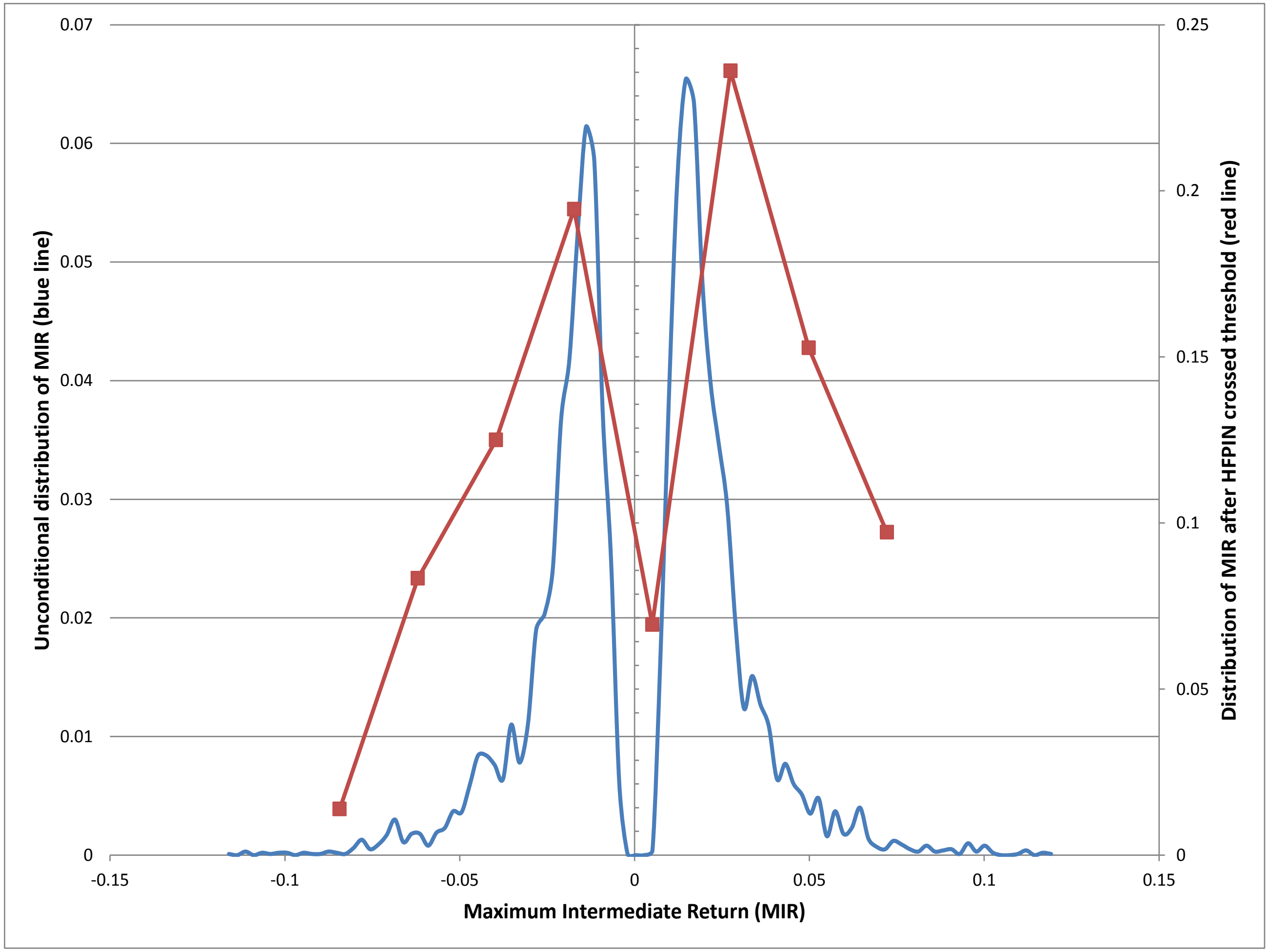
- For each particular *e*, Event Horizon h(*e*) is defined as

$$h(e) = \{h_0(e), h_1(e)\} = \max_{\substack{0 \leq h_0 < h_1 \\ 1 \leq h_1 \leq BpD}} \left| \frac{P_{\tau(e)+h_1}}{P_{\tau(e)+h_0}} - 1 \right|$$

- Similarly, Maximum Intermediate Return MIR(e) is defined

$$MIR(e) = \frac{P_{\tau(e)+h_1(e)}}{P_{\tau(e)+h_0(e)}} - 1$$

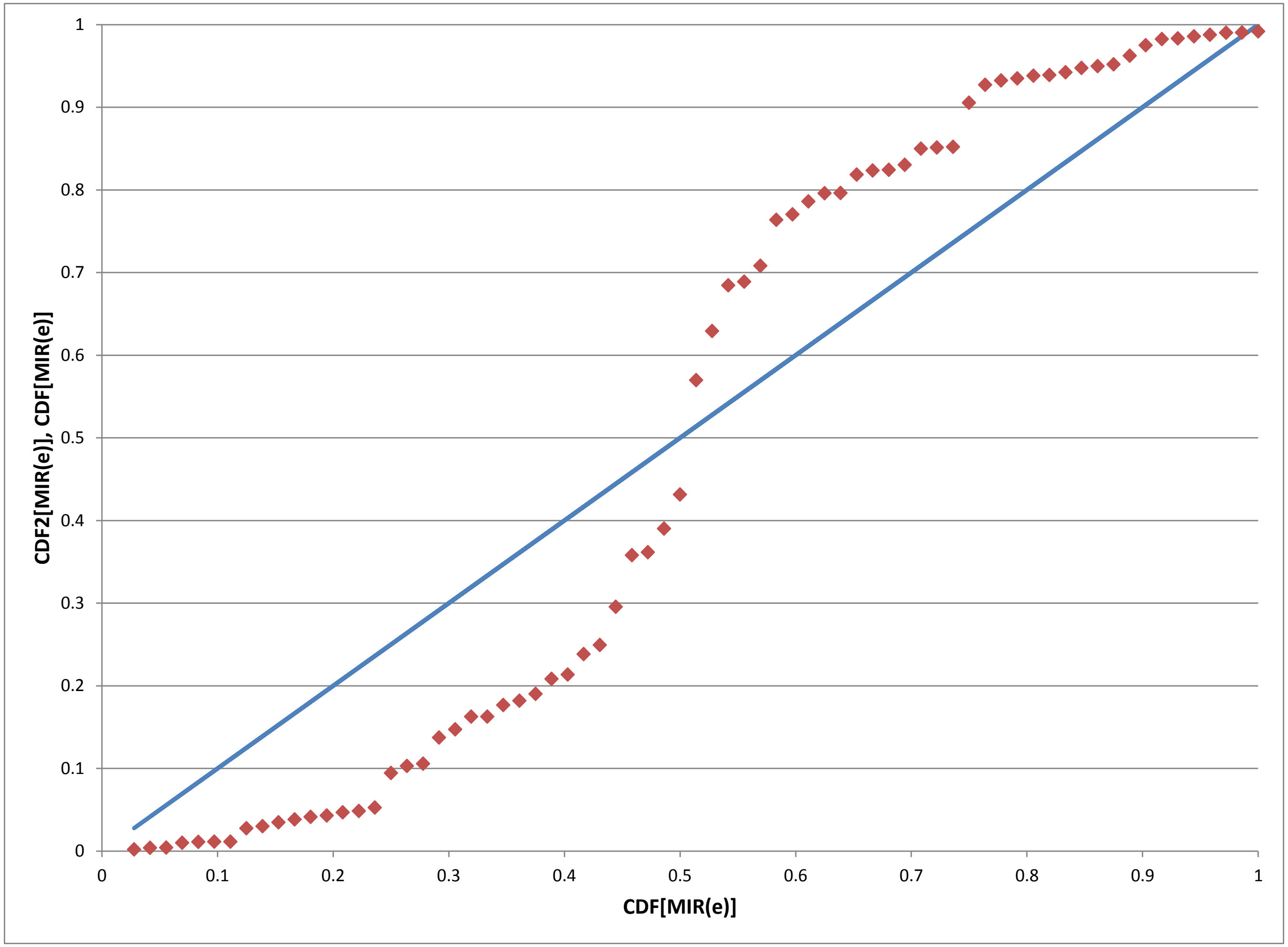
Forecasting Toxicity-induced volatility (2/3)



We have computed two distributions of probability: One for MIRs following an event *e (in red)*, and another one for MIRs at random starts (in blue).

Following an event *e*, most MIR (red) fall within one of the two tails of the unconstrained distribution (blue). High volatility occurred *after* HFPIN crossed the designated threshold

Forecasting Toxicity-induced volatility (3/3)



This qq-plot shows that both distributions are clearly different: HFPIN events are not random and indeed have consequences in terms of non-standard MIR).

This is consistent with most (red) $MIR(e)$ falling at the tails of unconstrained *MIR (blue)*.

SECTION V
Optimal Execution Horizon

Optimal Execution Strategies

- Almgren and Chriss [2000] is one of the most widely used models for execution.

- A key input for execution strategies is the execution horizon. This is typically set as exogenous, however it would be useful coming up with an estimate.

- Our goal: To determine the amount of volume needed to "disguise" a trade so that it leaves a minimum footprint on the trading range.

- This is not an execution strategy in itself, but a complement to Almgren and Chriss [2000] family of models.

Liquidity component

- Suppose that we wish to disguise a trade for *m* contracts within an execution horizon of *V* contracts. The impact on VPIN will be:

$$\frac{\left|\widetilde{V^B} - \widetilde{V^S}\right|}{V} \equiv \frac{\left|\frac{V^B}{V}(V - |m|) - \frac{V^S}{V}(V - |m|) + m\right|}{V} =$$

$$= \left|(2v^B - 1)\left(1 - \frac{|m|}{V}\right) + \frac{m}{V}\right|$$

- We call *footprint* the displacement of the order imbalance generated by our order, from $(2v^B - 1)$ to

$$\widetilde{OI} = \varphi[|m|]\left[(2v^B - 1)\left(1 - \frac{|m|}{V}\right) + \frac{m}{V}\right] + (1 - \varphi[|m|])(2v^B - 1)$$

Timing risk component

- At the same time, we cannot wait an unlimited amount of volume V to disguise m.
- For a security price S with St.Dev $\hat{\sigma}$ of price changes over volume buckets of size V_σ, the ΔS over a volume V is

$$\Delta S = \hat{\sigma}\sqrt{\frac{V}{V_\sigma}}\xi$$

with IID $\xi \sim N(0,1)$. This is bounded at a significance level λ by

$$P\left[Sgn(m)\Delta S > Z_\lambda \hat{\sigma}\sqrt{\frac{V}{V_\sigma}}\right] = 1 - \lambda$$

Footprint minimization

- A probabilistic loss function Π combines both components:

$$\Pi = \underbrace{\left|\varphi[|m|]\left[(2v^B-1)\left(1-\frac{|m|}{V}\right)+\frac{m}{V}\right]+(1-\varphi[|m|])(2v^B-1)\right|\left[\overline{S}-\underline{S}\right]}_{liquidity\ component}$$

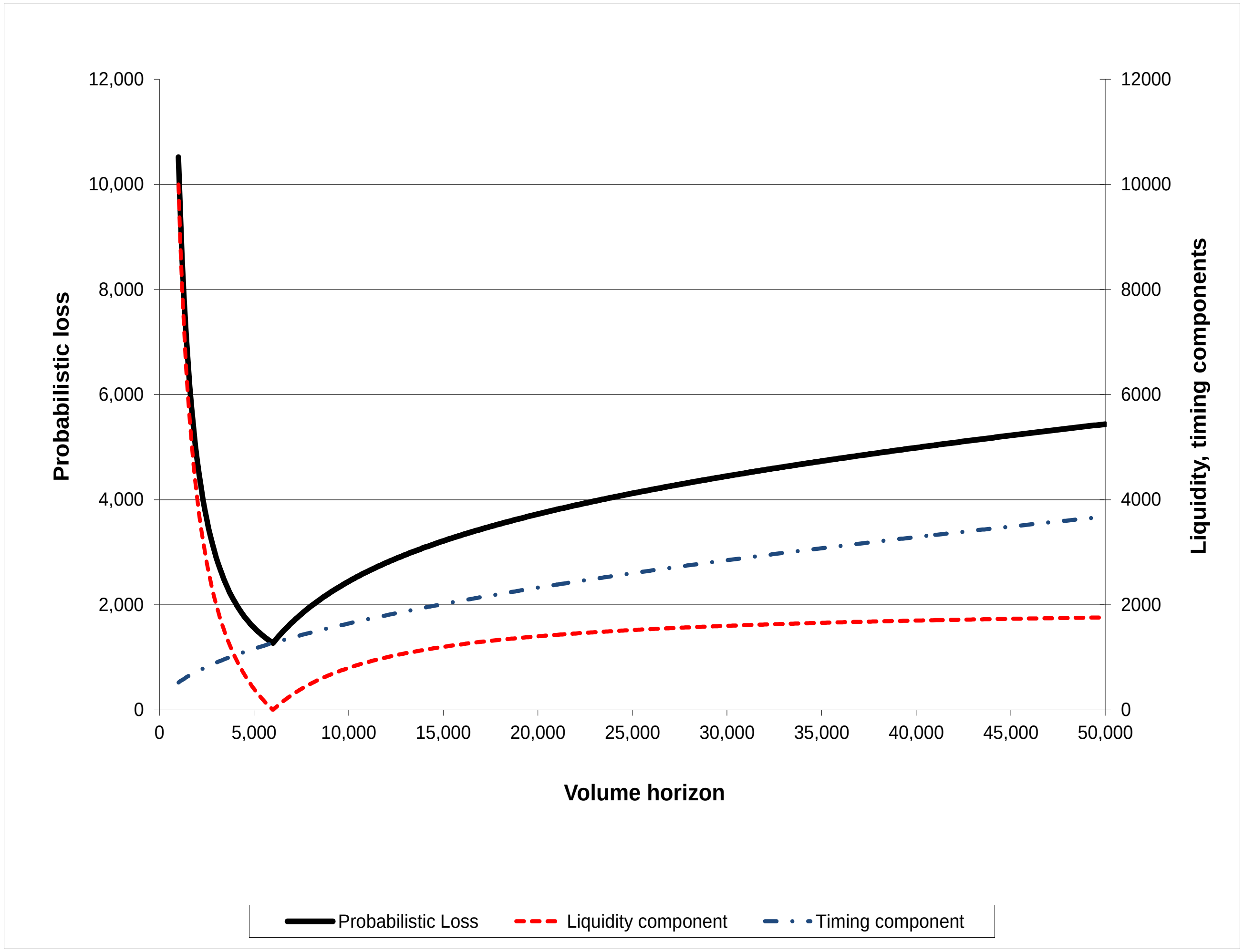
$\underbrace{-Z_\lambda\sqrt{\frac{V}{V_\sigma}}\hat{\sigma}}_{timing\ risk\ component}$. Π reaches a minimum when *V*

$$V^* = \begin{cases} \left(\dfrac{Z_\lambda\hat{\sigma}}{2\varphi[|m|]Sgn\left(\widetilde{OI}\right)[(2v^B-1)|m|-m]\left[\overline{S}-\underline{S}\right]\sqrt{V_\sigma}}\right)^{-2/3} & for\ \widetilde{OI}\neq 0 \\ \varphi[|m|]\left(|m|-\dfrac{m}{2v^B-1}\right) & for\ \widetilde{OI}=0 \end{cases}$$

$$\widetilde{OI} = \varphi[|m|]\left[\frac{m-(2v^B-1)|m|}{V}+(2v^B-1)\right]+(1-\varphi[|m|])(2v^B-1)$$

Scenario 1: $v^B = 0.4$

$\hat{\sigma} = 1{,}000$, $V_\sigma = 10{,}000$, $m = 1{,}000$, $[\overline{S} - \underline{S}] = 10{,}000$, $\lambda = 0.05$ and $\varphi[|m|]$=1.



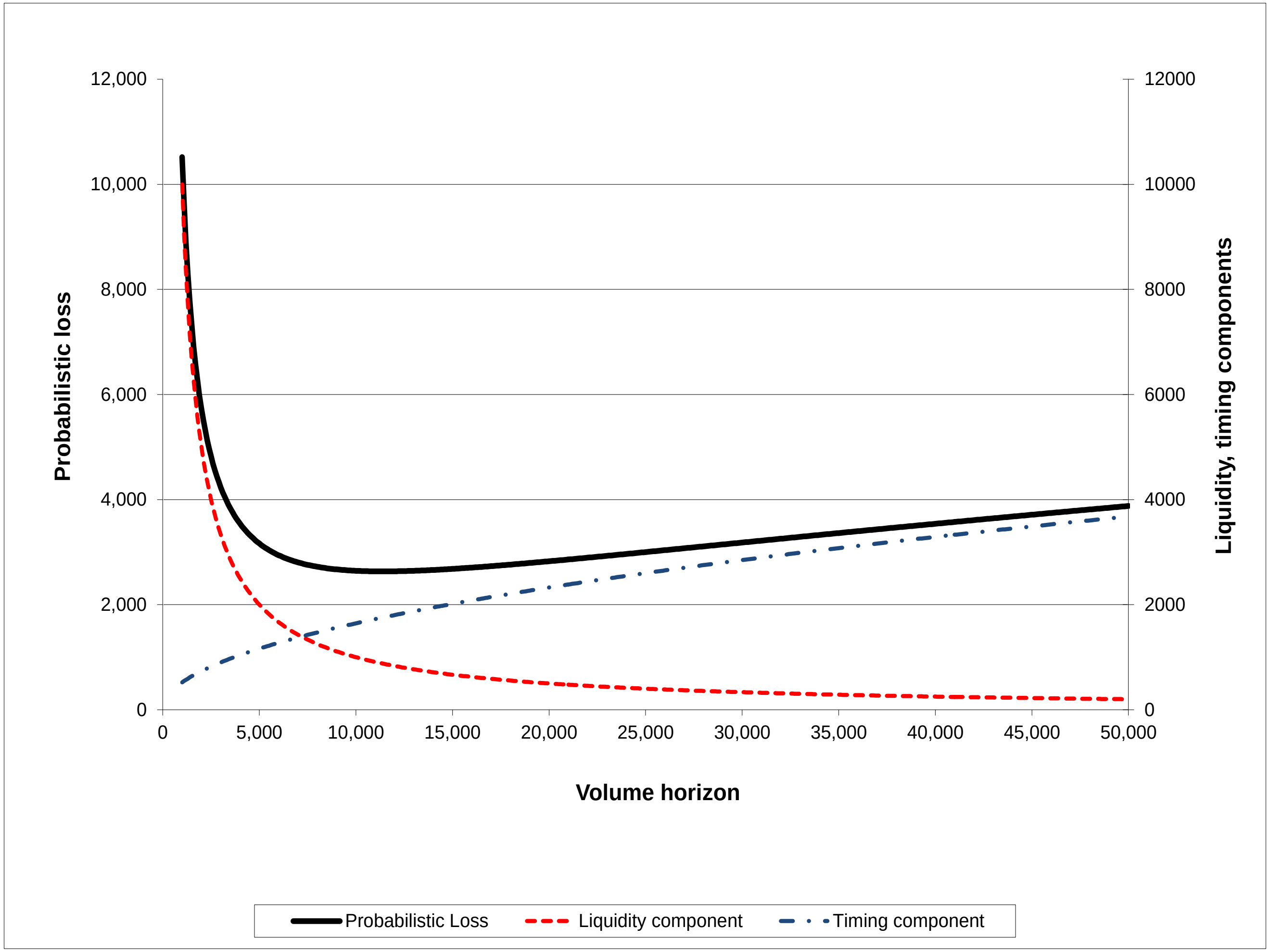
$V^* = 6{,}000$

We are buying in a selling market, thus our order contributes to narrowing the trading spread.

This evidences the fact that order's side, and not only size, determines the execution horizon.

Scenario 2: $v^B = 0.5$

$\hat{\sigma} = 1{,}000$, $V_\sigma = 10{,}000$, $m = 1{,}000$, $[\overline{S} - \underline{S}] = 10{,}000$, $\lambda = 0.05$ and $\varphi[|m|]$=1.

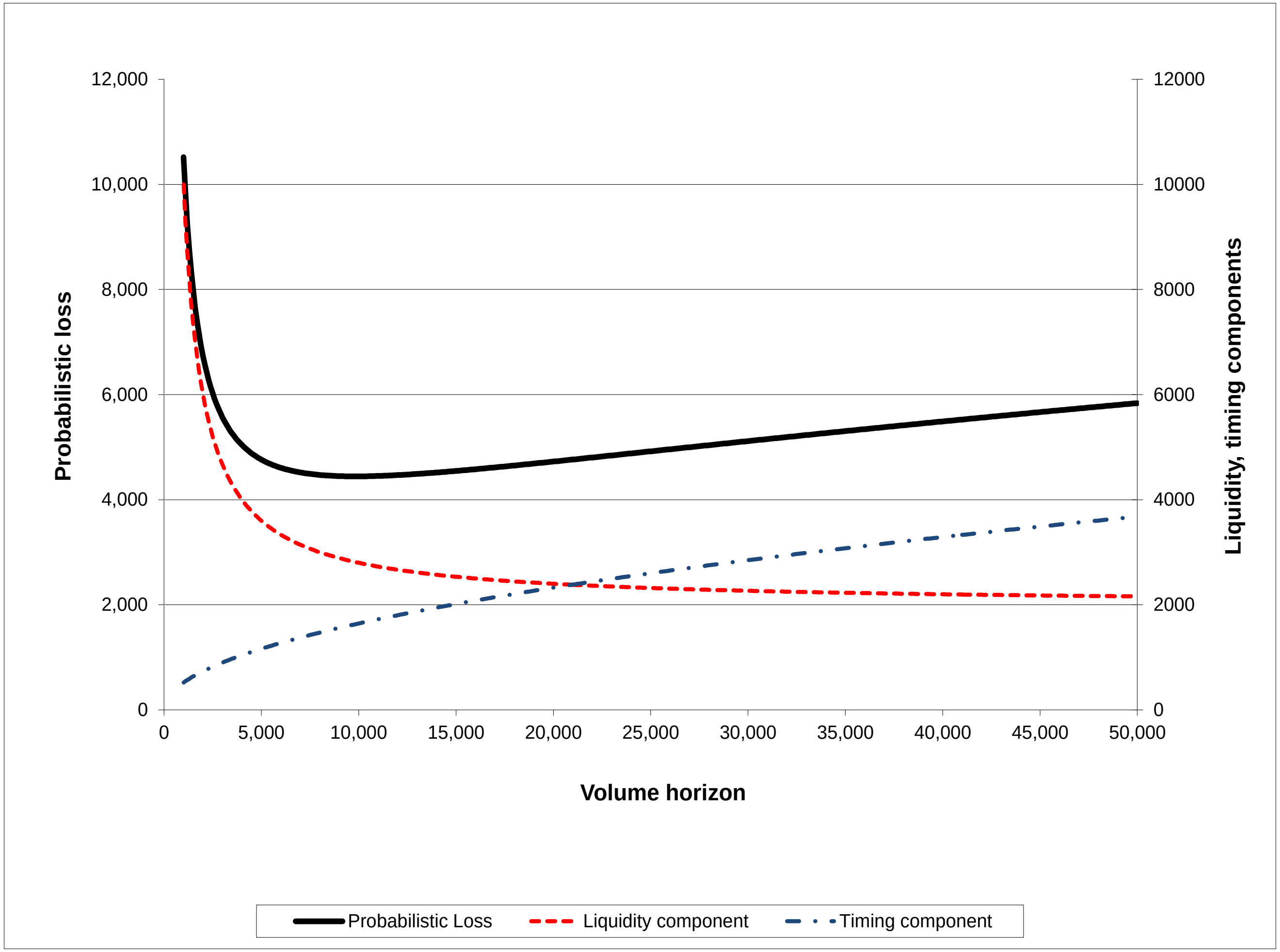


$V^* = 11{,}392$

We are buying in a balanced market. The liquidity component function is now convex decreasing, without an inflexion point, because the market is not leaning against us. The optimal V^* must be larger than in Scenario 1, but limited by greater timing risk with increasing *V*.

Scenario 3: $v^B = 0.6$

$\hat{\sigma} = 1{,}000$, $V_\sigma = 10{,}000$, $m = 1{,}000$, $[\overline{S} - \underline{S}] = 10{,}000$, $\lambda = 0.05$ and $\varphi[|m|]$=1.

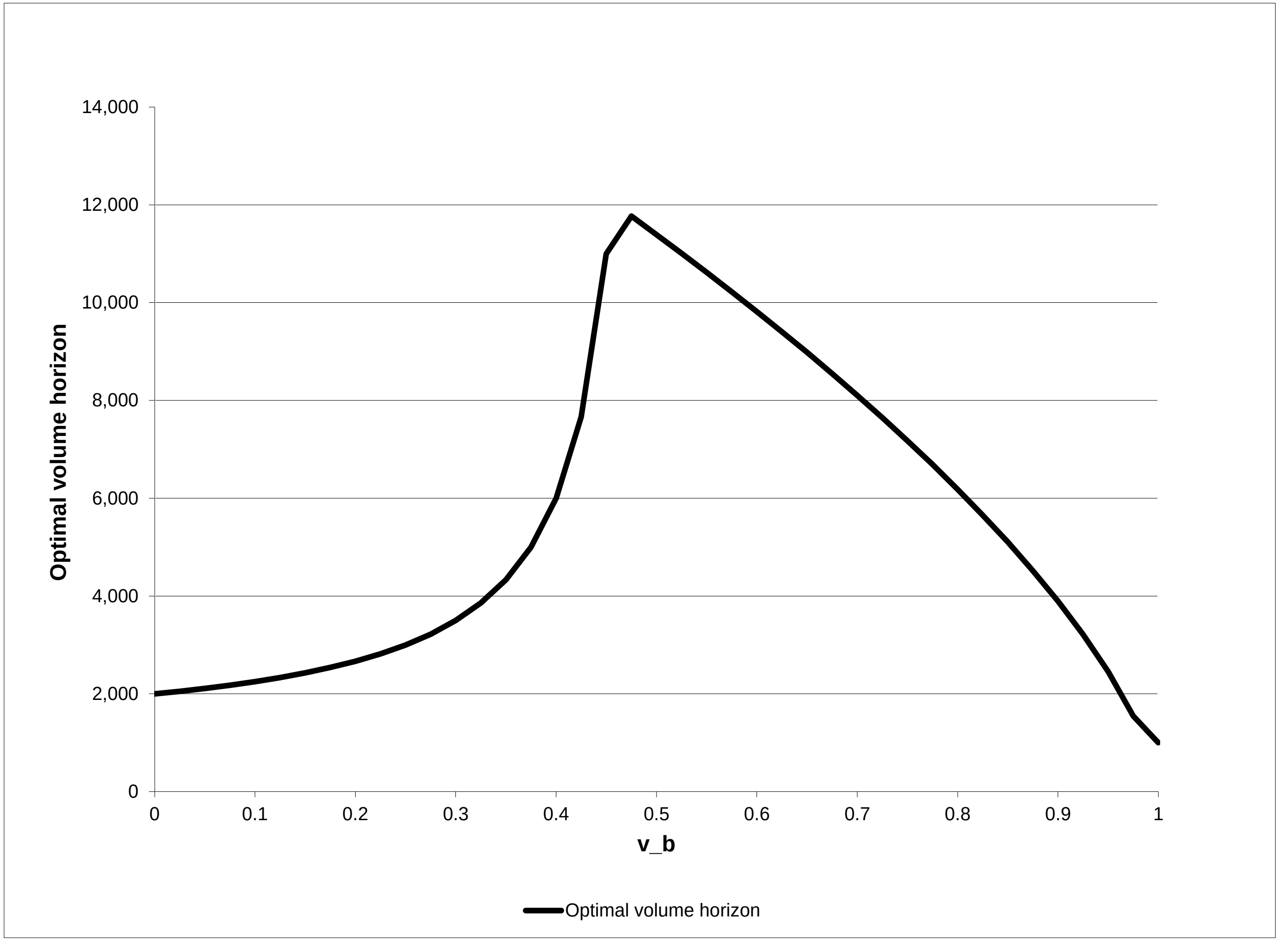


$V^* = 9{,}817$

Two forces contribute to this outcome: First, we are leaning with the market, thus we need a larger volume horizon than in Scenario I. Second, the gains from narrowing Σ are offset by the additional timing risk, and Π eventually cannot be improved further.

For all possible v^B scenarios…

$\hat{\sigma} = 1{,}000$, $V_\sigma = 10{,}000$, $m = 1{,}000$, $[\overline{S} - \underline{S}] = 10{,}000$, $\lambda = 0.05$ and $\varphi[|m|]$=1.



Optimal trading horizon for a buy order depends upon the expected fraction of buy orders in the market. When all orders are buys, v^B is 1, while if all orders are sells v^B is 0.

This explains why extreme order imbalances are typically followed by an increase in trading rates.

THANKS FOR YOUR ATTENTION!

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Bio

Marcos M. López de Prado is Head of Global Quant Research and High Frequency Futures Trading at *Tudor Investment Corp.* Formerly, a Partner at *PEAK6 Investments*, Head of Quantitative Equity Research at *UBS Wealth Management*, and a Portfolio Manager at *Citadel Investment Group*. He has been appointed Visiting Scholar at *Cornell University*, a Postdoctoral Research Fellow of RCC at *Harvard University* and a Research Affiliate at *Lawrence Berkeley National Laboratory* (U.S. Department of Energy's Office of Science). He received a Ph.D. in Financial Economics (2003), Sc.D. in Computational Finance (2011) from *Complutense University* and the National Graduation Award in Economics by the Government of Spain (National Valedictorian, 1998).

Dr. López de Prado is a member of the editorial board of the *Journal of Investment Strategies* (Risk Journals), and has co-authored several academic papers with Professors Maureen O'Hara and David Easley (Cornell University), which are listed among the most read in SSRN and have resulted in three international patent applications. His current Erdös number is 3 (with a valence of 2, through Bailey→Pomerance and Foreman→Komjáth), and would be happy to hear from potential co-authors with an Erdös number of 1.

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