

Dynamic Trading using short term and long term predictions

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Introduction

Active investors, such as hedge funds, mutual funds, proprietary traders, individuals and other asset managers, often have multiple strategies to predict returns. These predictors may have different mean reversion (alpha decay) rates. Short-term strategies will have a high mean reversion rate and long-term strategies have a low one. The investors may seek to exploit all the predictors to form a strategy that predicts returns more accurately, minimizes risks and also minimizes transactions costs. Garleanu and Pedersen analyze this problem in *“Dynamic Trading with Predictable Returns and Transaction Costs”*. The paper has not been published yet, but a pre-print is available on their web site.

1. Problem formulation

1.1 Preliminaries

An economy with S securities is considered, traded at each time $t = 1, 2, 3, \dots$. The securities' price changes between times t and $t+1$, $p_{t+1} - p_t$, are collected in a vector r_{t+1} given by:

$$r_{t+1} = \mu_t + \alpha_t + u_{t+1}$$

where μ_t is the “fair return” vector (given e.g. by CAPM), α_t are the predictable excess returns and u_{t+1} is an unpredictable noise term with variance $\text{var}_t(u_{t+1}) = \Sigma$.

As explained in the introduction, we wish to combine multiple predictors into one for the alphas. In this paper it is assumed that

$$\alpha_t = B f_t$$
$$\Delta f_{t+1} = -\Phi f_t + \epsilon_{t+1}$$

where f_t is a K -vector of factors that predict returns and B is a $S \times K$ matrix of factor loadings. Φ is a $K \times K$ positive-definitive matrix of mean-reversion coefficients for the factors and ϵ_{t+1} is the shock affecting the predictors.

It is natural to assume that the agent uses certain characteristics of each security to predict its returns. Hence, each security has its own return-predicting factors (whereas in the general model above all the factors could influence all the securities). In this case, we let alpha for security s be given by:

$$\alpha_t^s = \sum_i \beta^i f_t^{i,s}$$

where $f_t^{i,s}$ is characteristic i for security s and β^i is the predictive ability of characteristic i .

1.2. Transaction Costs

The paper we studied considers the effect of transaction costs on the trading strategy. It is assumed that when trading $\Delta x_t = x_t - x_{t+1}$ shares the transactions costs are given by:

$$TC(\Delta x_t) = \frac{1}{2} \Delta x_t^T \Lambda \Delta x_t$$

The intuition behind that is that trading Δx_t shares the (average) price is moved by $\frac{1}{2} \Lambda \Delta x_t$ and this results in a total trading cost of Δx_t^T times the price move, which gives TC . We will assume that $\Lambda = \lambda \Sigma$.

1.3 Optimization formulation

The investor needs a strategy that provides the best possible portfolio at each time step. To do that he solves the following optimization problem:

$$\max_{x_0, x_1, \dots} E_0 \left[\sum_t (1 - \rho)^t \left(x_t^T \alpha_t - \frac{\gamma}{2} x_t^T \Sigma x_t - \frac{1}{2} \Delta x_t^T \Lambda \Delta x_t \right) \right]$$

where ρ is a discount factor and γ is the risk aversion coefficient. This formulation tries to maximize the returns given the alphas, while taking into consideration the transaction costs and trying to minimize the risk involved.

2. Theoretical Results

To solve the above optimization problem, dynamic programming is used and the main result is the following:

$$x_t = \left(1 - \frac{a}{\lambda}\right) x_{t-1} + \frac{a}{\lambda} \text{target}_t$$

ie the optimal portfolio is a linear combination of the current portfolio and a moving 'target' portfolio. Parameter a depends on γ , λ and ρ and the target portfolio is function of a , γ , ρ , Σ , Φ , B and f_t , where f_t is what makes it time-dependent. Matrices Σ , Φ and B need to be estimated from the data using regression, whereas setting the values for γ , λ and ρ is up to the discretion of the manager.

The paper provides an expression for target_t which is rather complicated. In order to simplify it, the additional assumption is made that the mean reversion of each factor f^k only depends on its own level, that is, $\Phi = \text{diag}(\phi^1, \dots, \phi^K)$ is diagonal. Under this assumption, the target portfolio is given by:

$$\text{target}_t = (\gamma \Sigma)^{-1} B \left(\frac{f_t^1}{1 + \phi^1(1 - \rho) \frac{a}{\gamma}}, \dots, \frac{f_t^K}{1 + \phi^K(1 - \rho) \frac{a}{\gamma}} \right)$$

where we can clearly see that the weight of each factor depends on its mean reversion rate. Notice that predictors with slower mean reversion get more weight. This agrees with our intuition, since these predictors lead to a favorite positioning both now and in the future.

The paper also considers a static model, where the future is fully discounted ($\rho = 1$) and we are only interested in the current period. The investor simply solves:

$$\max_{x_t} \left(x_t^T \alpha_t - \frac{\gamma}{2} x_t^T \Sigma x_t - \frac{\lambda}{2} \Delta x_t^T \Sigma \Delta x_t \right)$$

The solution in this case is a specialization of the previous one:

$$x_t = \frac{\lambda}{\gamma + \lambda} x_{t-1} + \frac{\gamma}{\gamma + \lambda} (\gamma \Sigma)^{-1} \alpha_t$$

The optimal portfolio is again a linear combination of the previous position and a moving target portfolio, but now the expressions for the weights and the target portfolio are much simpler.

3. Application: Dynamic Trading of Commodity Futures

3.1 Methodology

Six different commodity futures are considered in order to test the performance of the trading strategy discussed. We collect data on the six commodities – Aluminum, Copper, Nickel, Zinc, Lead, and Tin – from London Metal Exchange (LME). Sample period ranges from 7/11/1995 to 4/24/2009 and we normalize the price series such that each commodity's price changes have annualized volatility of 10%.

Each commodity characteristic is its past returns at various time horizons. As such, in order to predict the 1-day, 1-year, and 5-year return factors for the commodities, pooled panel regression on the data set is run using the OLS method (the method of least squares) to obtain:

$$p_{t+1}^s = -0.002 + 0.0947 \times f_t^{5D,s} - 0.0161 \times f_t^{1Y,s} + 0.0084 \times f_t^{5Y,s} + u_{t+1}^s$$

Here, $f_t^{5D,s}$ is the average past 5 days' price changes divided by past month's standard deviation of price changes, $f_t^{1Y,s}$ is the average past year's price changes divided by the year's standard deviation of price changes, and $f_t^{5Y,s}$ is the same over the past 5 years. The coefficients of f_t are factor loadings, which we store in matrix B. Using regression, we also obtain mean reversion coefficients ϕ to be [0.1992, 0.0483, 0.107] for the three return factors.

The variance-covariance matrix Σ is estimated using the covariance of prices over the full sample. Absolute risk aversion is set at $\gamma = 10^{-9}$ which can be thought of as the relative risk aversion of 1 for an agent with \$1 billion under management. Transaction cost is taken to be $\lambda = 5 \times 10^{-7}$. In our experiments, we vary both γ and λ to see how it affects excess returns. The discount rate is set so that it corresponds to a 2% annualized rate, i.e. $\rho = 1 - e^{\frac{-0.02}{260}}$.

We consider two different trading strategies, both proposed by the same authors: the optimal dynamic strategy and the optimal static strategy (single-period optimization). For the static portfolio, the coefficient on x_{t-1} is chosen to be the same as the one for the optimal portfolio, which is numerically the same as choosing appropriate λ to maximize the portfolio's Sharpe Ratio.

Implementation of this portfolio strategy requires constant readjustment of portfolio weights. Also, large amount of assets are traded on a given trading day. In order to check if the portfolio is self-financing, we calculate and plot the rebalancing costs of the portfolio throughout the trading period. The mean rebalancing cost is calculated to be close to 0, leading to a conclusion that the portfolio is self-financing for the set of 6 commodities we have chosen.

3.2 Results

Following chart shows the excess returns, $\alpha \times \Delta x(t)$, for the optimal dynamic trading strategy and for its static counterpart. The dynamic strategy beats the static strategy for all trading days. The dynamic trading strategy trades slowly compared to the static one and trades towards the more persistent signals. Static, on the other hand, simply tries to control the trading speed but doesn't differentiate between the signals and incurs larger transaction costs. This is the theoretical explanation behind dynamic strategy performing better than the static one.

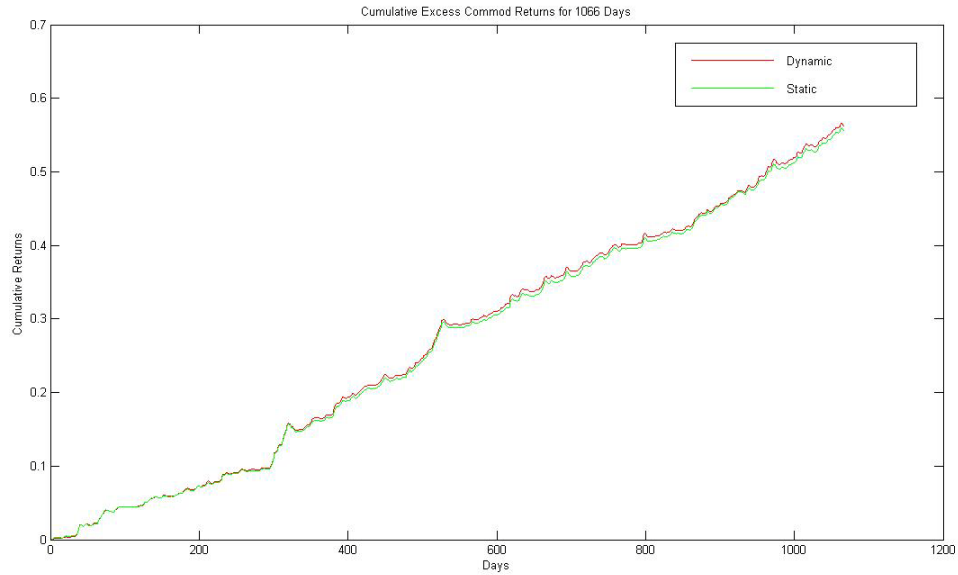


Figure: Excess returns for dynamic and static portfolios

We also vary absolute risk-aversion coefficient and the transaction cost to see their effects on the excess returns. When the transaction cost is increased, the excess returns go down, which is as expected. Similarly, as the agent becomes more risk-averse, the returns go down, consistent with the relationship between risk and reward.

The performance of each strategy measured by Sharpe Ratio is shown in the table below. We see that dynamic strategy achieves higher Sharpe Ratio than the static strategy.

Dynamic Strategy	0.49
Static Strategy	0.47

Table: Sharpe Ratio comparison for Dynamic and Static strategies

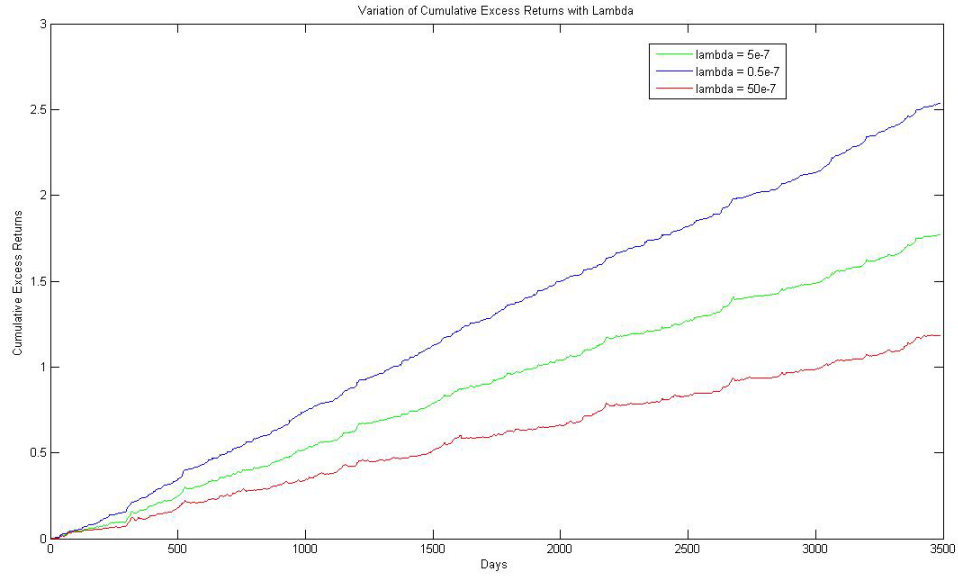


Figure: Excess returns for different transaction costs

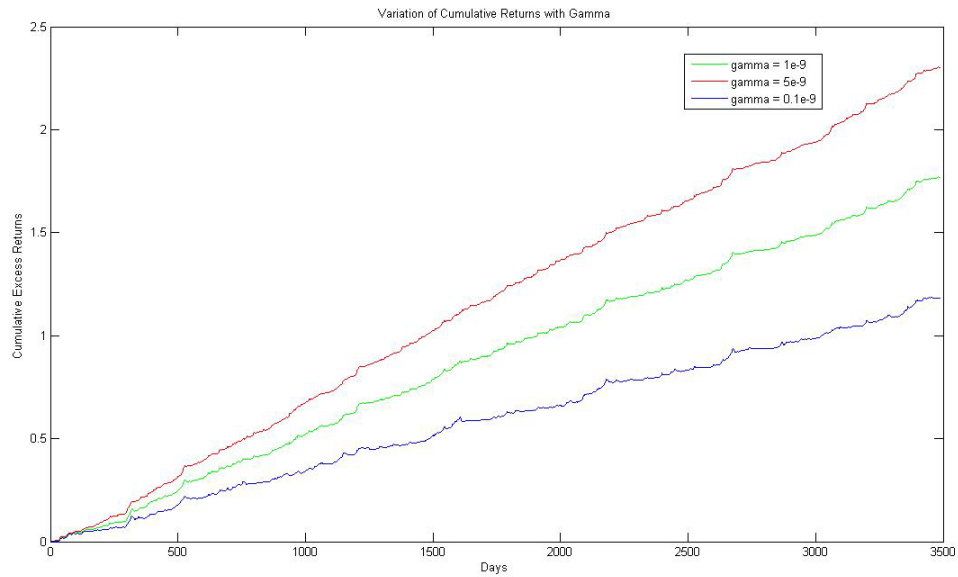


Figure: Excess returns for different risk-aversion coefficient

The plot of the rebalancing costs for different trading days is shown below. The costs fluctuate between positive and negative values, reflecting buying and selling of assets. The mean of the rebalancing costs at the end of the trading period is close to

0, which means the trading strategy for the given portfolio of commodities is self-financing.

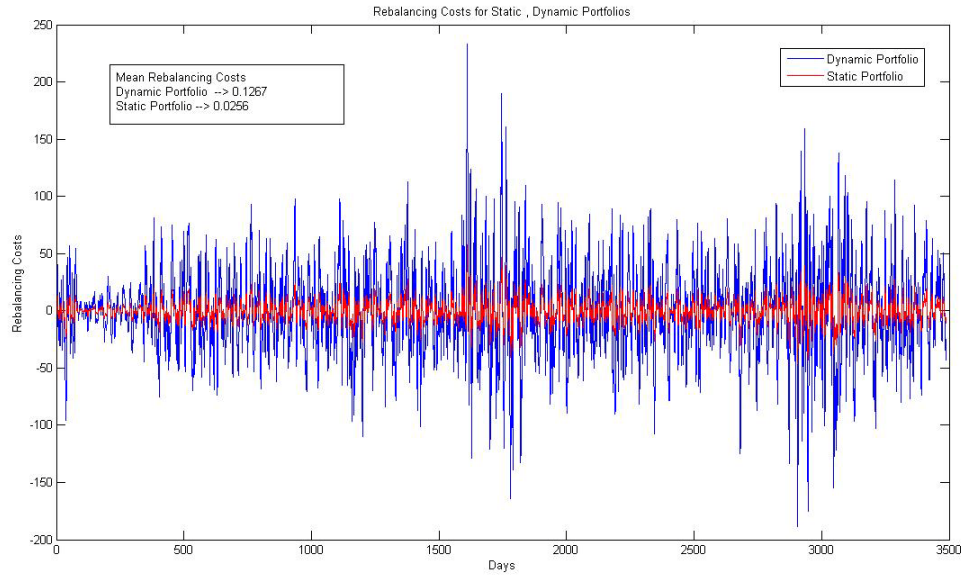


Figure: Rebalancing costs on different trading days

4. Evaluation of the Dynamic trading strategy with EVA's data

We now evaluate the dynamic trading strategy with EVA Fund's data. Here we try to construct an optimal dynamic trading strategy using short term and long term predictions. The short term return predictors were derived from the EVA's Statistical Arbitrage data and the long term predictors were derived from EVA's Equity Market Neutral (EMN) portfolio data.

The time period for the analysis was from 03/01/2005 to 27/03/2009 and we had the return values for 1089 securities. We had daily data for the Statistical Arbitrage portfolio and monthly data for the EMN portfolio. Since, our trading strategy requires both the predictors to be of the same frequency, we had to interpolate the EMN monthly data to get daily return values.

The logic for interpolation is as follows, say we are at the first day of the month and we have the monthly return values for the current month and previous month and we shall compute the daily return for the first day as $(1 + r_1)^{20} = (r_{\text{current}}/r_{\text{prev}})$. And solve for r_1 . Now, in order to get the daily return for the second day of the month, we shall use the value of r_1 obtained earlier and do $(1+r_1)*(1+r_2)^{19} = (r_{\text{current}})/(r_{\text{prev}})$. And solve for r_2 . This way we can obtain the daily return values for all the days in the month.

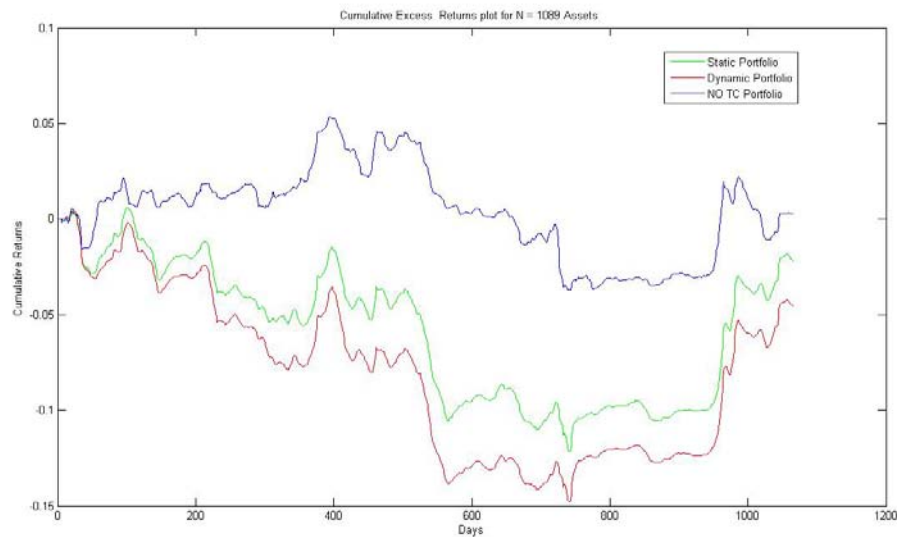
4.1 Determining the Mean reversion coefficients

As with the commodities data, we determine the mean reversion coefficients by regression. We first obtain the B matrix by doing OLS on the predictors with the normalized returns. We then determine the mean reversion coefficients by regressing F_t on F_{t-1} , where F is the matrix that contains all the predictors.

4.2 Performance with all the assets included in the portfolio

As a first step we evaluated the performance of the dynamic trading strategy including all the 1089 assets in our portfolio. We found the results to be pretty unsatisfactory. Upon further analysis, we realise that the covariance between the assets plays a huge role in determining the returns of the strategy. And when we have a large number of assets, we will get poorer values of covariance since we are trying to estimate a large number of parameters using a limited amount of data points. Thus, our first conclusion was that we should not be using all the 1089 assets in our portfolio. Our optimal portfolio will have just a subset of these assets.

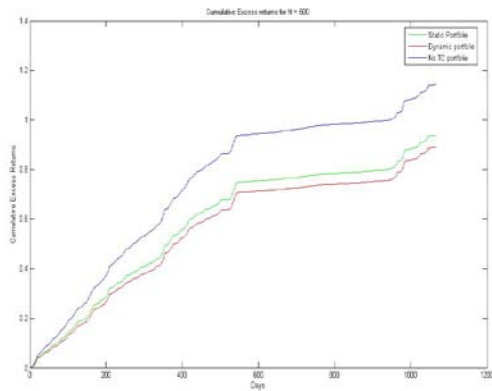
The following plot shows the performance of the strategy with all the assets included in the portfolio. As we see, the excess returns are very unsatisfactory for all the trading strategies i.e., the dynamic, the static and the one with no transaction costs.



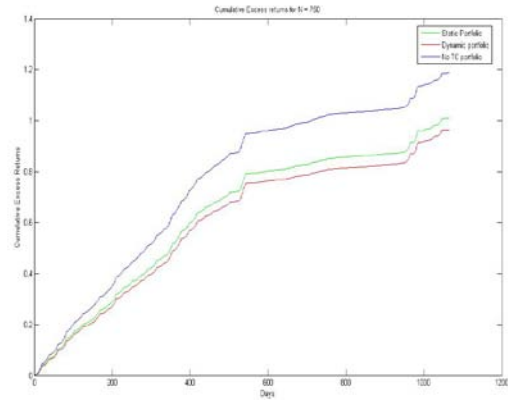
Thus, we see the need to reduce the size of the portfolios. To this end, we tried to plot the excess returns generated by the optimal trading strategy for various values of N (the number of assets). The plots are as shown below:

For N = 500 Assets

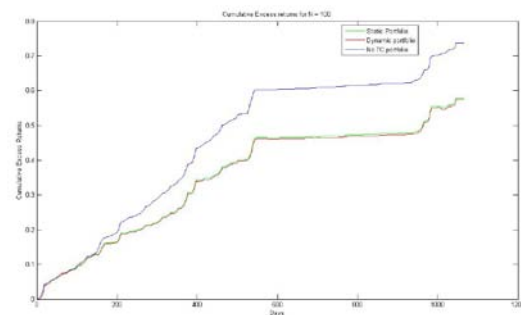
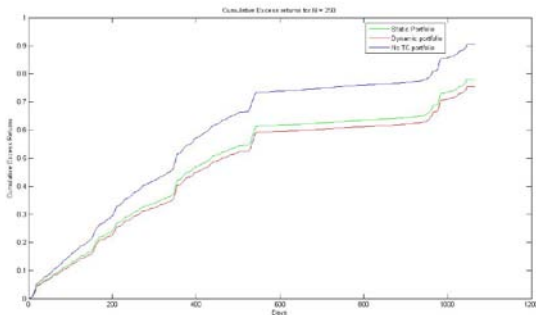
N = 750 Assets



For N = 250 Assets



For N = 100 Assets



From the above plots we found that as we decreased the number of assets in the portfolio, the excess returns kept getting better and better. This led us to the conclusion that the optimal number of assets that we should have in our portfolio is less than 100.

4.3 Performance of the Dynamic Trading Strategy Versus EVA's Stat-Arb and EMN Strategies

In this section we shall evaluate the performance of the dynamic trading strategy and see how it compares with EVA's Stat-Arb and EMN strategies. That is, for a given number of randomly chosen assets, how many times does the dynamic trading strategy outperform the Stat-Arb and EMN strategies? This is an important indicator as this shows the effectiveness of the Dynamic Trading strategy.

To this end, we first fix a number of assets that we plan to have in our portfolio and then we take a random subset of the assets from the 1089 stocks and run both the dynamic trading strategy and EVA's Stat-Arb and EMN strategies on these assets. We then repeat this for 10000 simulations and find out how many times the overall returns from the dynamic trading strategy are higher than that of EVA's Stat-Arb and EMN strategies.

There are two ways in which one could measure the performance. One is the overall returns and another is in terms of the Sharpe Ratios. We evaluate the performance of the dynamic trading strategy using both these metrics.

The results from our simulation are as follows (for N = 19 Assets):

4.3.1 Performance in Terms of Excess Returns

Trading Strategy	Stat-Arb	EMN
Dynamic Trading Strategy	3.4%	91.6%
Static Trading Strategy	5.5%	91.8%
NO TC (Target Portfolio)	17.7%	99.1%

Thus, we find that the Dynamic Trading Strategy outperforms the Stat-Arb strategy only 3.4% of the times. This shows that the dynamic trading strategy on the whole does not work well with equities as it does with Commodities.

Also, the performance of the dynamic trading strategy decreases even further as we increase the number of assets in the portfolio. For example for N = 100 Assets, following are the performance results:

Trading Strategy	Stat-Arb	EMN
Dynamic Trading Strategy	0.0%	98%
Static Trading Strategy	0.0%	98%
NO TC (Target Portfolio)	0.0%	99%

4.3.2 Performance evaluation in terms of sharpe ratios

We can also evaluate the performance of the dynamic trading strategy in terms of sharpe ratios i.e., find out if the dynamic trading strategy

For N = 19 Assets the results are as follows:

Trading Strategy	Stat-Arb	EMN
Dynamic Trading Strategy	8.564%	87.34%

Static Trading Strategy	12.84%	86.24%
NO TC (Target Portfolio)	12.46%	80.19%

Once again we find that the dynamic trading strategy does not “consistently” beat the Stat-Arb and EMN Strategies in terms of Sharpe Ratios. However, the results are better than the one we obtained for excess returns.

For N = 100 Assets the results are as follows:

Trading Strategy	Stat-Arb	EMN
Dynamic Trading Strategy	0.0%	99%
Static Trading Strategy	0.0%	99%
NO TC (Target Portfolio)	1.2%	91%

Thus, once again we see that increasing the number of assets has a detrimental effect on the performance of the dynamic trading strategy.

4.4 Determining the Optimal Portfolio

We know from the previous section, that the dynamic trading strategy does not outperform EVA’s Stat-Arb and EMN strategies for all assets. Hence, there is a need to determine the optimal set of stocks for which one could obtain higher returns using the dynamic trading strategy.

In this section, we describe an algorithm which we can use to find such an optimal portfolio of assets that maximizes the excess returns.

To this end , we used the following algorithm:

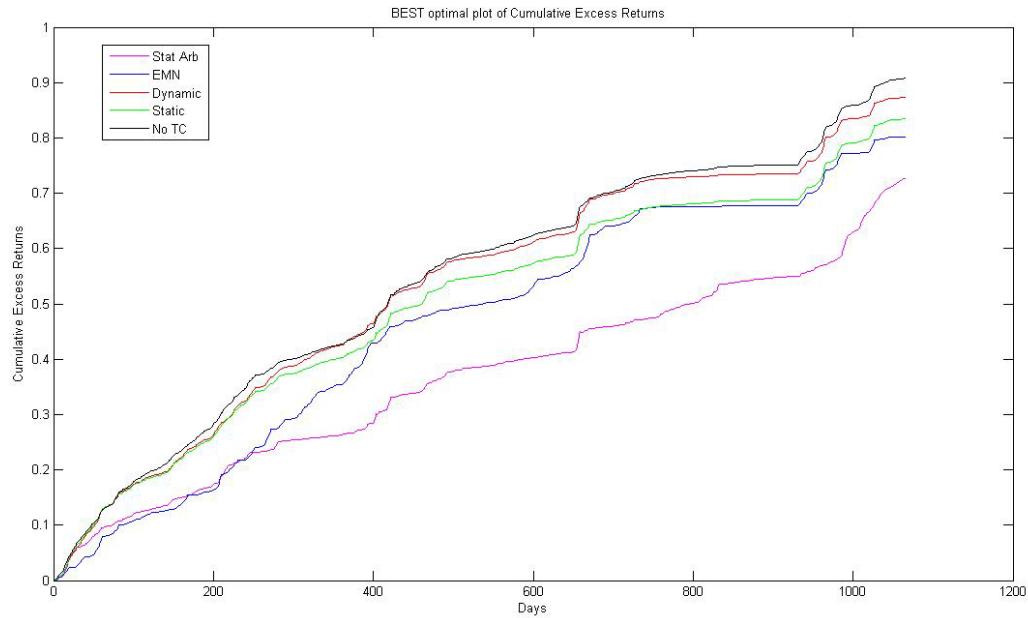
- 1) Fix the number of assets (Num_assets) in the portfolio
- 2) Choose a random subset of assets from the 1089
- 3) Get the cumulative excess returns, sharpe ratio and rebalancing costs for the portfolio constructed from step (2)
- 4) Repeat steps (2) and (3) with N = 10000 simulations and find the optimal portfolio that achieves maximum returns on a given optimization criterion (like highest excess return or sharpe ratio etc)
- 5) Repeat steps 1 through 4 for different values of Num_assets

Thus, using the above algorithm one could obtain a portfolio of assets which would give us highest excess return or highest sharpe ratio.

4.5 Optimal Portfolio obtained by maximizing Excess Return

We know from earlier analysis that the value of Num_assets must be less than 100. Hence we simulated for different values of Num_assets less than 100 to obtain the optimal portfolio of assets .

When we optimized on maximizing the excess return, we obtained the following plot for the best portfolio. We found the optimal number of assets to be $N = 19$



The Keys of the set of assets that gave maximum excess returns are:

841 849 45 1034 823 214 323 1089 843 225 621 631 605 181 839 984 752 117 693

In the above plot, we find that the dynamic trading strategy nearly equals the target portfolio (the one with no transaction costs). Also, we find that all the trading strategies Dynamic, Static and No Transaction costs produce better excess returns than the ones which rely solely on either short term or long term prediction. This means that, it makes sense to trade using a dynamic trading strategy that uses both short and long term prediction than relying solely on short term or long term predictions.

4.5.2 Sharpe Ratios

However, the excess returns generated by this strategy have a large standard deviation which results in a pretty low value of Sharpe Ratio. The Sharpe Ratio values for this portfolio is:

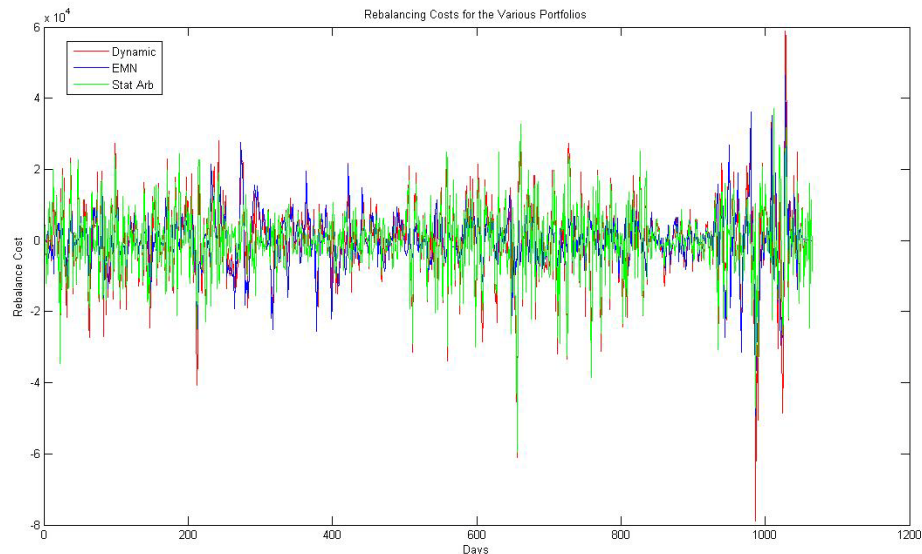
Portfolio	Dynamic	Static	No TC	EVA's Stat-Arb	EVA's EMN
Sharpe Ratio	0.26131	0.2904	0.34306	0.5102	0.2123

4.5.3 Rebalancing Costs

Whenever we analyze a dynamic trading strategy, it is very important to take the rebalancing costs in to account. We have to ensure that we are not getting higher excess returns at the expense of paying more money in rebalancing the portfolio. In the dynamic strategy, we rebalance the portfolio by calculating the new values of X at each instance. So, there is definitely a rebalancing cost involved.

We can say that our strategy is optimal if on an average we do not pay a very high amount of rebalancing costs i.e., the rebalancing costs average out over time, so that we do not make or lose money by rebalancing daily.

The rebalancing costs for the above portfolio is plotted below:



The mean rebalancing costs are as follows:

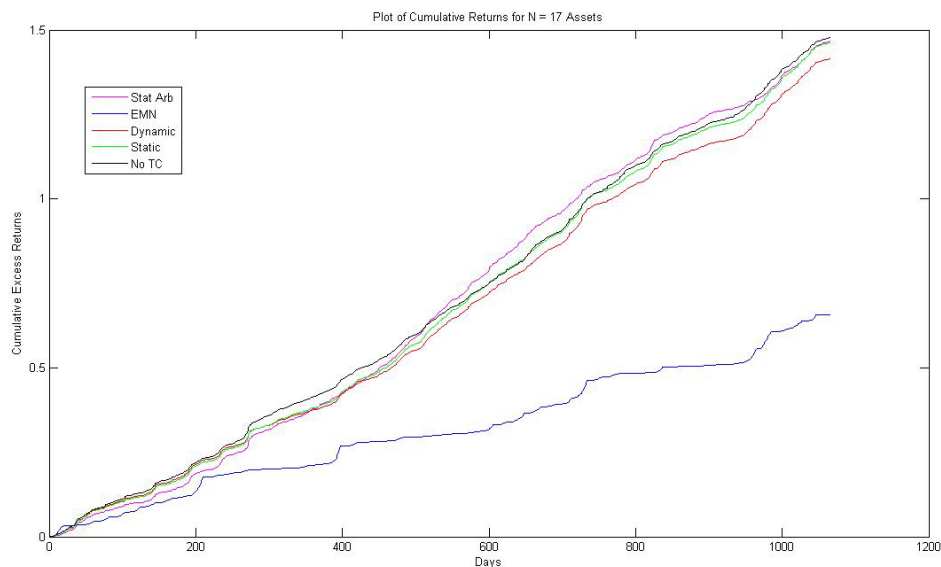
Portfolio	Dynamic	Static	No TC	EVA's Stat-Arb	EVA's EMN
Rebalancing Cost	-\$24.21	-\$17.67	-\$207.20	\$-32.23	7.24

We find the rebalancing costs for the NO TC portfolio to be very high , this is because one would be rebalancing the portfolio more when we know that there are no transaction costs involved in the process. Thus the $X_i - X_{i-1}$ (change in the number of shares) would be higher for the target(no TC) portfolio.

On the whole we find the rebalancing costs for all the portfolios to be reasonable. In the sense the values are not extremely high. Also, we find that the rebalancing costs for the dynamic strategy to be comparable to that of the Stat-Arb and EMN strategies. So, we don't stand to lose a lot of money by investing in the dynamic trading strategy.

4.6 Optimal Portfolio obtained by maximizing Sharpe Ratio

In this section we describe another method of obtaining the optimal portfolio. Instead of maximizing the excess returns we maximize the Sharpe Ratio. Following is the plot for the optimal portfolio thus obtained



The Optimal Set of assets to be included in the portfolio are:

537 773 582 879 934 747 520 1034 898 72 663 1001 382 1019 954 881 133

From the above plot we find that the excess returns obtained using both the dynamic and the Stat-Arb strategies are more or less similar.

4.6.1 Sharpe Ratios

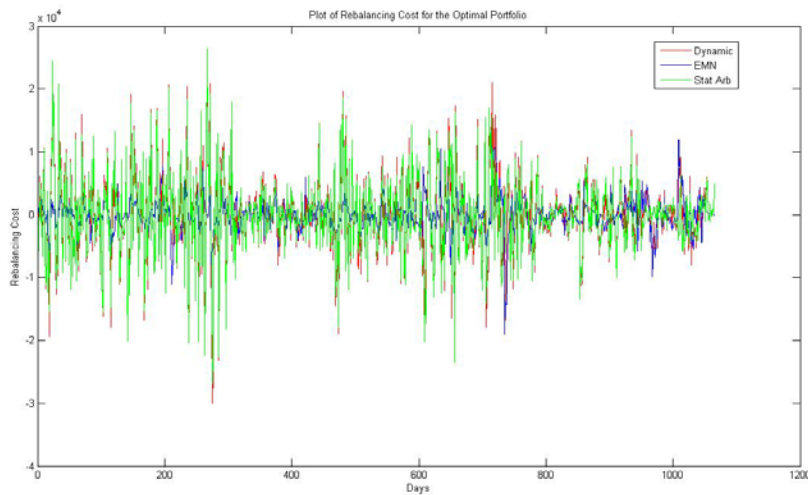
We find that the Sharpe ratio for the Static strategy is higher than that of the Stat-Arb strategy. Following are the Sharpe Ratios for the various strategies.

Portfolio	Dynamic	Static	No TC	EVA's Stat-Arb	EVA's EMN
Sharpe Ratio	1.0843	1.1252	1.1436	0.9841	0.4071

Thus, we find that the Sharpe ratio for the Dynamic Trading strategy is higher than that of the Stat-Arb and EMN strategies. Hence, one can say that for the optimal set of assets obtained above it is more profitable to trade using the dynamic strategy.

4.6.2 Rebalancing Costs

Here we plot the rebalancing costs for the optimal portfolio obtained by maximizing the Sharpe Ratio.



The mean rebalancing costs are as follows:

Portfolio	Dynamic	Static	No TC	EVA's Stat-Arb	EVA's EMN
Rebalancing Cost	\$2.2524	\$3.5372	-\$71.3071	\$14.31	-\$14.69

Thus, we find that the optimal portfolio obtained by maximizing the Sharpe Ratios has a lesser rebalancing cost when compared to the Stat-Arb and EMN portfolios.

Thus, one gets a higher Sharpe ratio with lower rebalancing costs. Hence, it makes sense to trade using the dynamic trading strategy for this optimal set of assets.

5. Conclusion

We find that the dynamic trading strategy, which combines the short term and long term predictions, gives us consistently good results with commodities. One of the reasons for this could be the fact that since, all the commodities belong to the same sector they tend to be more correlated with each other.

However, when we try to implement the dynamic trading strategy for a diverse set of stocks, the results are less convincing. We find that the dynamic trading strategy does NOT consistently outperform the Stat Arb and EMN strategies for all stock portfolios. Also, the performance of the dynamic trading strategy deteriorates further as we increase our portfolio size.

But, one could construct an optimal portfolio of stocks for which the dynamic trading strategy would give a higher value of Sharpe Ratio with lower rebalancing cost. Thus, there is a need to find an optimal portfolio of stocks for which one could obtain a higher excess return by using the dynamic trading strategy.

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