Algorithmic and High-frequency trading: an overview

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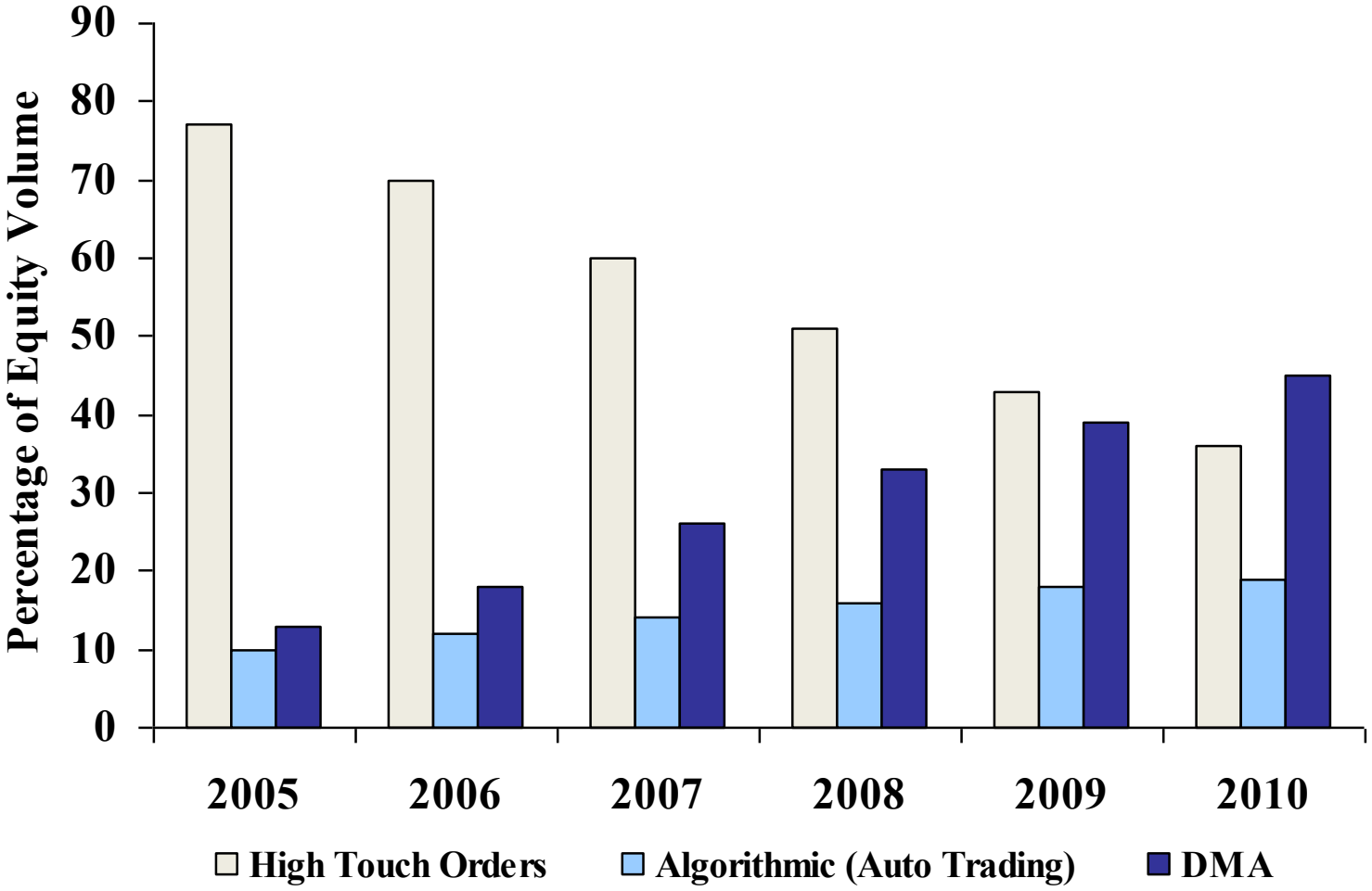
US Equities markets: percentage of orders generated by algorithms



The market in numbers

- US Equities volumes: 5 and 10 billion shares per day
- **1.2 – 2.5 Trillion shares per year**
- **Annual volume: USD 30 – 70 trillion**
- At least 30% of the volume is algorithmic: 360 a 750 billion shares/year
- Typical large ``sell side'' broker trades between 1 and 5 USD Tri per year using algos
- Each day, between 15,000 and 3,000 orders are processed
- An algorithmic execution strategy can be divided into 500 – 1,000 small daughter orders

Algorithmic trading

- **Algorithmic trading:** the use of programs and computers to generate and execute (large) orders in markets with electronic access.

- Orders come from institutional investors, hedge funds and Wall Street trading desks

- The main objective of algo trading is not necessarily to maximize profits but rather to control execution costs and market risk.

- Algorithms started as tools for institutional investors in the beginning of the 1990s. Decimalization, direct market access (DMA), 100% electronic exchanges, reduction of commissions and exchange fees, rebates, the creation of new markets aside from NYSE and NASDAQ and ***Reg NMS*** led to an explosion of algorithmic trading and the beginning of the decade.

  Today, brokers compete actively for the commission pool associated with algorithmic trading around the globe – a business estimated at USD 400 to 600 million per year.

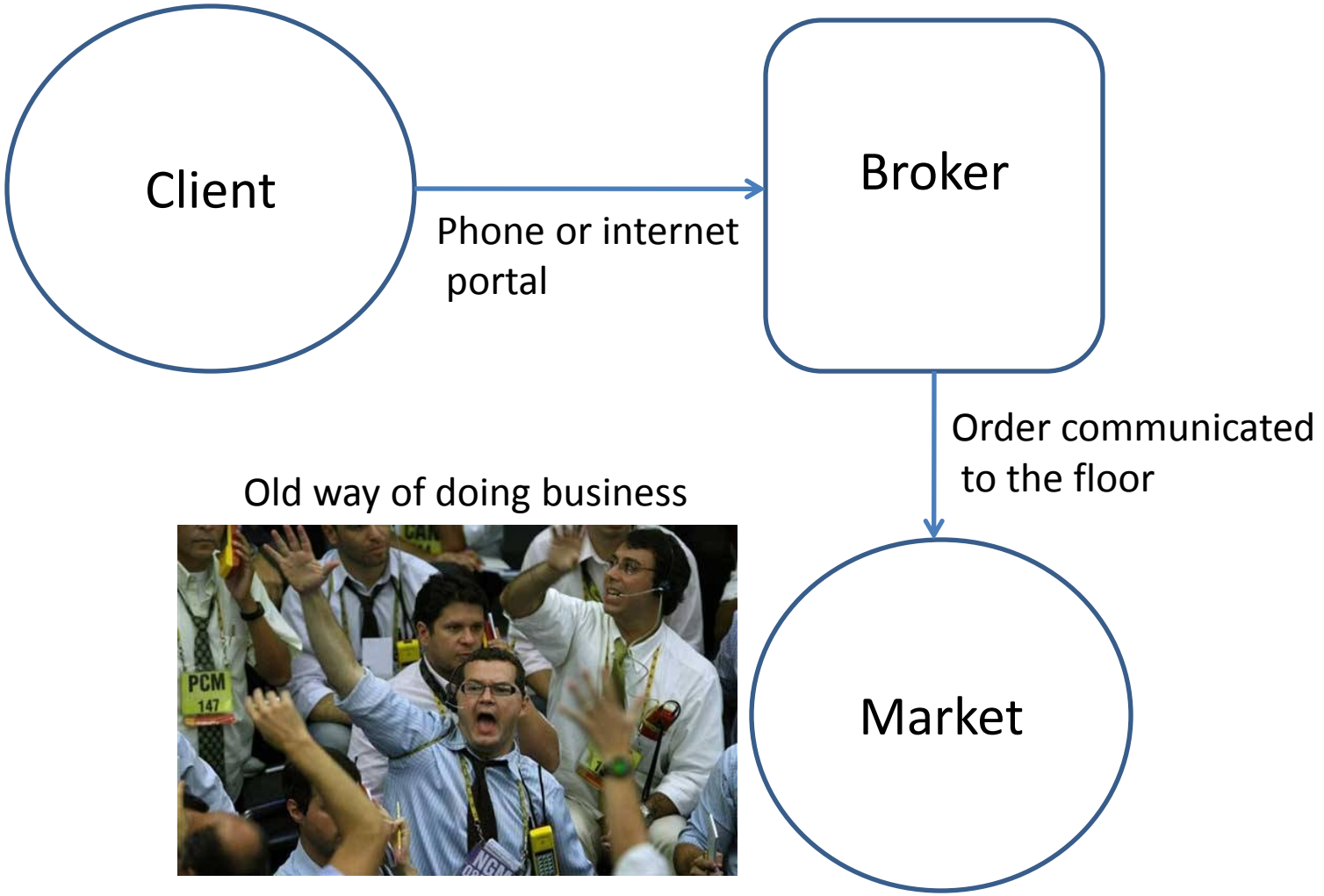
Why Algorithms?

- Institutional clients need to trade large amounts of stocks . These amounts are often larger than what the market can absorb without impacting the price.
- The demand for a large amount of liquidity will typically affect the cost of the trade in a negative fashion (``slippage'')
- Large orders need to be split into smaller orders which will be executed electronically over the course of minutes, hours, day.
- The procedure for executing this order will affect the average cost per share, according to which algorithm is used.
- In order to evaluate an algorithm, we should compare the average price obtained by trading with a market benchmark (``global average'' of the daily price, closing price, opening price, ``alpha decay'' of a quant strategy, etc).

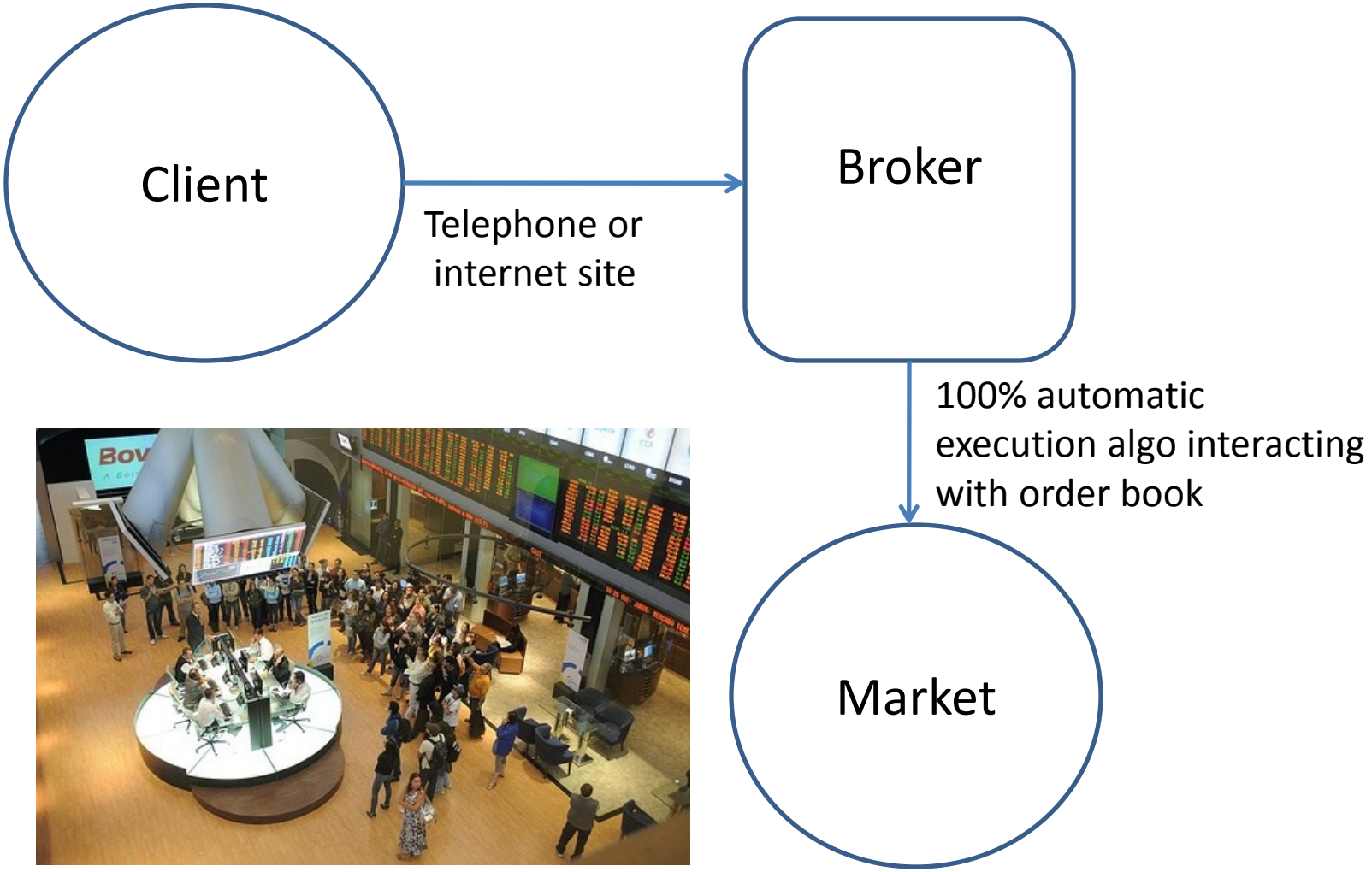
Main issues in Algorithmic Trading

- The decision of how to split the order in smaller pieces is just one of several issues.
- Once an algo is chosen the smaller orders need to be executed electronically
- Execution strategies interact with the market and decide how to place orders (Limit, Market,etc) and at what prices
- Objective: to achieve the ``best price'' for each daughter order
- Recent changes in the US equity market structure (in particular, different liquidity sources) make things more interesting and complicated
- Dark Pools (liquidity pools that do not show the order book), ECNs (electronic communications networks), autonomous liquidity providers

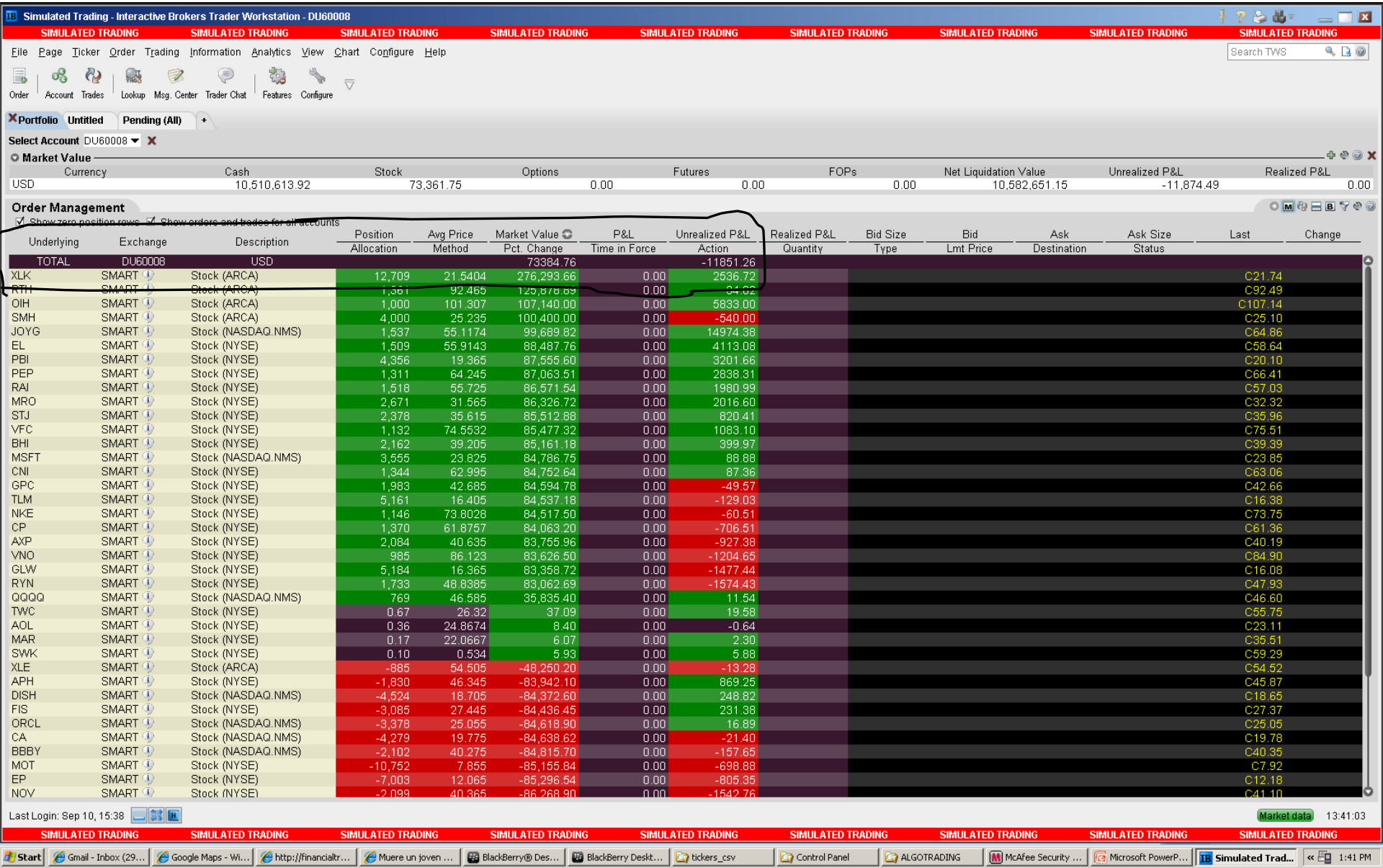
1. ``Ancient'' brokerage model

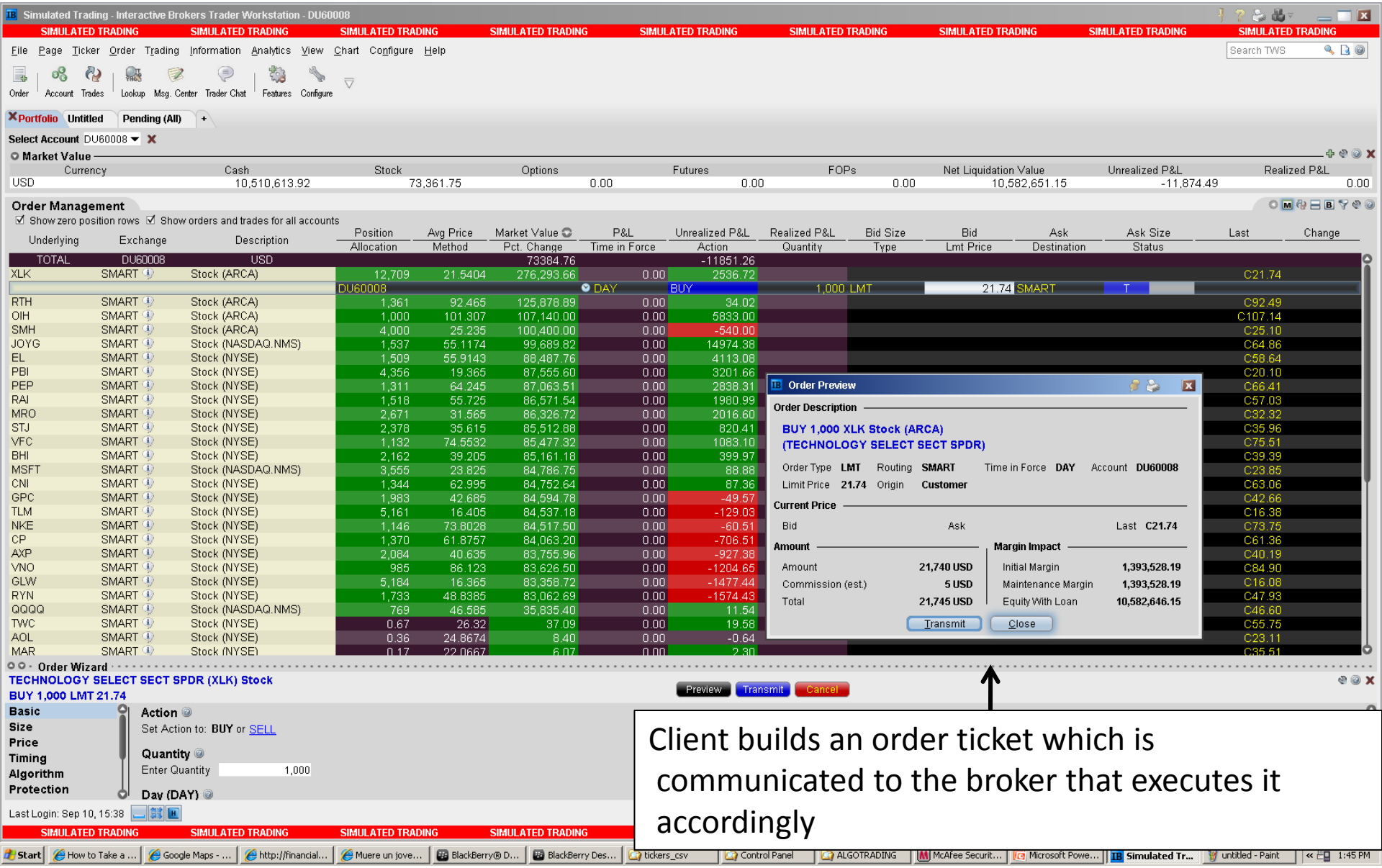


2. Electronic market



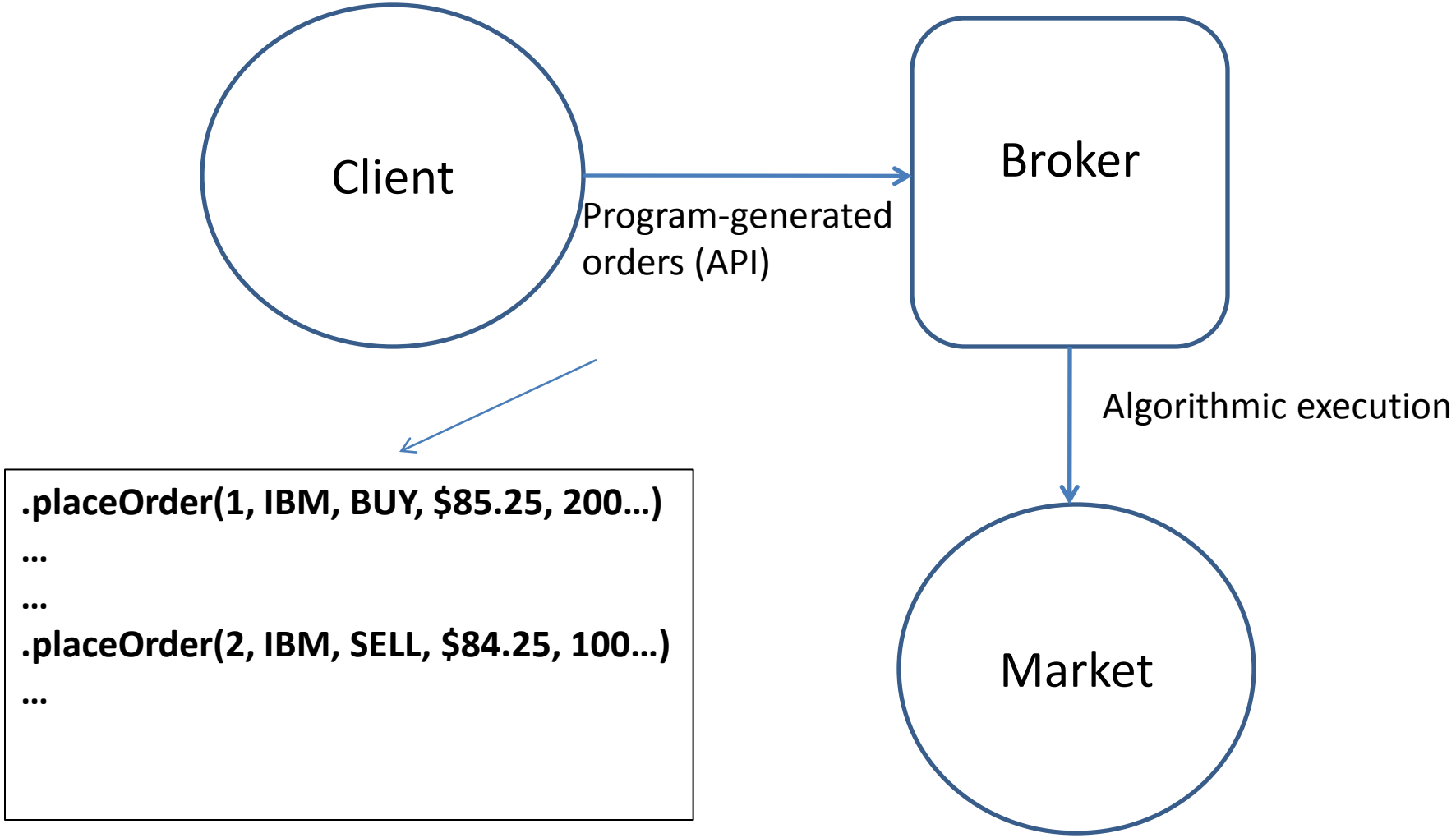
Electronic order-management and execution system (client-broker)



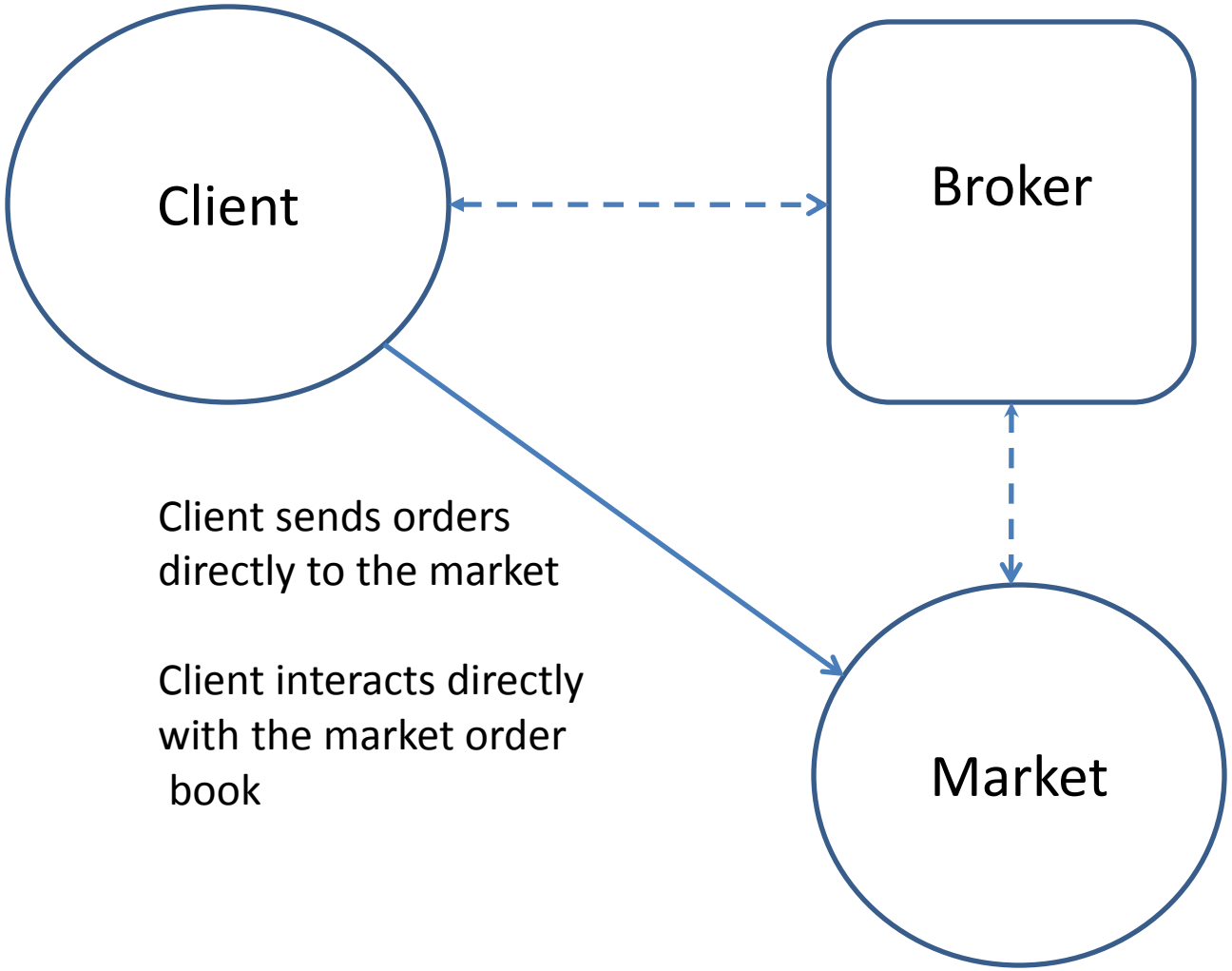


Client builds an order ticket which is communicated to the broker that executes it accordingly

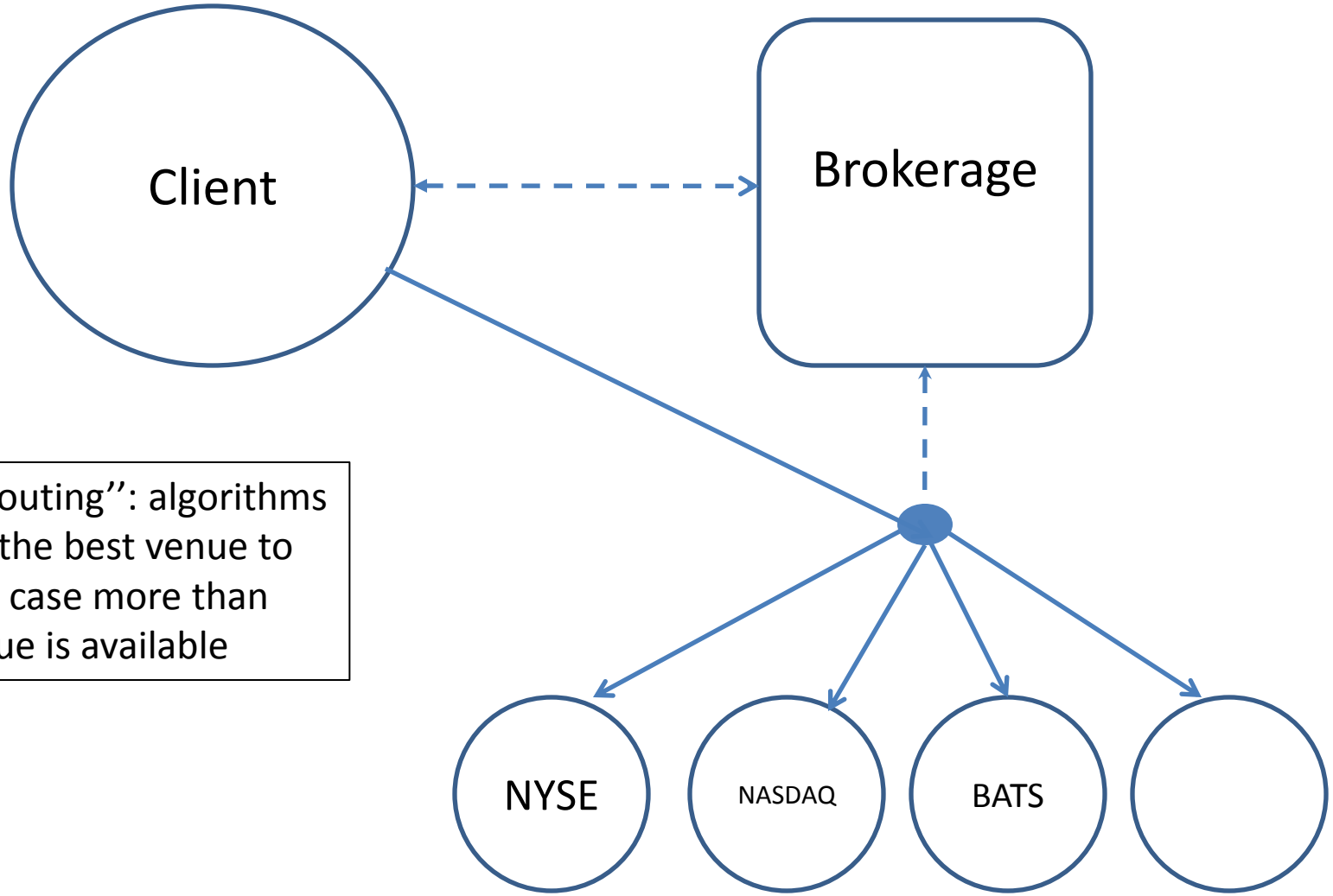
3. Electronic execution model with API



4. Direct Market Access (DMA)



ECNs, Dark Pools, Multiple Execution Venues



``Smart routing'': algorithms look for the best venue to trade, in case more than one venue is available

A few trading venues for US equity markets

- ARCA-NYSE: electronic platform of NYSE (ex- Archipelago)
- BATS: (Kansas)
- BEX: Boston Equity Exchange
- CBSX: CBOE Stock Exchange
- CSXZ: Chicago Stock Exchange
- DRCTEDGE: Direct Edge (Jersey City, NJ)
- ISE: International Securities Exchange
- ISLAND: Acquired by Nasdaq in 2003
- LAVA: belongs to Citigroup
- NSX: National Stock Exchange (Chicago)
- NYSE: New York Stock Exchange
- TRACKECN: Track ECN

Reg NMS (``National market system'')

Order Protection Rule (Trade-thru rule) - protects visible liquidity at the top of book of automated market centers (SROs + ADF participants) from being traded through by executions outside each market's BBO.

Access Rule - caps access fees for top of book access at $.003

Sub-Penny Rule - prohibits market centers from accepting quotes or orders in fractions under $.01 for any security priced greater than $1.00.

Market Data Rule - changes the allocation of market data revenue to SROs for quotes and trades

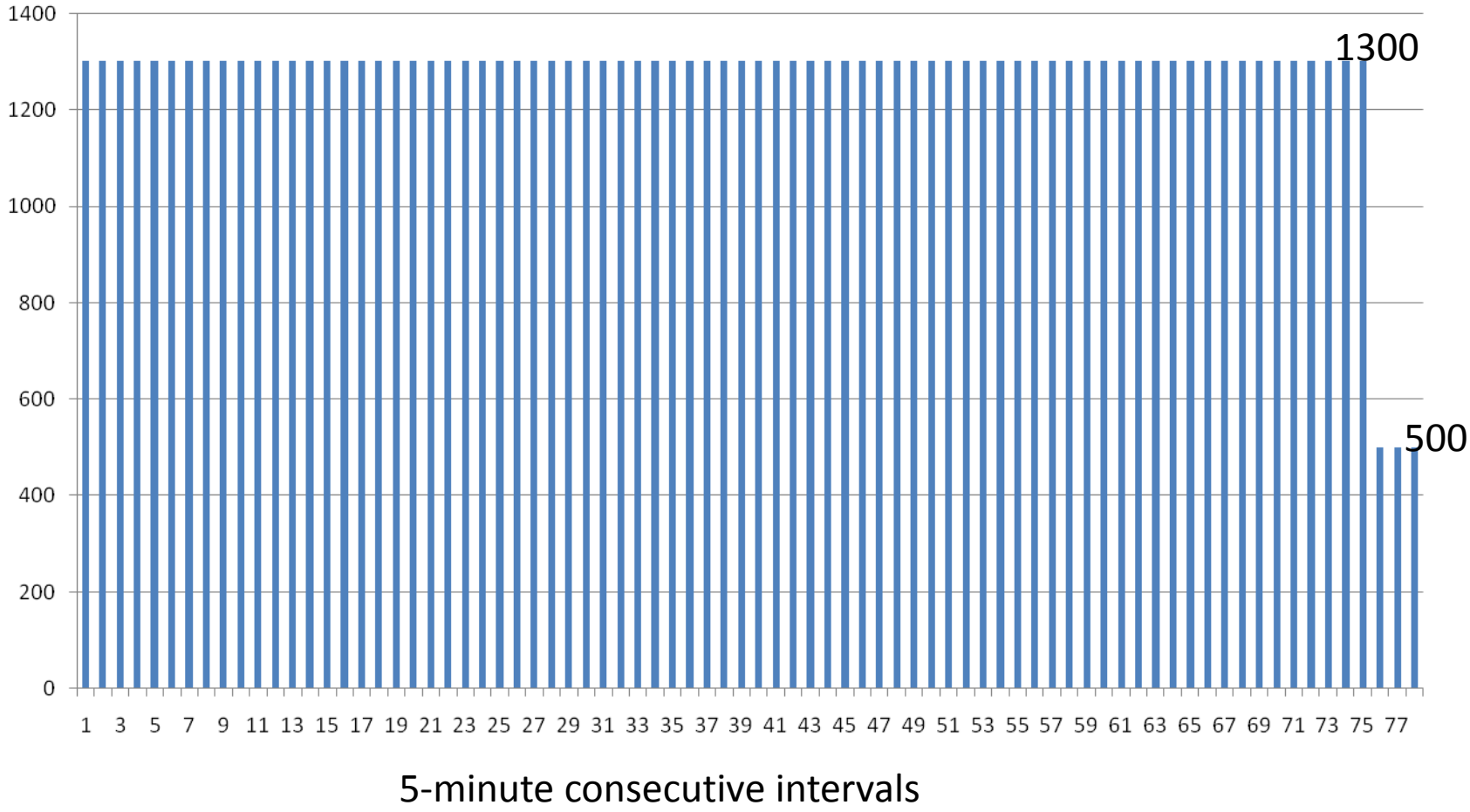
SRO: NYSE, NASD, FINRA
ADF: Alternative Display Facility/ consolidation of NYSE/NASDAQ

The three steps in algorithmic trading

Time-weighted average price (TWAP)

Equal amount of shares in each period of time.

Example: 100,000 shares TWAP/all day



Volume is greater in the beginning and at the end of the day

Volume-weighted average price (VWAP)

Volume changes in the course of the day (less volume in the middle).

VWAP: To execute a large order, the way in which we split it
depends on the time of day (minimize impact)

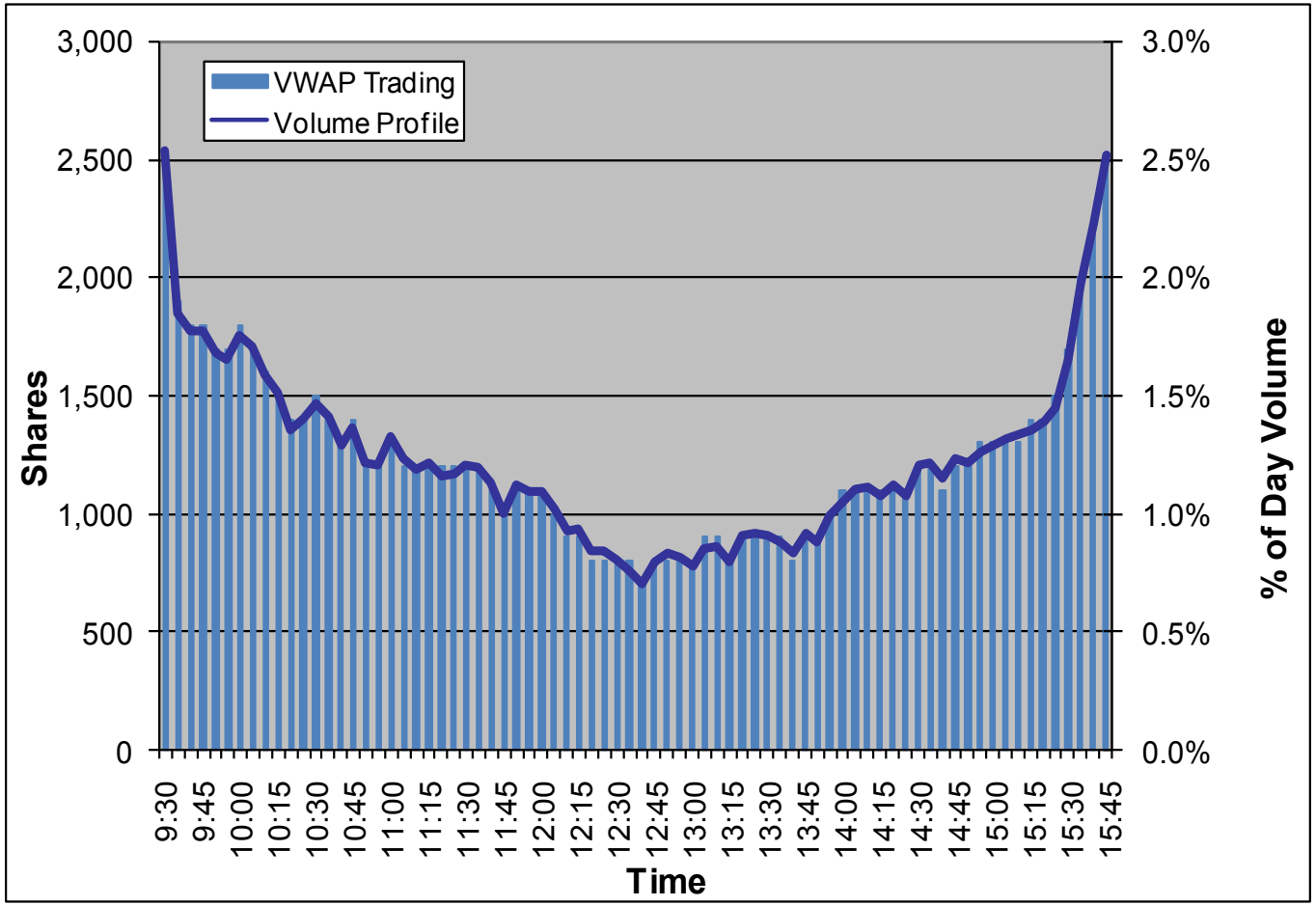
Objetive: obtain an average price ``weighted by volume''

Algorithm:

1. estimate the average volume traded in every 5 minute interval
2. In each time-interval, execute an amount proportional to the normative volume for that interval

Properties:

1. the algorithm always concludes (trade sizes are known in advance)
2. volume function is estimated using historical data. This may not correspond exactly to *ex-post* VWAP.



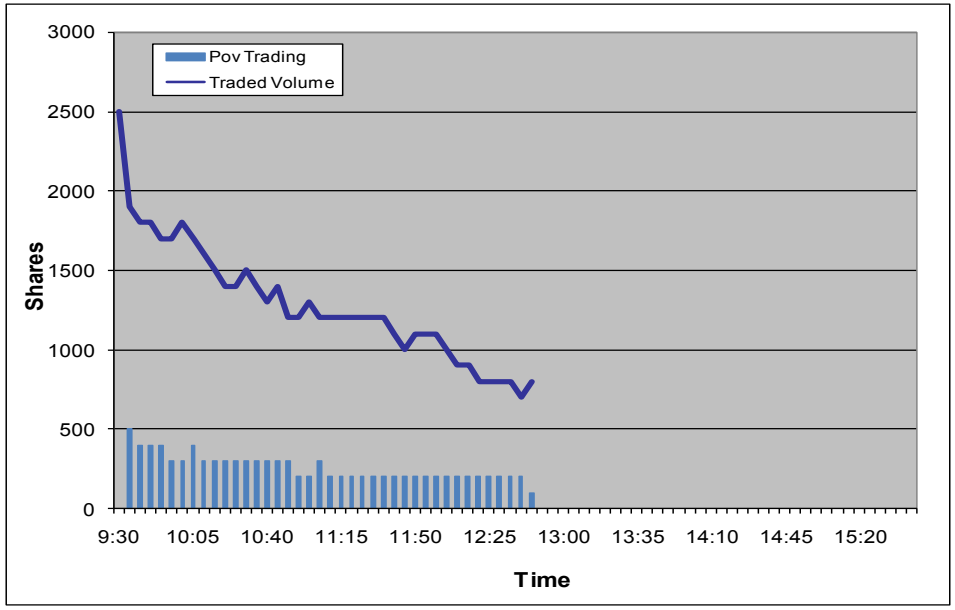
$$VWAP(t_1, t_2) = \frac{\sum_{t=t_1}^{t_2} \delta V(t) P(t)}{\sum_{t=t_1}^{t_2} \delta V(t)}$$

Percentage of Volume (POV)

- The PoV (Percentage of Volume) algorithm addresses the problem of VWAP by using the actual traded volume of the day as benchmark. The idea is to have a contant percentage participation in the market along the trading period.

- If the quantity that remains to be traded is **Q**, and the participation ratio is γ , the algo algo computes the volume **V** traded in the period (**t- ΔT, t)** and executes a quantity

  q = min(Q,V*γ)



$V(t)$ = total volume traded in the market up to time t

$Q(t)$ = number of shares that remain to be traded. ($Q(0)$ = initial quantity)

$$Q(t+\delta t) - Q(t) = -\min\left[\gamma\left(V(t) - V(t-\delta t)\right), Q(t)\right]$$

$$\begin{cases} \dfrac{dQ}{dt} = -\gamma \dfrac{dV}{dt} & ;\ Q(t) > \gamma \dfrac{dV}{dt}\delta t \approx 0 \\ \dfrac{dQ}{dt} = 0 & ;\ Q(t) \le \gamma \dfrac{dV}{dt}\delta t \approx 0 \end{cases}$$

$$\frac{dQ}{dt} = -\gamma \frac{dV}{dt} \quad \therefore \quad Q(T) - Q(0) = -\gamma \cdot V(T) \quad \therefore \quad Q(0) = \gamma \cdot V(T)$$

$$\frac{dQ}{dt}\,p(t) = -\gamma \frac{dV}{dt}\,p(t) \quad \therefore \quad \int_0^T \left|\frac{dQ}{dt}\right| p(t) = \gamma \int_0^T \frac{dV}{dt}\,p(t)$$

$$\frac{\int_0^T \left|\dfrac{dQ}{dt}\right| p(t)}{Q(0)} = \frac{\int_0^T \dfrac{dV}{dt}\,p(t)}{V(T)}$$

POV is similar to WVAP if ratio is small

(Or is it? More later ☺)

Almgren-Chriss (``Expected Shortfall'')

Market impact combined with ``urgency in execution'' (price risk)

$$dp(t) = -av(t)dt + \sigma dZ(t) \qquad v(t) = -\frac{dQ(t)}{dt}$$

Dynamic price model with price impact (`permanent impact')

$$\overline{p}(t) = p(t) - b|v(t)|$$

Execution price (`temporary impact')

$$E = -\mathbf{E}\left\{\int_0^T \overline{p}(t)\frac{dQ(t)}{dt}dt\right\} = -\mathbf{E}\left\{\int_0^T p(t)\frac{dQ(t)}{dt}\right\} + b\int_0^T \left(\frac{dQ(t)}{dt}\right)^2 dt$$

Expected execution cost

$$V = \sigma^2 \int_0^T (Q(0) - Q(t))^2 dt$$

Execution risk

$$\min_Q \{E + \lambda V\}$$

Optimization problem

Analytic solution

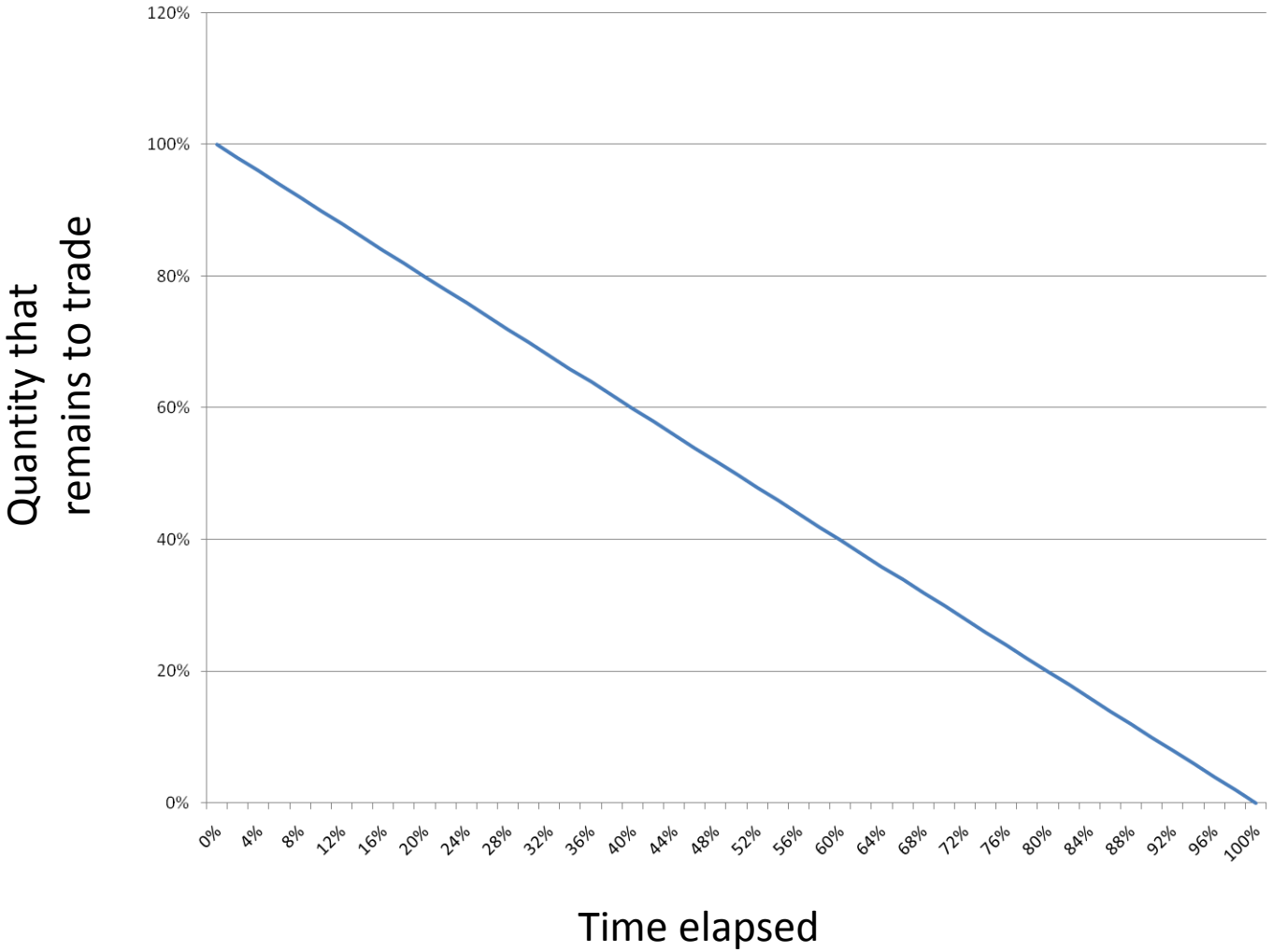
$$Q(t) = Q(0)\frac{\sinh\left(\sqrt{\frac{\lambda\sigma^2}{a+b}}(T-t)\right)}{\sinh\left(\sqrt{\frac{\lambda\sigma^2}{a+b}}T\right)}$$

$$\frac{Q(t)}{Q(0)} = \frac{\sinh(\Omega(1-\tau))}{\sinh\Omega}, \qquad \Omega = T\sqrt{\frac{\lambda\sigma^2}{a+b}}, \quad \tau = \frac{t}{T}$$

Omega: proportional to execution time, varies directly with risk-aversion and volatility, inversely to market impact elasticities

Omega = (price risk)/(impact risk)

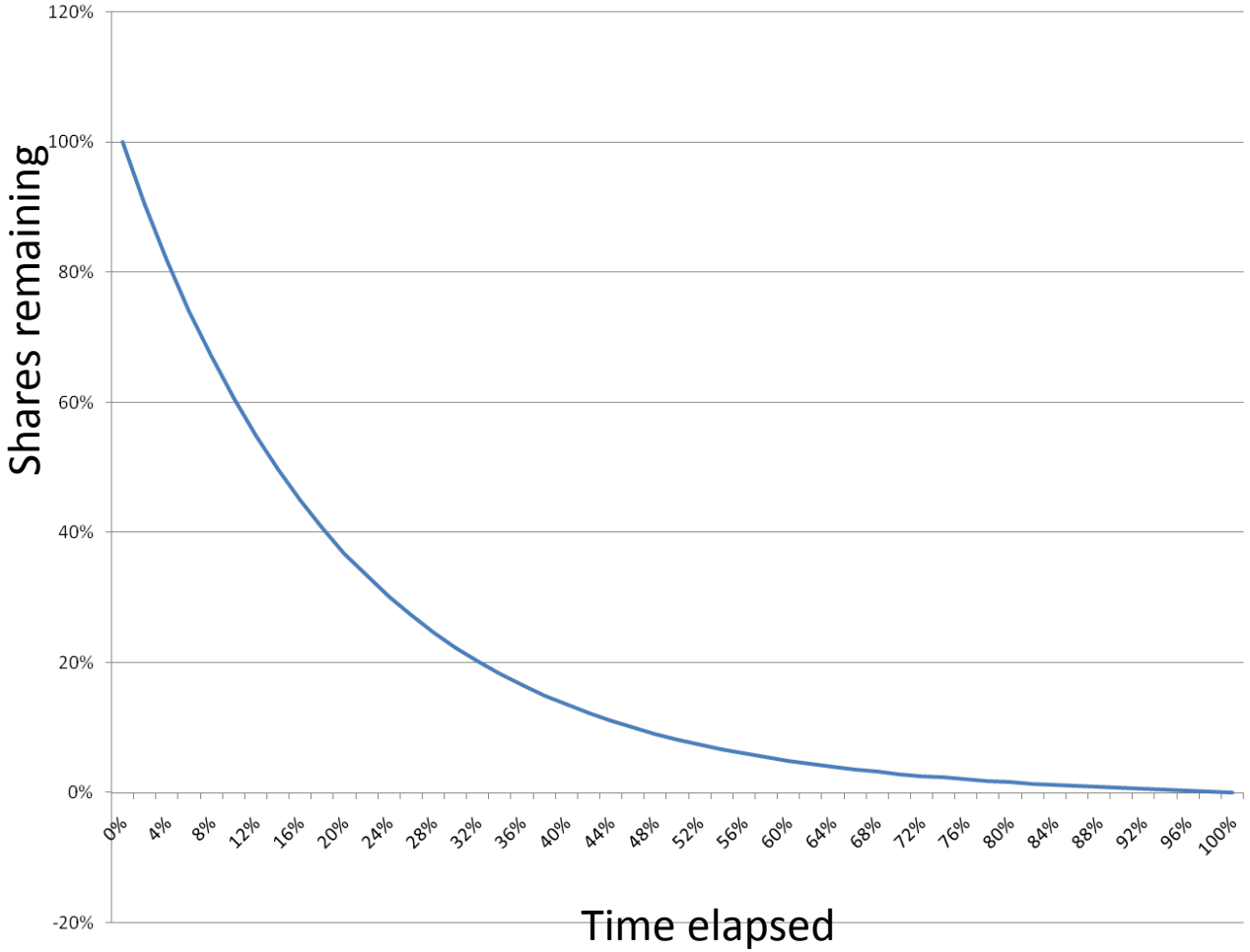
Case $\Omega = 0$, TWAP (VWAP)



Impact risk >> price risk

Algorithm= TWAP or VWAP

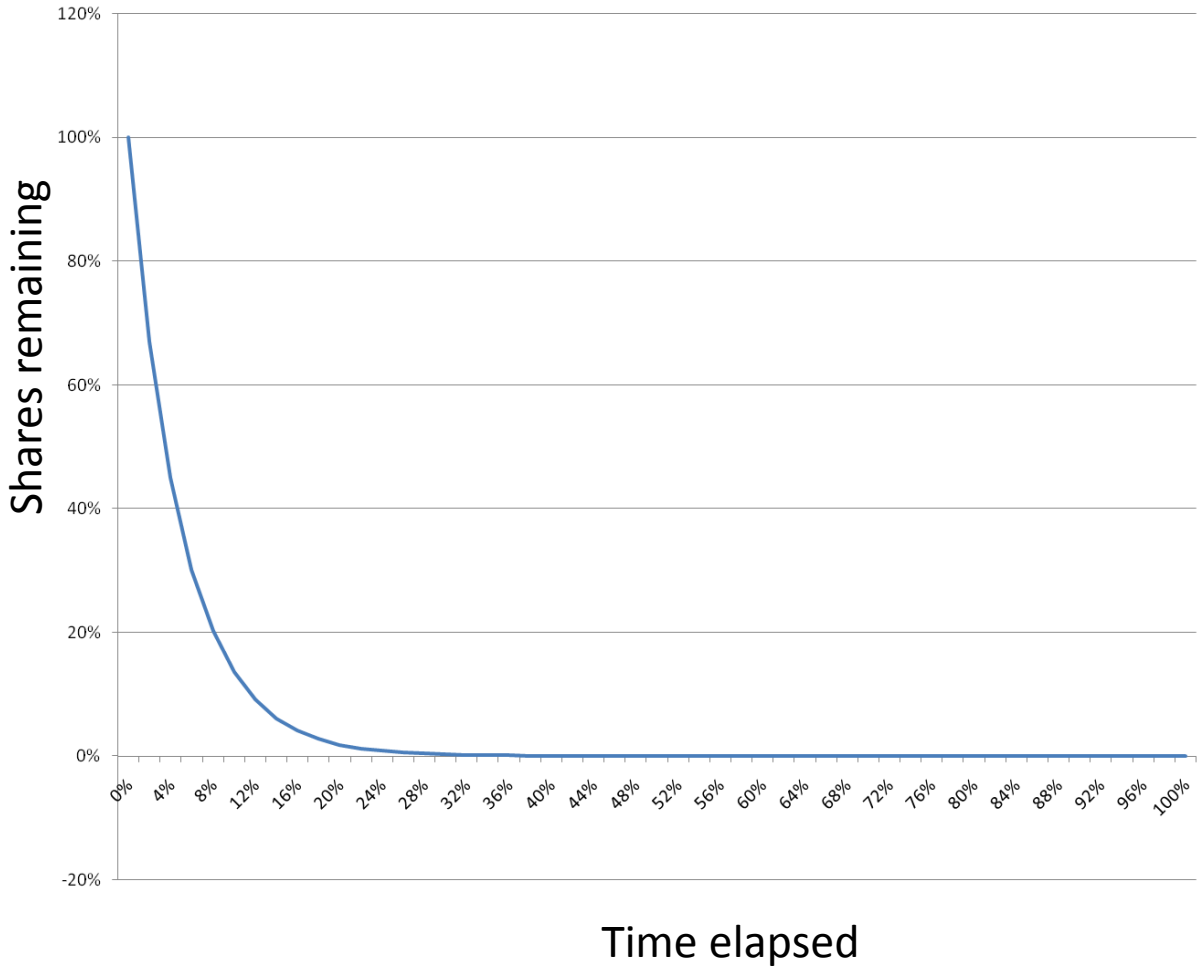
Case $\Omega = 10$



Significant market risk

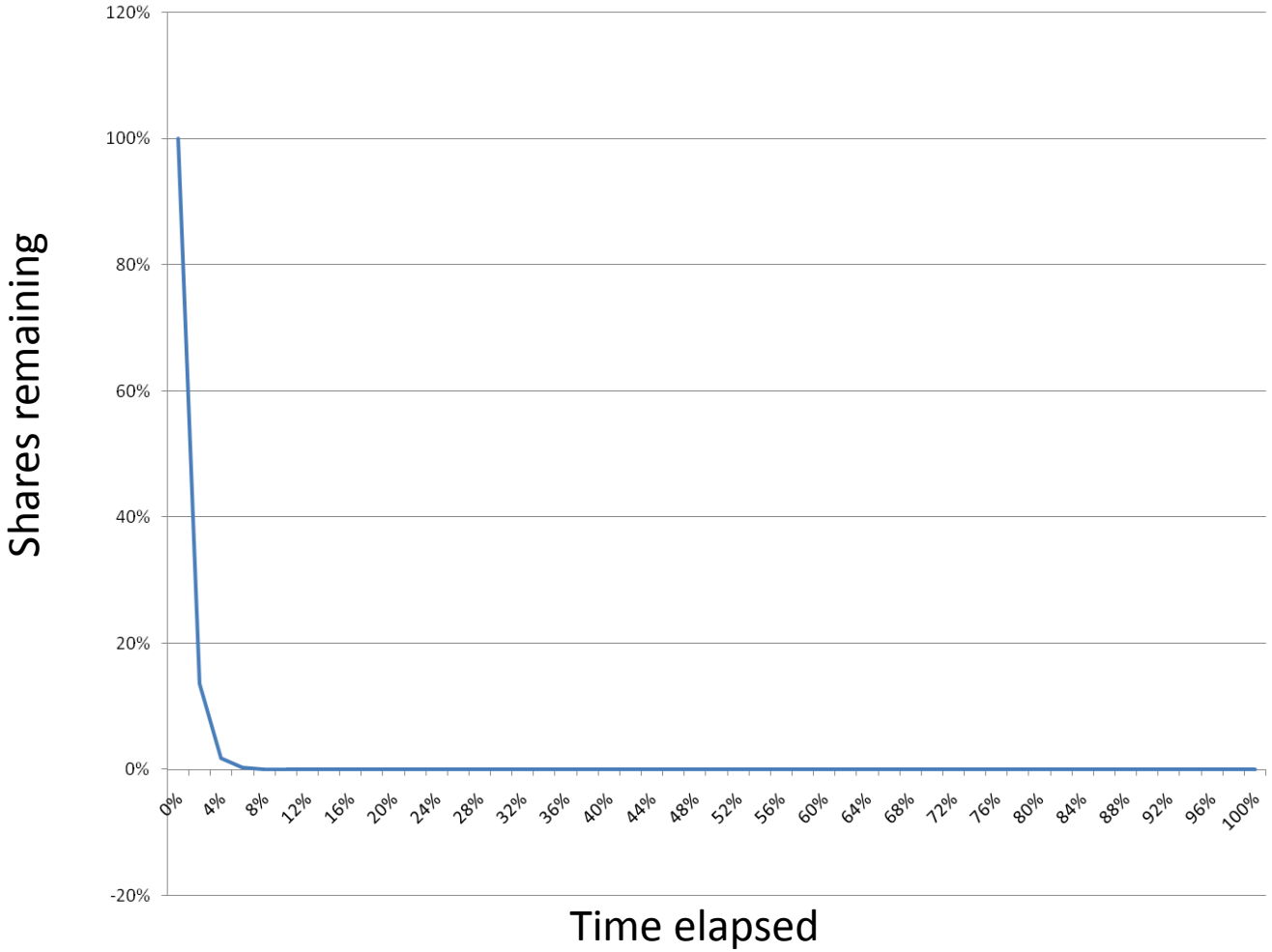
Execution must be faster

Case $\Omega = 20$



Faster execution

Case $\Omega = 100$



``Slam'' the market!

Generalizations of Almgren-Chriss order-splitting algorithm

- Incorporate intraday volume in the impact model (modification of VWAP)
- Incorporate drift in the price model (momentum)
- Incorporate exchange fees, rebates and other costs
- Almgren-Chriss & generalizations are now part of the standard toolkit that execution brokers offer to clients

Examples of quant strategies that make use of algorithms

- Index and ETF arbitrage
- Statistical arbitrage (``Stat Arb'')
- Liquidity providing (``Market making'')
- Volume providing (``High-frequency, selective, market-making'')
- High frequency trading and price forecasting

ETFs

-- ETF: similar to mutual funds (holding vehicles) but which trade like stocks

-- Short-selling, margin financing allowed.

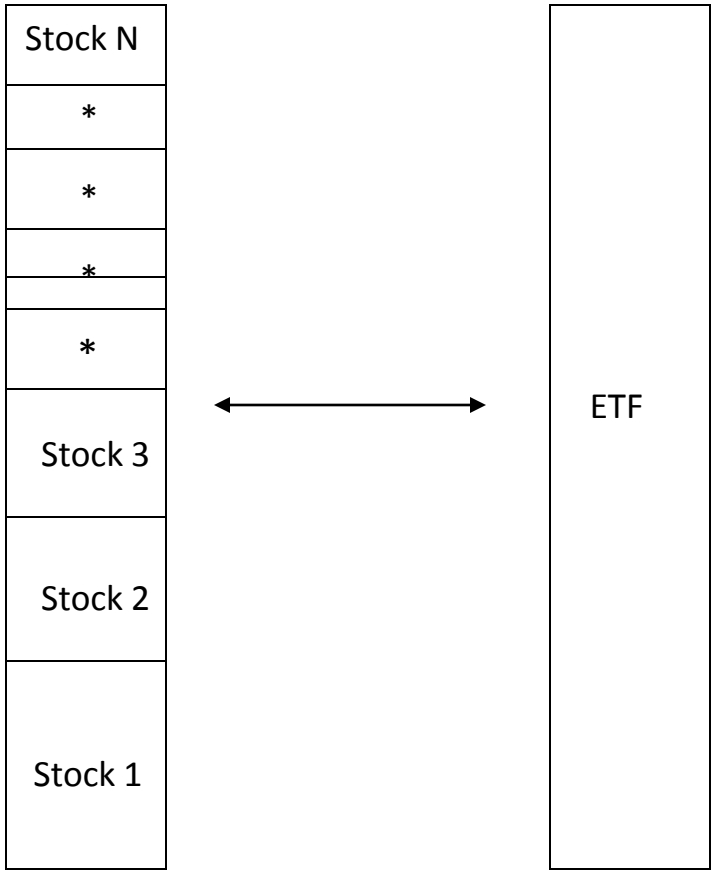
-- Began like equity index & basket trackers, then generalized to currencies and commodities

-- **Authorized participants** may create or redeem ETF shares at NAV, enforcing the theoretical relationship between the ETF and the underlying basket

-- ``creation units'': 25K to 100K shares

-- Authorized participants are typically market-makers in the ETFs (but not always).

Arbitrage of ETFs against the underlying basket



1. Buy/sell ETF
against the underlying share holdings

2. Creation/redemption of ETFs
to close the trade

This requires high-frequency algorithmic trading to lock-in arbitrage opportunities

Statistical Arbitrage

Long-short shares/etfs – market neutral

Sector	ETF	Num of Stocks	Market Cap unit: 1M/usd		
			Average	Max	Min
Internet	**HHH**	22	10,350	104,500	1,047
Real Estate	**IYR**	87	4,789	47,030	1,059
Transportation	**IYT**	46	4,575	49,910	1,089
Oil Exploration	**OIH**	42	7,059	71,660	1,010
Regional Banks	**RKH**	69	23,080	271,500	1,037
Retail	**RTH**	60	13,290	198,200	1,022
Semiconductors	**SMH**	55	7,303	117,300	1,033
Utilities	**UTH**	75	7,320	41,890	1,049
Energy	**XLE**	75	17,800	432,200	1,035
Financial	**XLF**	210	9,960	187,600	1,000
Industrial	**XLI**	141	10,770	391,400	1,034
Technology	**XLK**	158	12,750	293,500	1,008
Consumer Staples	**XLP**	61	17,730	204,500	1,016
Healthcare	**XLV**	109	14,390	192,500	1,025
Consumer discretionary	**XLY**	207	8,204	104,500	1,007
Total		**1417**	**11,291**	**432,200**	**1,000**

January, 2007

Statistical Arbitrage (II)

systematic component

idiosyncratic component

$$\frac{dS_i(t)}{S_i(t)} = \beta_i \frac{dI(t)}{I(t)} + \varepsilon_i(t)$$

Stock return is compared to the return on the **corresponding sector ETF** (regression, co-integration)

Residuals: modeled as a mean-reverting process

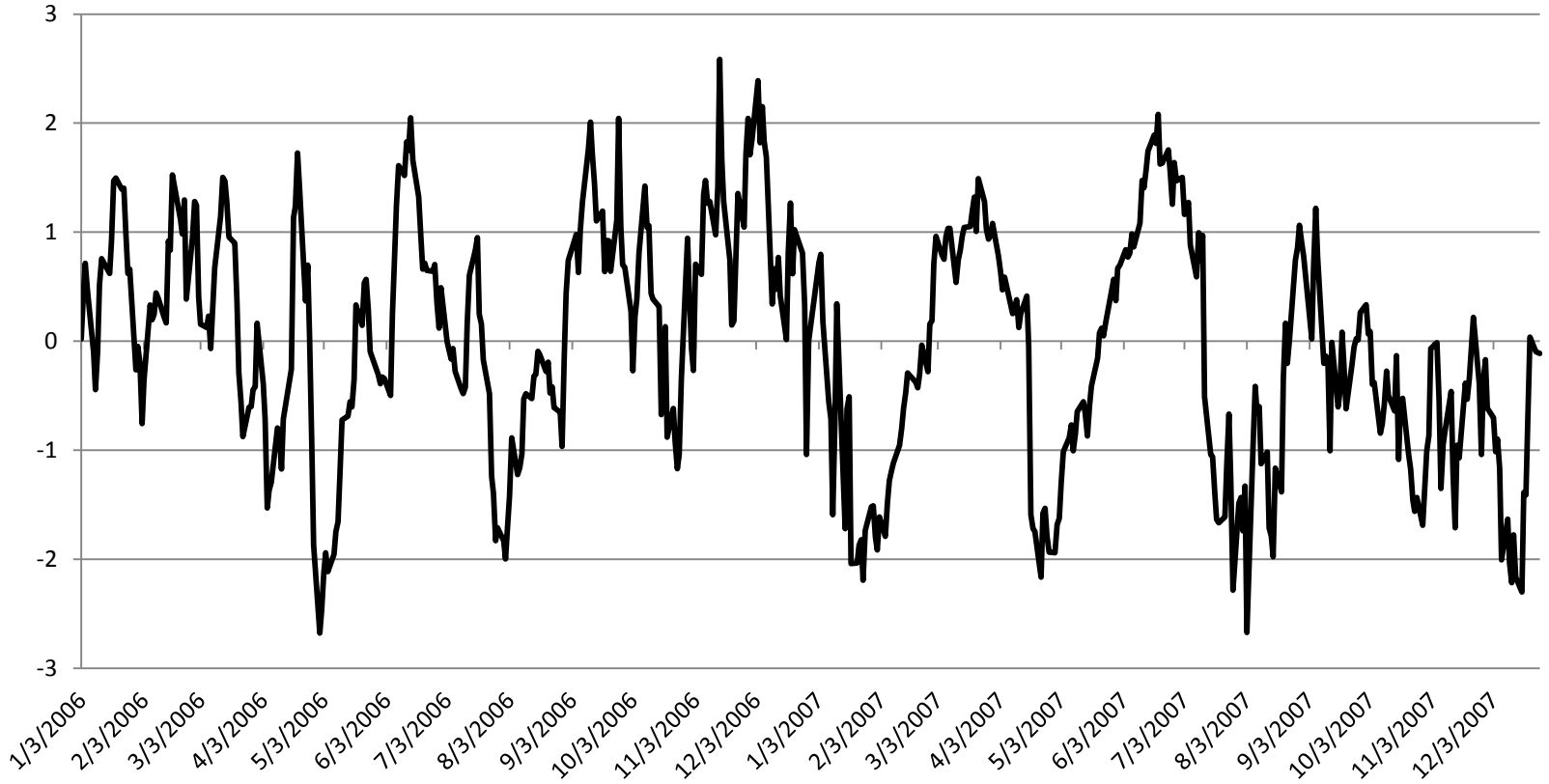
$$\varepsilon_i(t) = \alpha_i dt + dX_i(t)$$

$$dX_i(t) = \kappa_i(m_i - X_i(t))dt + \sigma_i dW_i(t)$$

Ornstein-Ulembeck (AR-1)

Example of sampling window =3 months (~ 60 business days)

X(t) process for JPM/XLF (Financial sector ETF from State Street)



Constructing Stat Arb strategies

-- Diversified universe of stocks, ``good choice '' of shares/ETF pairs

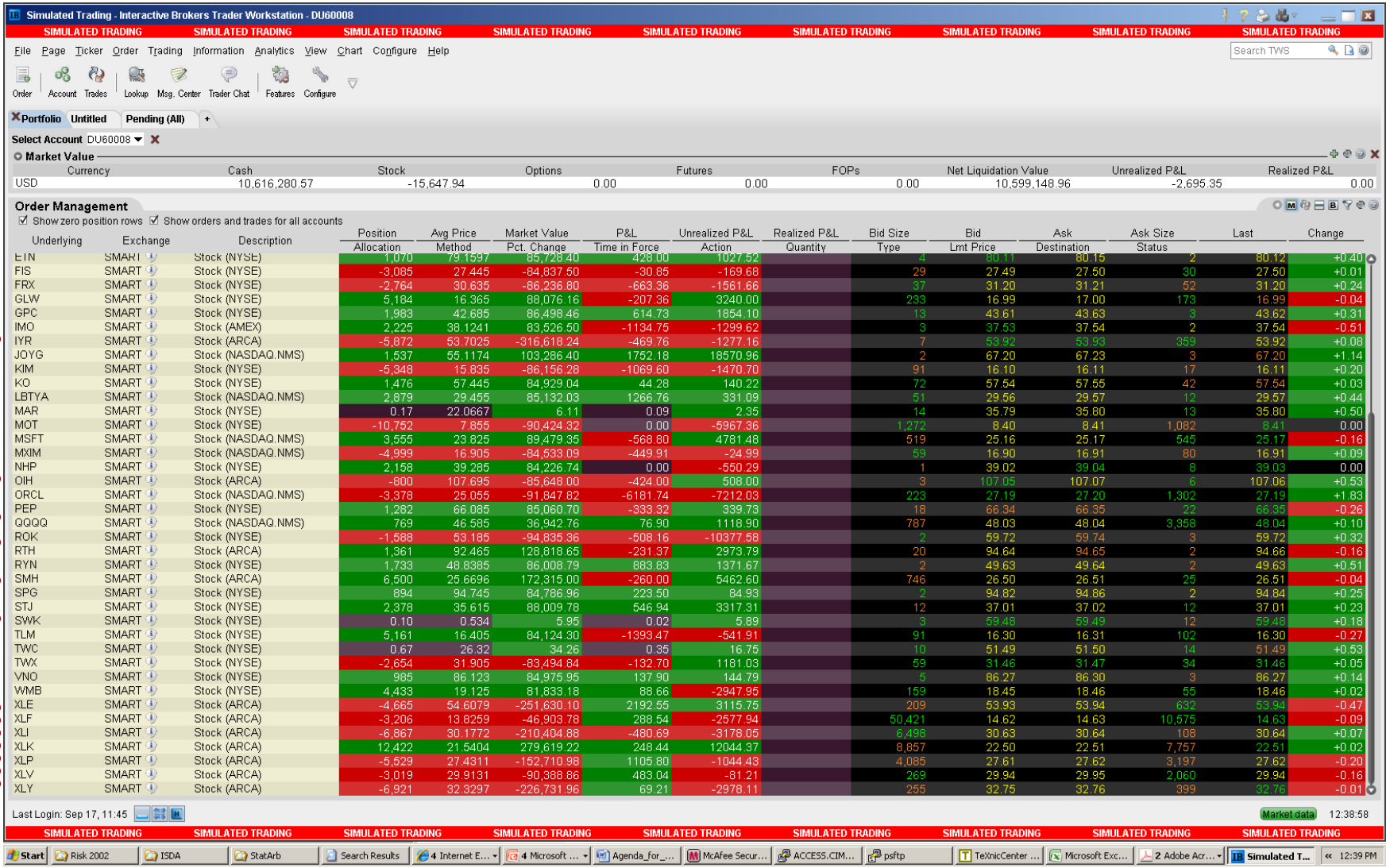
-- Buy or sell the spread (pair) according to the statistical model

-- Risk-management using real-time VaR

-- Execution: VWAP

-- Taking volume into account is important to avoid ``adverse selection''
(the reason for divergence of X(t) in practice)

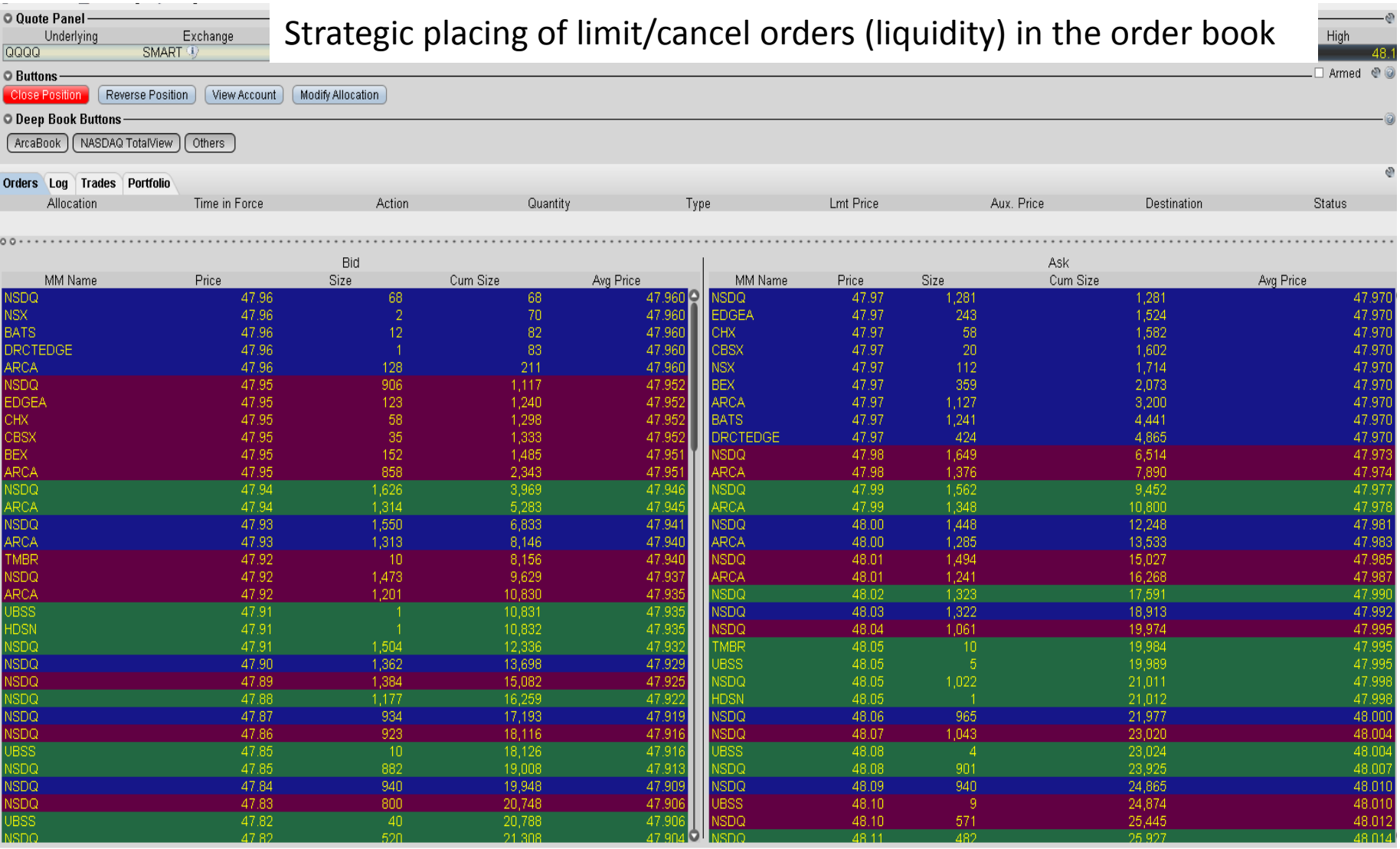
Example of Stat-Arb portfolio



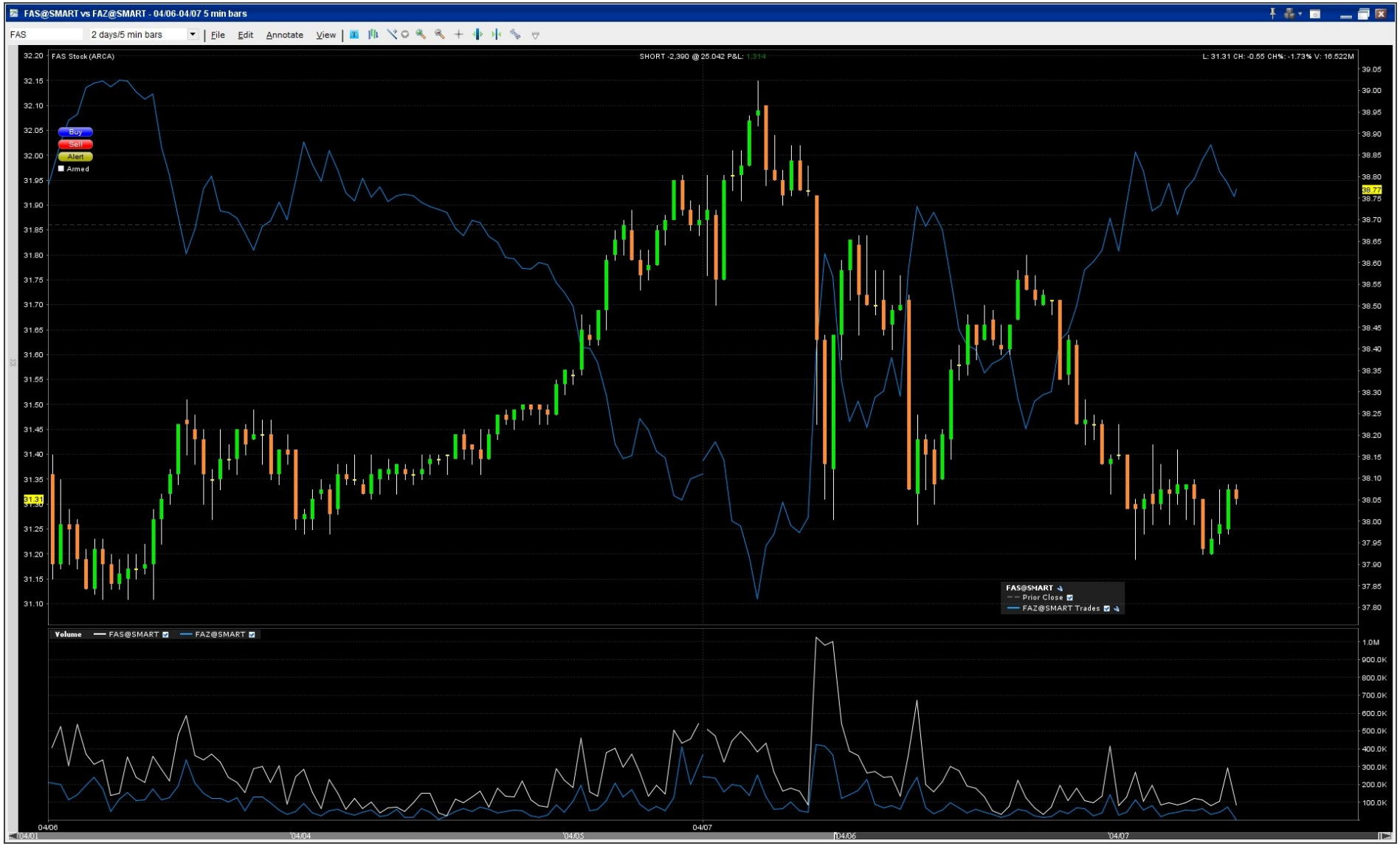
- ETF

Liquidity providing (high frequency)

Strategic placing of limit/cancel orders (liquidity) in the order book



HF Pairs trading? Intraday evolution of FAZ & FAZ (inverse leveraged ETFs)



Algorithmic trading and the ``flash crash'' (May 6, 2010)

The reasons behind the ``crash of 2:15'' were studied in a joint CFTC/SEC report available online.

Institutional trader sold **75,000 S&P E-mini contracts in 15 minutes PoV.**

* Drop in S&P futures, SPY etf, etf components
* Withdrawal of autonomous MMs; ``stub quotes''
* HFTs provide a lot of volume but not a lot of liquidity (`hot potato trading')

Forecasting prices in HF?

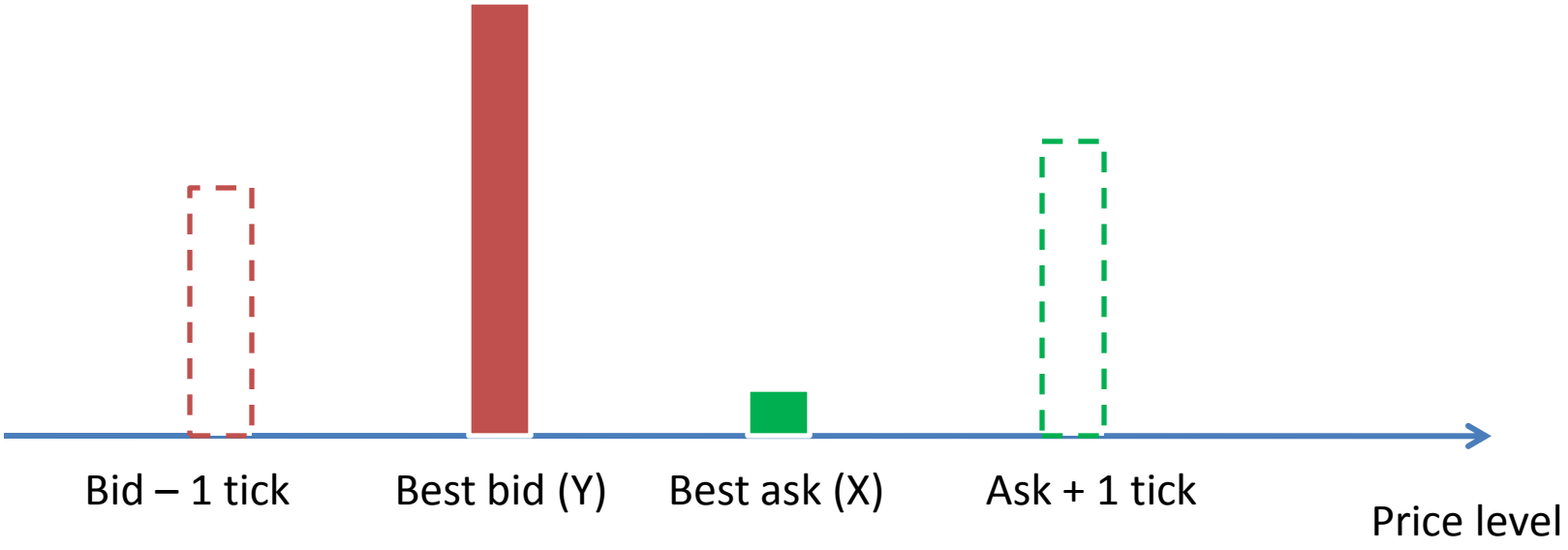
- Models for the dynamics of order books
- Modeling **hidden liquidity** in the market (not visible in the OB)
- Computing the probabilities of price changes (up or down) given liquidity on the bid side and ask-side (Avellaneda, Stoikov, Reed, 2010: pre-published in SSRN, Oct-10)

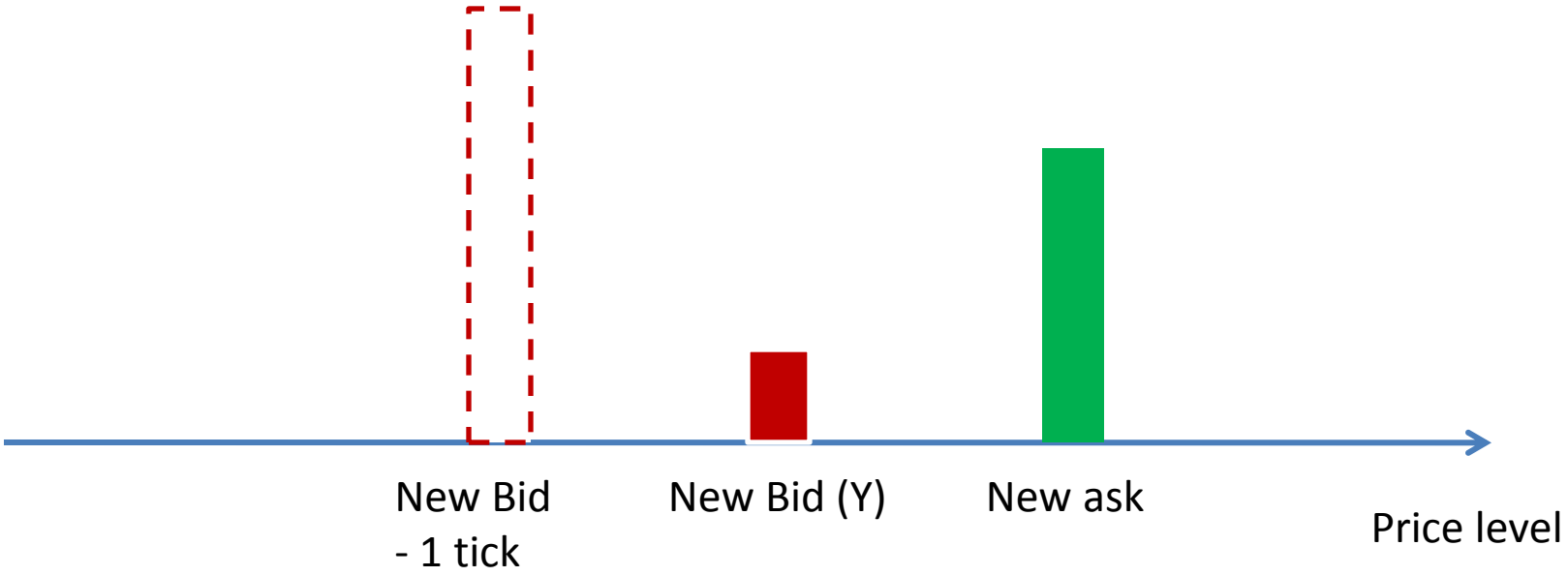
Bid	Q(bid)=x	Ask	Q(ask)=y
100.01	527	100.03	31

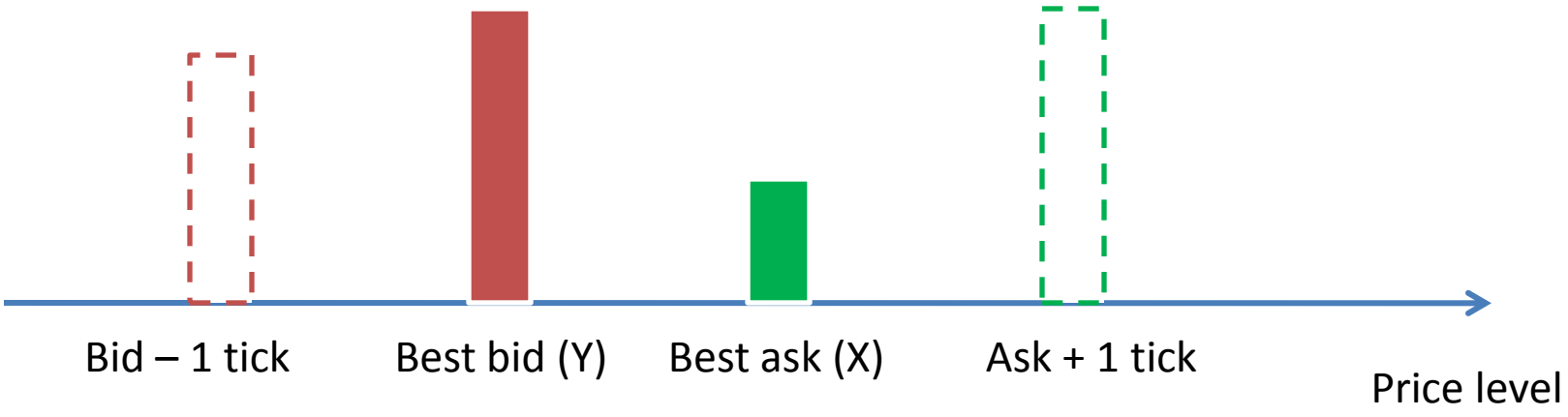
Simple formula that we are testing with HF data

$$P(\uparrow) = \frac{H + x}{2H + (x + y)}$$

H= ``hidden liquidity''

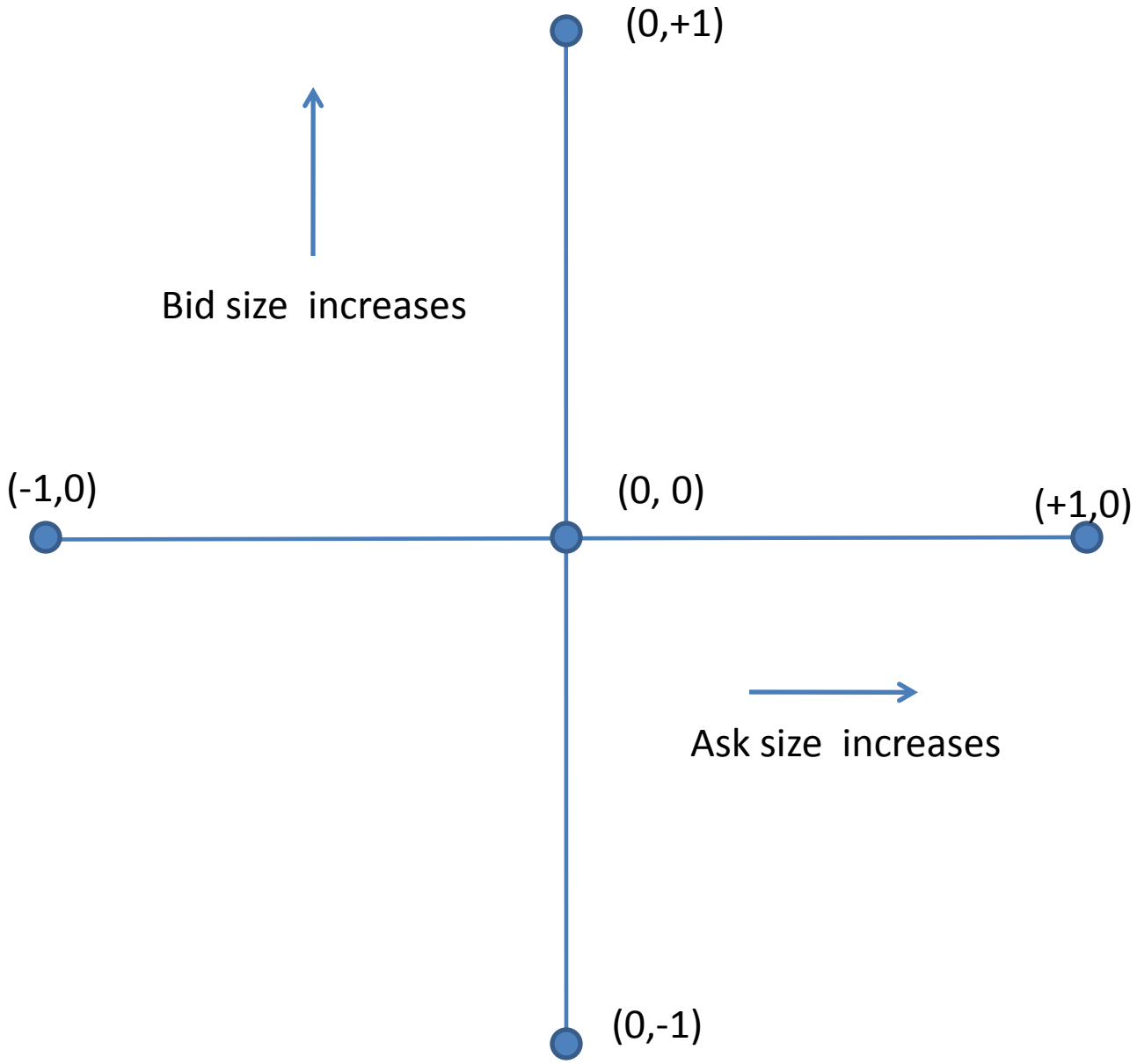

Bid – 1 tick
Best bid (Y)
Best ask (X)
Ask + 1 tick
Price level


New Bid
- 1 tick
New Bid (Y)
New ask
Price level

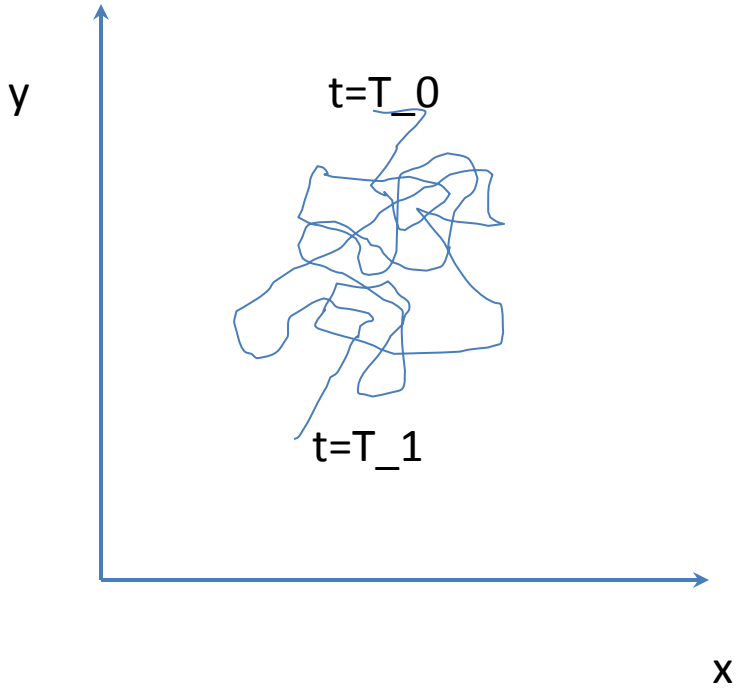

Bid – 1 tick
Best bid (Y)
Best ask (X)
Ask + 1 tick
Price level

Level 1 Quotes

Quote size depletion may be a precursor for a price move.
Does imbalance predict prices?


(0,+1)
Bid size increases
(-1,0)
(0, 0)
(+1,0)
Ask size increases
(0,-1)

Mathematical framework: Diffusion Approximation for Quote Sizes (Level I)



X= bid size
Y = ask size

$$X_t = \sigma W_t$$
$$Y_t = \sigma Z_t$$
$$E\left(dW_t dZ_t\right) = \rho dt$$

A price change occurs when (i) one of the sizes vanishes and (ii) either there is a new bid or a new ask level

Probability that the Ask queue depletes before the Bid queue

$$u(x,y)=\frac{1}{2}\left(1-\frac{\tan^{-1}\left(\sqrt{\frac{1+\rho}{1-\rho}}\,\frac{y-x}{x+y}\right)}{\tan^{-1}\left(\sqrt{\frac{1+\rho}{1-\rho}}\right)}\right)$$

$$\rho=0 \quad \Rightarrow \quad u(x,y)=\frac{2}{\pi}\tan^{-1}\left(\frac{x}{y}\right)$$

$$\rho=-1 \quad \Rightarrow \quad u(x,y)=\frac{x}{x+y}$$

$$p\uparrow(x,y,H)=u(x+H,y+H)$$

Probability of an upward price change.

H=`hidden liquidity'.

Estimating hidden liquidity in different exchanges (ability to forecast price moves)

Sample data

symbol	date	time	bid	ask	bsize	asize	exchange
QQQQ	1/4/2010	9:30:23	46.32	46.33	258	242	T
QQQQ	1/4/2010	9:30:23	46.32	46.33	260	242	T
QQQQ	1/4/2010	9:30:23	46.32	46.33	264	242	T
QQQQ	1/4/2010	9:30:24	46.32	46.33	210	271	P
QQQQ	1/4/2010	9:30:24	46.32	46.33	210	271	P
QQQQ	1/4/2010	9:30:24	46.32	46.33	161	271	P

Estimated H across markets

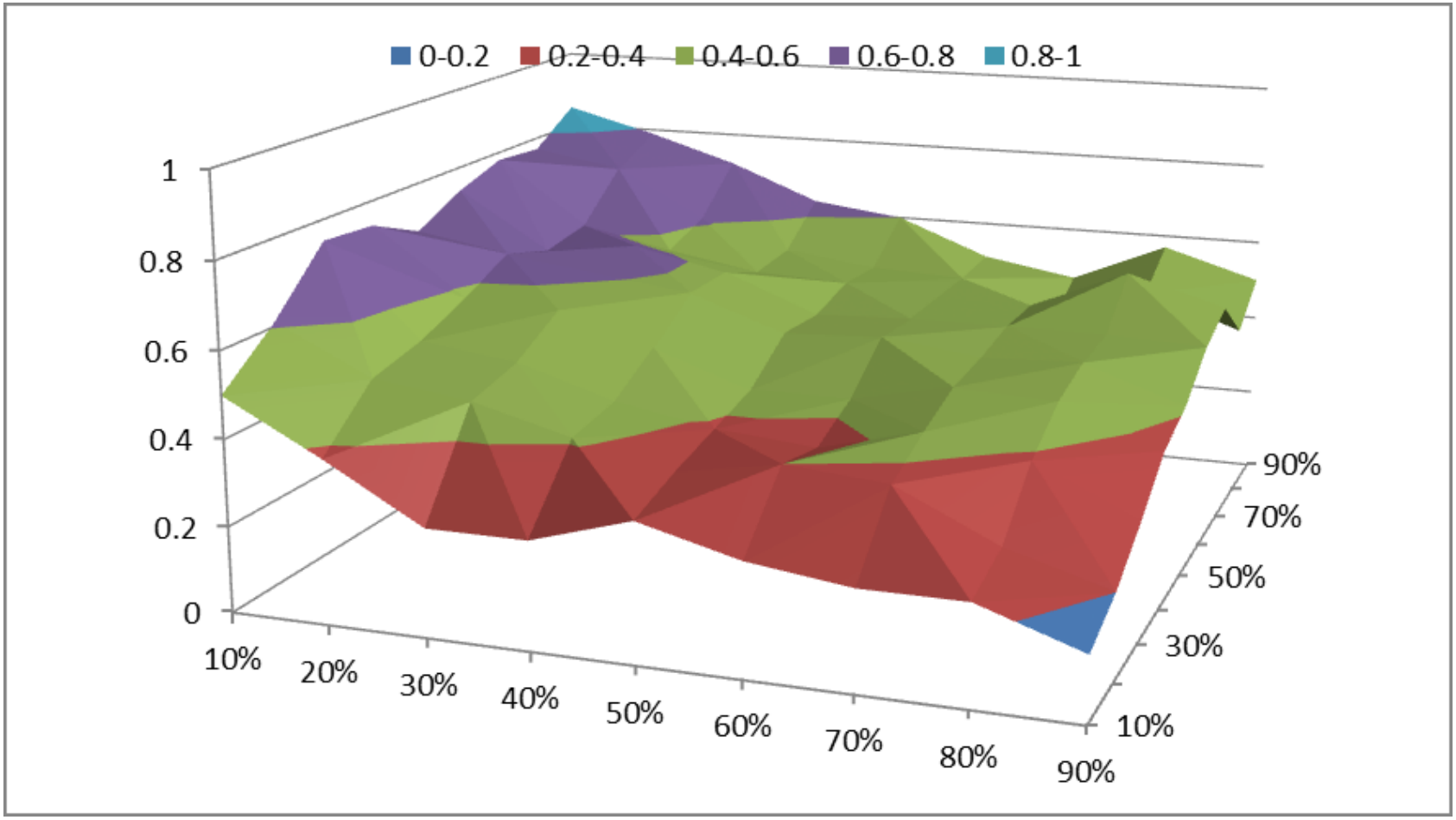
Ticker	NASDAQ	NYSE	BATS
XLF	0.15	0.17	0.17
QQQQ	0.21	0.04	0.18
JPM	0.17	0.17	0.11
AAPL (s=1)	0.16	0.9	0.65
AAPL (s=2)	0.31	0.6	0.64
AAPL (s=3)	0.31	0.69	0.63

Estimation Procedure

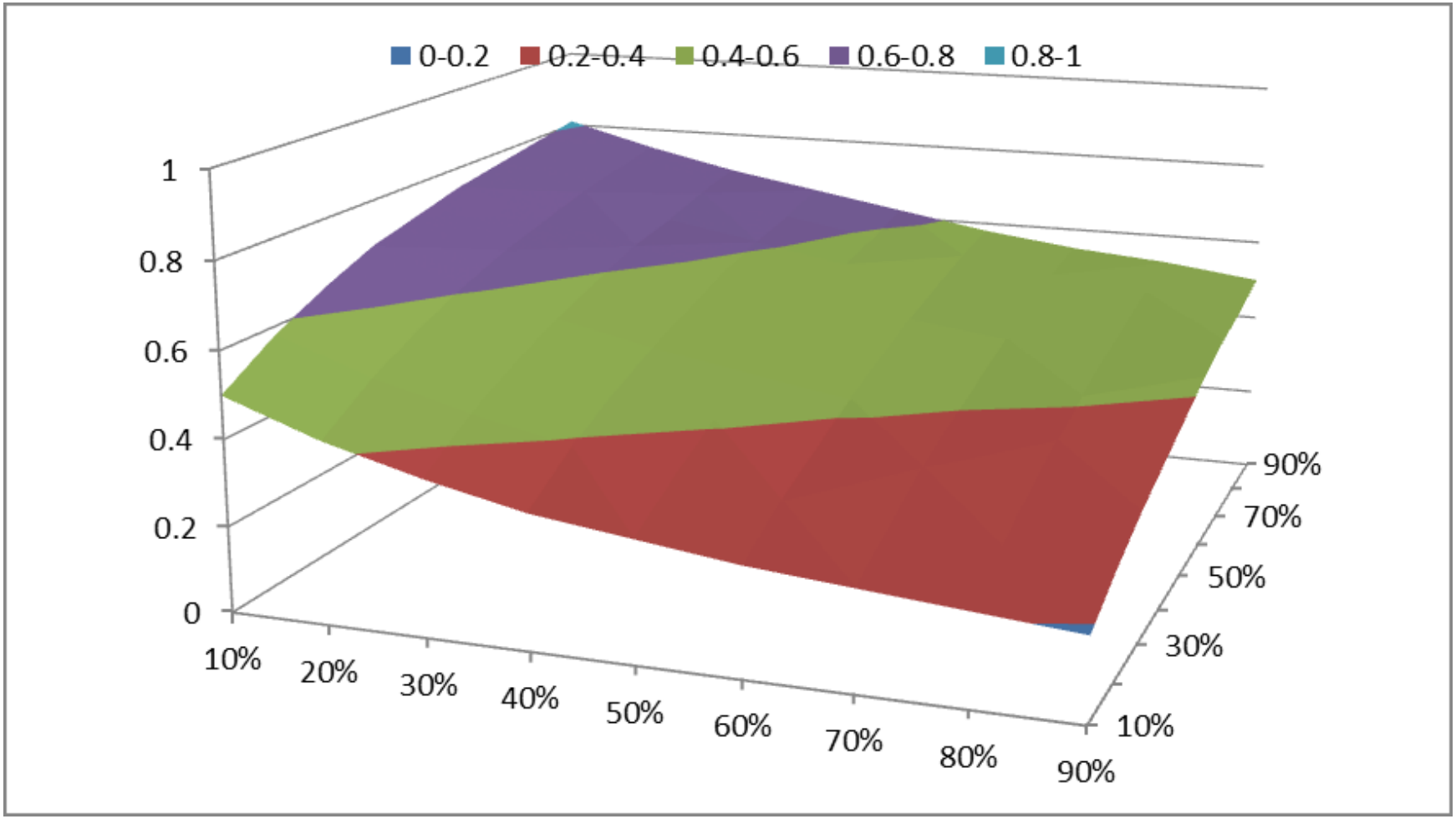
- Separate the data by exchange
- One trading day at a time
- Bucket the quotes (bid size, ask size) by deciles
- For each bucket (i,j) compute the frequency of price increases u_ij
- Count the number of occurrences of each bucket d_ij
- Perform a least-squares fit with the model

$$\min_{H,\rho} \sum_{ij=1}^{10} d_{ij} \left[u_{ij} - \frac{1}{2} \left(1 - \frac{\tan^{-1}\left(\sqrt{\frac{1+\rho}{1-\rho}} \, \frac{j-i}{j+i+2H} \right)}{\tan^{-1}\left(\sqrt{\frac{1+\rho}{1-\rho}} \right)} \right) \right]^2$$

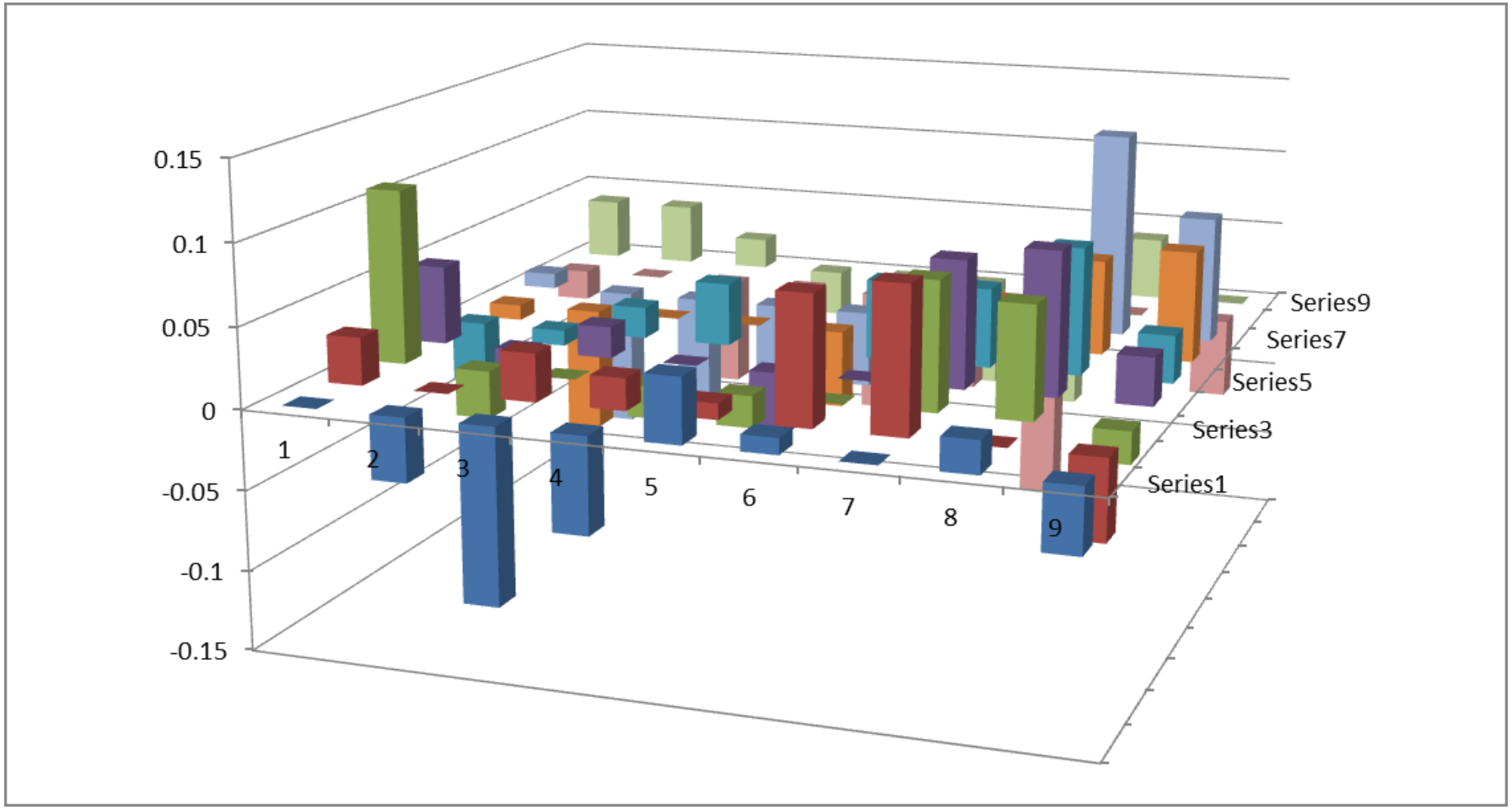
Empirical Probabilities for upward price move conditional on the quote (XLF)



Fitted model (XLF)



Difference between empirical and fitted probabilities

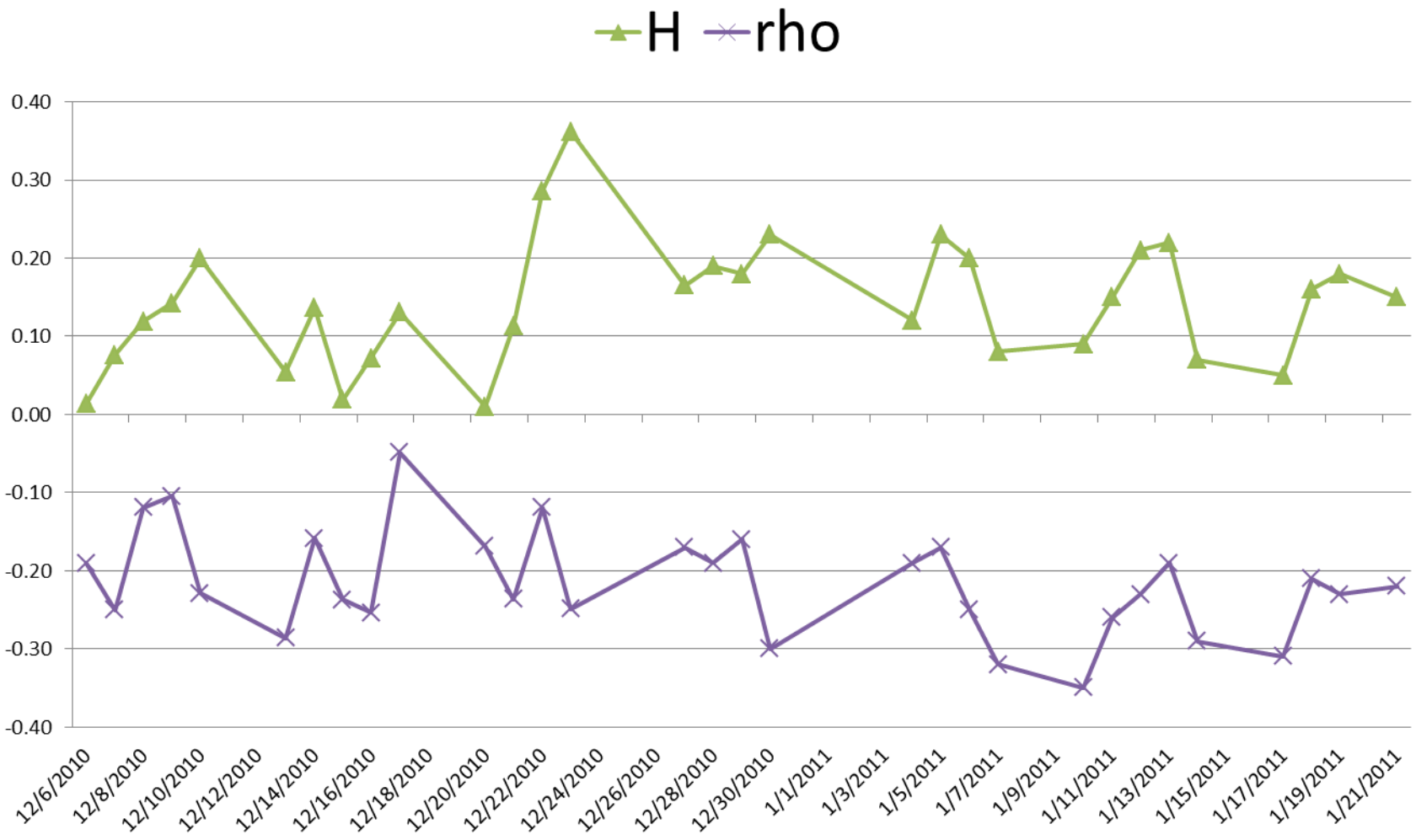


Estimating hidden liquidity (H) across exchanges

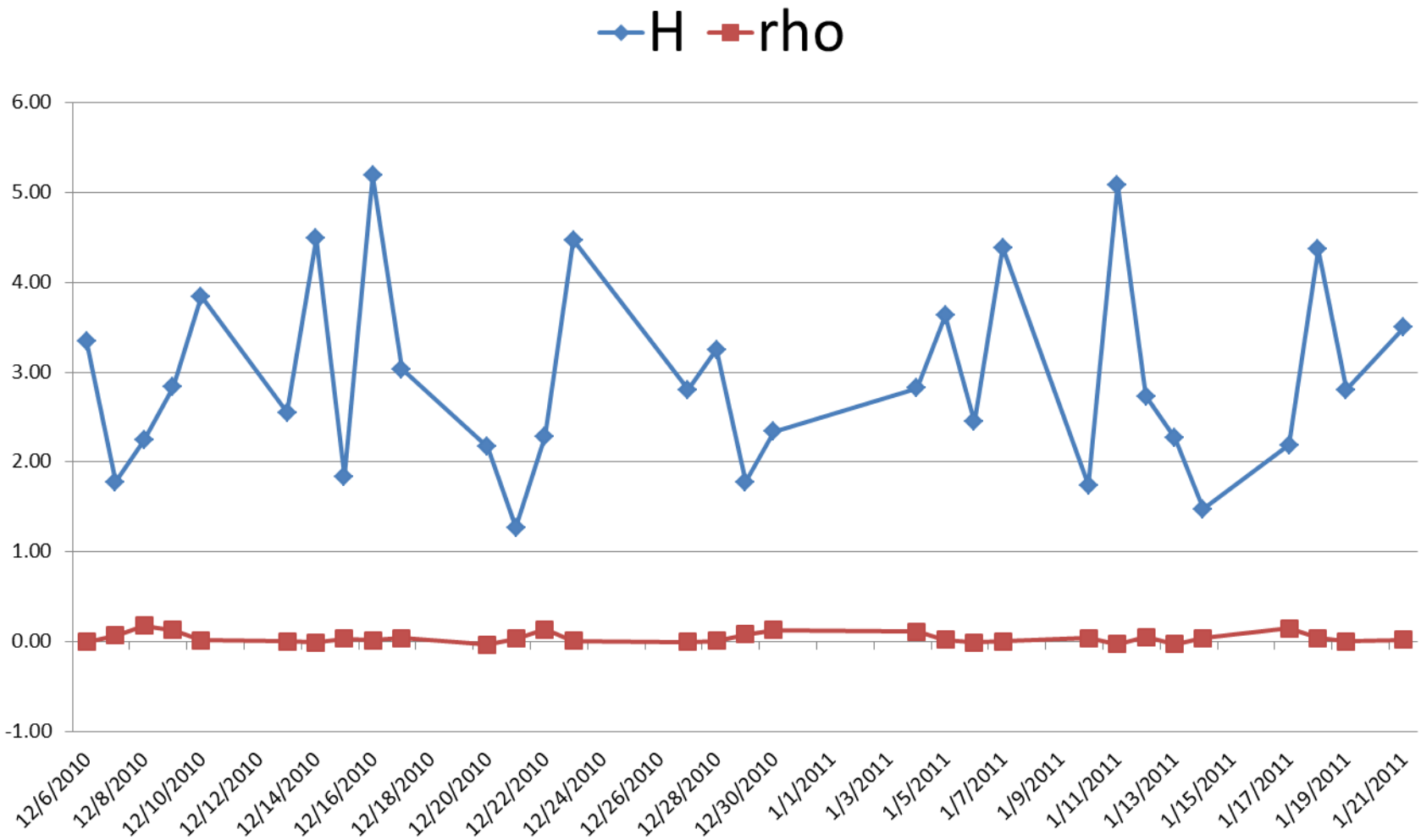
Ticker	NASDAQ	NYSE	BATS
XLF	0.15	0.17	0.17
QQQQ	0.21	0.04	0.18
JPM	0.17	0.17	0.11
AAPL (s=1)	0.16	0.9	0.65
AAPL (s=2)	0.31	0.6	0.64
AAPL (s=3)	0.31	0.69	0.63

Is H stable?

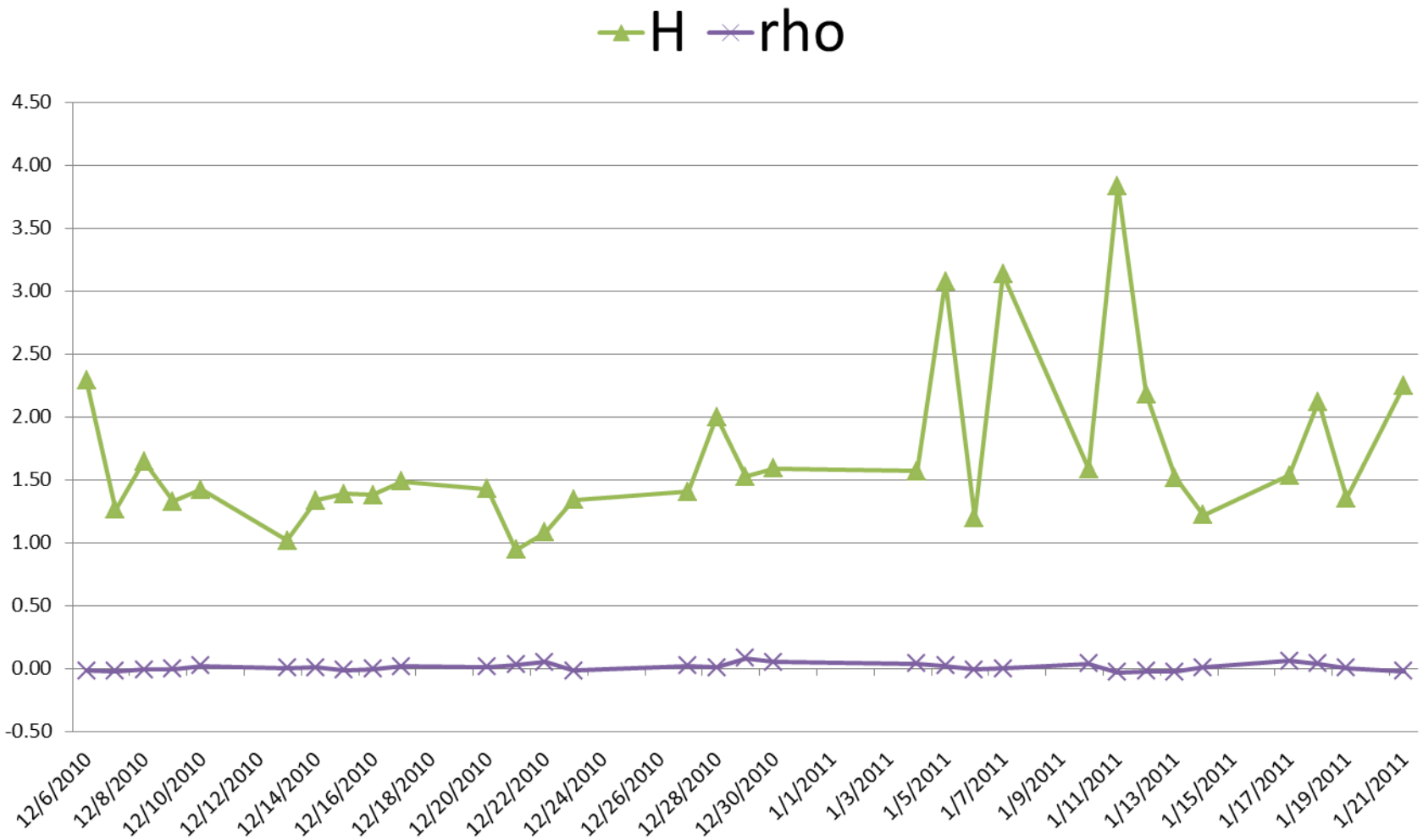
USD-BRL Futures (DOLc1)



Bovespa Index Futures (INDc1)



Mini Bovespa Index Futures (WINc1)



Conclusions

- Over 50% of all trades in the US equity markets are algorithmic. Algorithmic execution of block trades is an important tool allowing for systematic and disciplined execution of size

- The main idea is to split large orders into smaller ones according to available market liquidity, generally following volume (TWAP, VWAP, PoV)

- Algorithmic trading is essential to implement quant strategies such as stat arb and ETF arb

- With DMA and low-latency trading, we see the emergence of autonomous market-makers

- HFT traders provide volume but not necessarily liquidity when needed. Neither do the autonomous MMs (flash crash). Can we detect ``good liquidity'' ?

- Regulation on HFT and electronic market-making is being drafted and implemented as we speak. Recently, **stub quotes were forbidden by the SEC**. Other measures to regulate HF trading will follow.

- Algorithmic trading, DMA, autonomous market-making and HFT are here to stay and are rapidly expanding to new markets in Asia and Latin America.