Quantitative Finance

Automatic one two three

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In this paper, we show how market-technical trends can be calculated automatically from underlying price processes using a stop and reverse process. The basic tool is a so called minmax process indicating all relevant minima and maxima. For the existence of the minmax process, we give a constructive proof. Several successful trend-following trading strategies can be implemented automatically based on this 1-2-3-trend indicator.

1. Introduction

Going back to Charles H. Dow the notion of *trends* is central for trading financial markets (cf. Russell 1981). We use the following (market-technical) trend definition:

Definition 1.1 (**Market-technical Trend**) A financial market is in an **up-trend** if the last (at least two) consecutive relevant lows as well as highs are rising. To be precise, we assume that all lows are monotonically increasing, the highs are even assumed to be strictly monotonically increasing. Similarly, one speaks of a **down-trend**, when the last relevant lows as well as highs are falling (with lows even strictly monotonically decreasing). If neither up- nor down-trend are present, the market is called **trendless**.

Remark 1.2 The above definition allows lows in an up-trend on the same level, whereas each new high in an up-trend has to be strictly above the preceding one. If in an active up-trend the next high is on the same level (or even below) the level of the last high, this is usually a bearish sign. We will denote this situation as 'trend in question' (cf. Definition 3.1 in Section 3 for more details)

Definition 1.3 (**Movement and Correction**) In an up-trend the phase between a minimum and the next (higher) maximum is called **movement**, while the phase between the maximum and the next (higher) minimum is called **correction**. In a down-trend movement and correction are defined analogously (mirrored).

Although this definition at first glance is very intuitive, it is problematic in two ways.

Firstly, the notion of relevant lows or highs is subjective to the observer. For some observers only major extrema are relevant, whereas other observers sense minor extrema also as relevant. This sensitivity problem is not only human. Even a mathematically precise definition cannot find a way around this, as figure 2 shows.

A continuous process dents a new minimum in the left graph until the minimum can clearly be recognized as a relevant minimum next to two relevant maxima (right graph). Since this process is continuous, the decision from when on this new minimum is relevant has to be subjective.

The other problem with the definition of a relevant extremum is the time scaling. Clearly on a 10-min bar chart, much more extrema occur as relevant as on a daily or even weekly bar chart. We therefore take the position that the answer to the question, whether a market is in a trend, depends on the time scaling that we look at.

For instance, a market might be in the correction of a clear up-trend on a daily basis, while at the same time this market has a down-trend on an hourly basis (cf. figure 3).

For further reference, we introduce the numbering of the relevant extrema in a trend according to e.g. Voigt (2008).

Definition 1.4 (**1-2-3-trend Numbering**) In an up-trend the initial minimum is marked with 1, the proceeding maximum with 2 and the next (higher) minimum with 3. Further higher maxima (minima) are also numbered with 2 (3) (cf. figure 1). The same numbering, but mirrored, is used for down-trends (cf. figure 3). In the sequel we will often refer to point 2 or point 3, meaning the price level at these extrema.

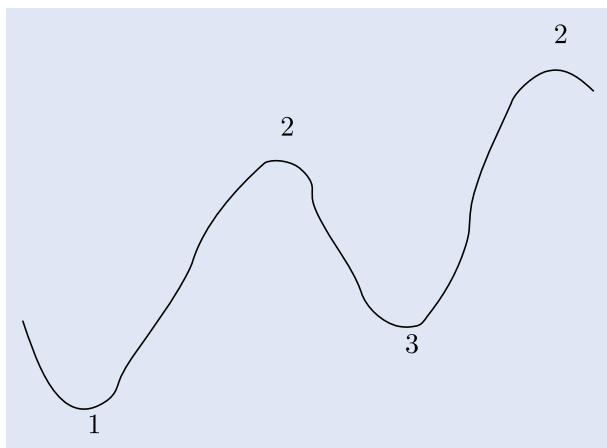*Email: maier@instmath.rwth-aachen.de; www.smp-fe.de

Figure 1.   Up-trend with numbering (cf. Definition 1.4).

Since an up-trend needs, besides two rising minima, also two rising maxima, the up-trend is only initialized after the minimum 3 has formed and the market rises in the sequel above the prior maximum 2. Although the new maximum has not yet been established, it is already clear that it will be above the last maximum 2, giving this level specific importance.

We summarize the proceeding:

Conclusion 1.5. *A precise definition of the phases of a market (up-trend, down-trend and trendless) can be given, once*

   (1) *the time scaling is fixed*
   (2) *depending on that scaling a mathematically exact definition of alternating relevant minima and maxima is given.*

As already outlined in figure 2, the second point in Conclusion 1.5 has to have a subjective solution, i.e. there may be several ways on how to define precisely what the alternating series of relevant minima and maxima is. Nevertheless, once this alternating 'minmax process' is fixed, the trend is well defined according to Definition 1.1.

The knowledge of relevant minima and maxima has furthermore importance for any sort of pattern trading. For instance, Dempster and Jones (2002) considered automatic trading of channel patterns, where they had to automatically find a sequence of two minima and two maxima. Conversely to our approach, they used a simple local definition for extrema. For example, a maximum was obtained if the top of the bar in question was higher than the top of the previous r bars and at least as high as the tops of the next s bars (r and s may vary).

Also, Silagadze (2011) already observed the necessity to recognize relevant minima and maxima for technical analysis of financial price series and proposed an indicator with inherent smoothing properties to do so. The extrema of this indicator, however, do not automatically yield the period and level of the extrema of the price series.

In the following section, we will outline how we intend to construct such a series of alternating minima and maxima based on a SAR process (stop and reserve process). To build the SAR process, we use the MACD indicator or Renko charts. The knowledge of a minmax process can then be used for recognizing market-technical trends according to Definition 1.1, yielding a so-called **1-2-3-trend indicator**. This indicator

can be used for trading trends or movements of a trend as will be shown in Section 3. An empirical study of statistical data obtained from the derived 1-2-3-trend indicator is given in Hafizogullari *et al.* 2013. Further statistical material on mechanical trading systems constructed from this trend indicator can be found in Tronnier (2010), Wache (2010), as well as in Hafizogullari and Platen (2011) and Hafizogullari *et al.* (2012a,b,c). Whereas Wache's 2010 mechanical trading approach was based on the movement when point 2 is trespassed, Tronnier's 2010 approach was to enter a trade in the trend direction when the correction has notably ended.

Hafizogullari and Platen (2011) implemented counter-trend trades, based on our automatic trend recognition, whereas Hafizogullari *et al.* (2012a,b,c) used a breakout system. The statistical data presented in the publications Hafizogullari *et al.* (2012a), Hafizogullari *et al.* (2012b,c), Hafizogullari and Platen (2011), Tronnier (2010) and Wache (2010) altogether prove that our trend indicator can be applied very successfully with high significance.

Notation: *In order to precisely define the relevant indicators in the following, we assume the underlying price process is given as a series with finitely many period based data*

$$\texttt{Open}[n], \texttt{Close}[n], \texttt{High}[n], \texttt{Low}[n],$$
$$n = 0, \ldots, M = \texttt{CurrentBarIndex}(),$$

where $n = 0$ is the current period, $n = 1$ is the period before the current period and $n = M$ is the first period loaded of in total $M+1$ periods. Here n is a relative period number since for a fixed bar this number changes with each new period loaded. For all other indicator series, we will use the same convention. Parameters will be denoted with a $ sign, e.g. $param.

2. MinMax process based on SAR processes

The goal of this section is the construction of a minmax process, i.e. a series of alternating (relevant) minima and maxima of the underlying price process. Before we do so, we need some SAR process also based on the underlying price process that indicates whether the prices currently move up or down. Later in this section, we will use the up-moves to look for relevant maxima and the down-moves to look for relevant minima. Therefore, the SAR process may be viewed as tact giving for the minmax process.

If we code the SAR process in a series called `Direction`, we have at the current period $n = 0$

$$\texttt{Direction}[0] := \begin{cases} +1, & \text{in case the prices move up} \\ -1, & \text{in case the prices move down.} \end{cases}$$

Again, the notion 'prices move up or down' is subjective, but we will give in the sequel several examples of how such a SAR process might be constructed and afterwards we will show in Subsection 2.3 how the SAR process might be used to construct the minmax process. A standard example of a SAR process is the parabolic indicator of Welles J. Wilder (see figure 4, cf. eg. Müller and Lindner 2007).
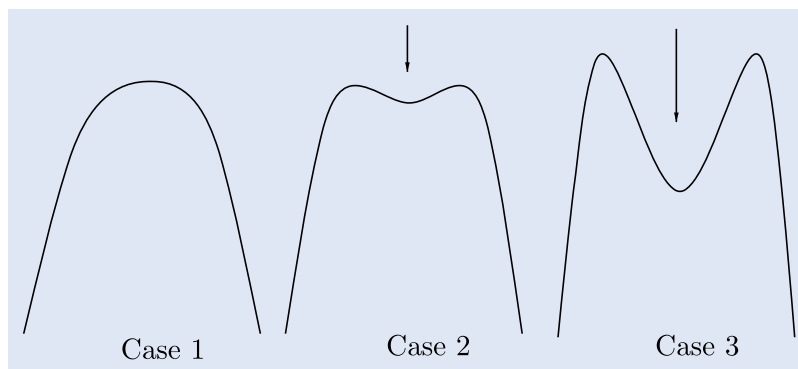
Figure 2.   Continuation from one maximum to three extrema.
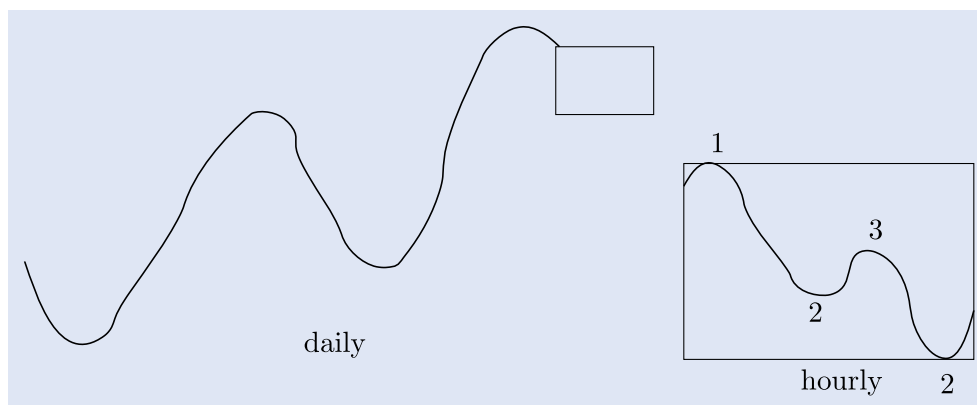
Figure 3.   Trend phases depend on the time scaling.

Figure 4.   Price chart and parabolic SAR.

2.1. *MACD based SAR processes*

The with figure 4 associated MACD (moving average convergence/divergence) indicator with standard parameters (26,12,9) is given below in figure 5.

It appears that the market is rising when the MACD lies above the signal line (with series `MACD` and `Signalline`) and falling when the MACD is below the signal line. An upward crossing of the MACD through the signal line stands commonly as a bullish, a downward crossing as a bearish signal. Therefore, it makes sense to search for relevant highs when the MACD is above the signal line (marked with 'H') and for relevant lows when the MACD is below the signal line (with 'L'), cf. again the price chart figure 4 for these extrema. This leads us to the first MACD-based direction process (cf. figure 6).

$$\texttt{MACDDirection[0]}$$
$$:= \begin{cases} +1, & \text{in case } \texttt{MACD[0]} \geq \texttt{Signalline[0]} \\ -1, & \text{in case } \texttt{MACD[0]} < \texttt{Signalline[0]}. \end{cases} \quad (1)$$

Not all the signals produced by the MACD (or similarly other crossing moving averages) are as clear as in figure 5. Sometimes MACD and signal line are for a longer period quite close, producing several 'false signals'. To avoid the fact that the direction process produces such false signals, we ask for a
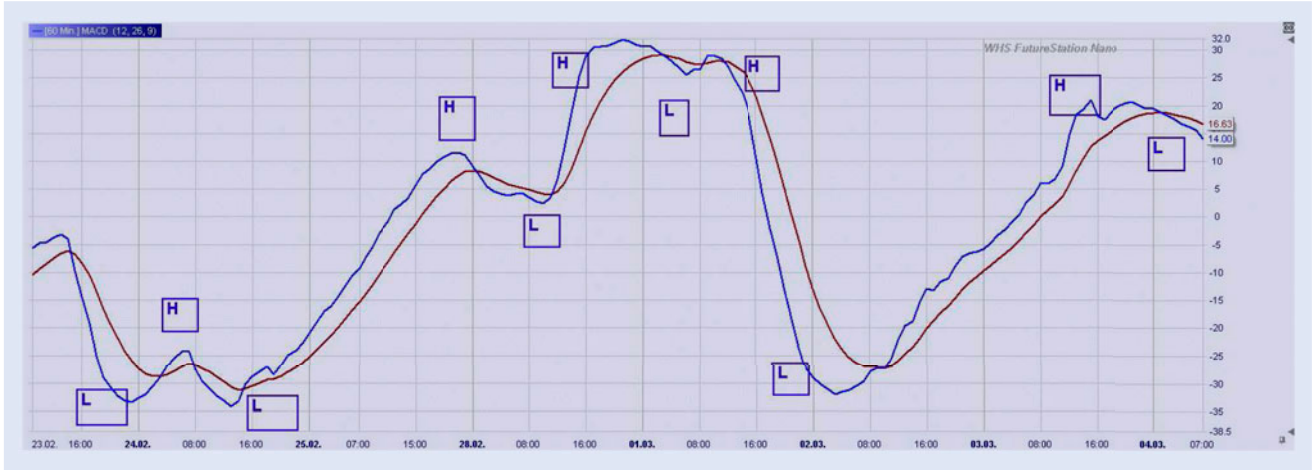
Figure 5.   MACD (blue) and signal line (red) with H(igh) and L(ow) periods.

change of the direction process that the distance of MACD and signal line is above some minimal threshold, i.e. for a change from -1 to $+1$, we ask that

$$\mathtt{MACD}[0] - \mathtt{Signalline}[0] \geq \delta \qquad (2)$$

and for a change from $+1$ to -1 that

$$\mathtt{MACD}[0] - \mathtt{Signalline}[0] \leq -\delta, \qquad (3)$$

where $\delta > 0$ might be constant ($\delta = \$\delta_0$) or relative to the price process, e.g. as percentage of the close price or a multiple of the average true range of the last $\$spanne$ periods

$$\text{i.e.} \quad \delta = \big(\$\delta_0/100\big) \cdot \mathtt{Close}[0] \quad \text{or}$$
$$\delta = \$\delta_0 \cdot \mathtt{ATR}(\$\mathtt{spanne}), \qquad (4)$$

for some parameters $\$\delta_0 > 0$. The direction process that changes sign whenever (2) or (3) are fulfilled will be denoted by the series $\mathtt{DistMACDDir}$ depending on $\delta > 0$ and the three weights of the MACD, i.e.

Definition 2.1 **(Distance MACD Direction)**

$\mathtt{DistMACDDir}[0]$
$$:= \begin{cases} 1, & \text{if } (2) \text{ or } \big(\mathtt{DistMACDDir}[1] = 1 \text{ and not } (3)\big) \\ -1, & \text{if } (3) \text{ or } \big(\mathtt{DistMACDDir}[1] = -1 \text{ and not } (2)\big). \end{cases}$$
$$(5)$$

Sometimes MACD and signal line are quite close for some time such that the minimal threshold from above cannot be reached, although the sign of $\mathtt{MACD} - \mathtt{Signalline}$ has already changed for some periods. An alternative direction process that takes care of this uses the difference of MACD and signal line in an integrated form. Let

$\mathtt{Integral}[0]$
$$:= \begin{cases} 0, & \text{in case } \mathtt{MACD}[0] = \mathtt{Signalline}[0] \\ \sum\limits_{n=0}^{n_0} \big(\mathtt{MACD}[n] - \mathtt{Signalline}[n]\big), & \text{otherwise}, \end{cases}$$

where $n_0 \geq 0$ is chosen maximal such that

$$\mathtt{MACD}[n] - \mathtt{Signalline}[n] \text{ has the same sign for}$$
$$n = 0, \ldots, n_0.$$

The associated direction process changes from -1 to $+1$ in case

$$\mathtt{Integral}[0] \geq \delta, \qquad (6)$$

where δ is constructed as in (4). Similarly, we ask for a change in the direction process from $+1$ to -1 that

$$\mathtt{Integral}[0] \leq -\delta. \qquad (7)$$

The direction process thus generated following (6) and (7) is called series $\mathtt{IntegMACDDir}$:

Definition 2.2 **(Integral MACD Direction)**

$\mathtt{IntegMACDDir}[0]$
$$:= \begin{cases} 1, & \text{if } (6) \text{ or } \big(\mathtt{IntegMACDDir}[1] = 1 \text{ and not } (7)\big) \\ -1, & \text{if } (7) \text{ or } \big(\mathtt{IntegMACDDir}[1] = -1 \text{ and not } (6)\big). \end{cases}$$
$$(8)$$

Remark 2.3 **(MACD Parameters)**   Besides the two parameters $\$\delta_0$ and $\$spanne$ there are the three standard parameters $\mathtt{fastparam} = 12$, $\mathtt{slowparam} = 26$ and $\mathtt{triggerparam} = 9$. In order to make the MACD Direction process faster or slower, it is sufficient for our purposes to scale these three parameters simultaneously, i.e. we use a parameter $\$\mathtt{TimeScale}$ and set

$$\begin{aligned} \mathtt{fastparam} &= 12 \cdot \$\mathtt{TimeScale} \\ \mathtt{slowparam} &= 26 \cdot \$\mathtt{TimeScale} \\ \mathtt{triggerparam} &= 9 \cdot \$\mathtt{TimeScale}. \end{aligned}$$

2.2. Renko-based SAR processes

The following SAR process is based on the idea of Renko charts. In the original version, Renko charts derive from the underlying price process a chart filled with 'bricks' of a before fixed box size.

> In order to draw one or more bricks in
> up-direction (up-bricks)
> $\mathtt{Close}[0]$ has to be at least one box size
> above the high of the last brick.   (9)

Similarly,

> in order to draw one or more bricks in
> down-direction (down-bricks)
> $\mathtt{Close}[0]$ has to be at least one box size
> below the low of the last brick.   (10)

The box size may be constant ($\mathtt{Boxsize} = \$\eta_0$), or relative to the price process as percentage of the close price or
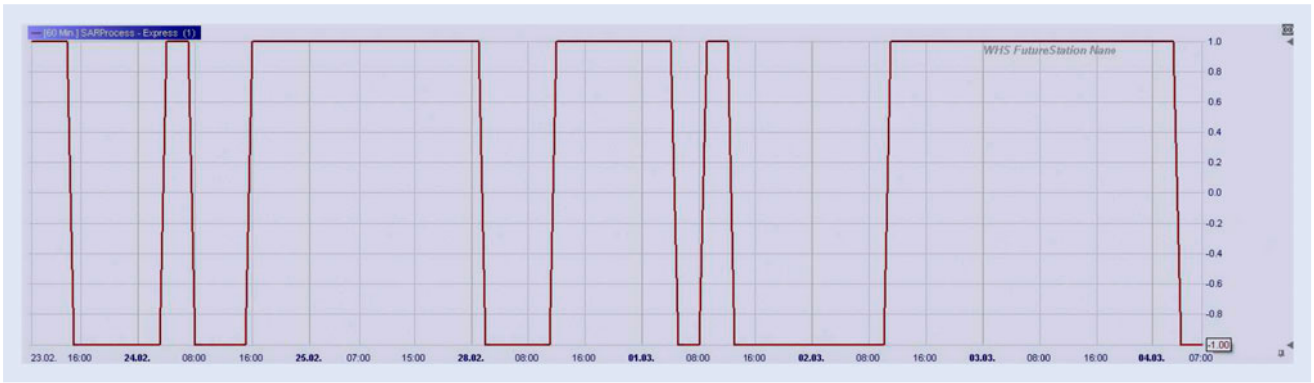
Figure 6.   MACD-based direction process for figures 4 and 5.
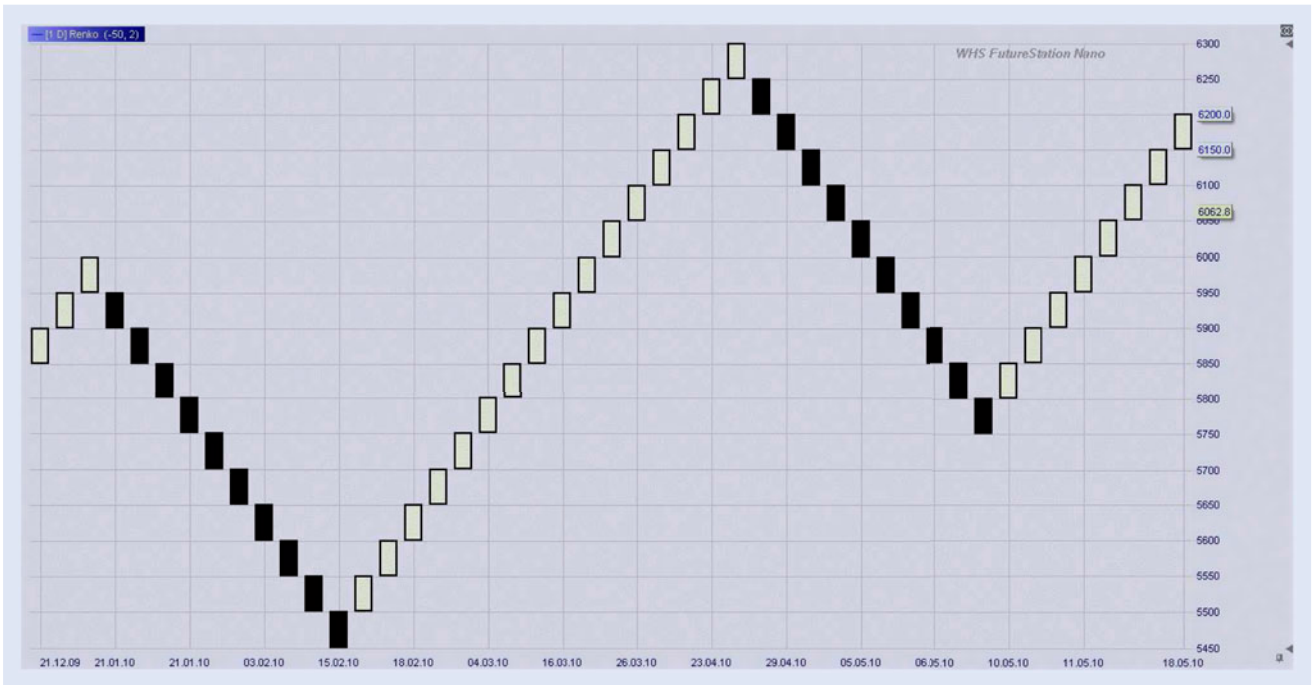
Figure 7.   Renko Chart.

multiple of the average true range, i.e.

$$\text{Boxsize} = \left(\$\eta_0/100\right) \cdot \text{Close}[0] \quad \text{or}$$
$$\text{Boxsize} = \$\eta_0 \cdot \text{ATR}(\$\text{spanne})$$

for some parameter $\$\eta_0 > 0$. The natural direction series associated with this idea is

$$\text{RenkoDirection}[0]$$
$$:= \begin{cases} 1, & \text{in case the last brick is an up-brick} \\ -1, & \text{in case the last brick is a down-brick.} \end{cases}$$

In figure 7, we see a Renko chart with $\text{Boxsize} = 50$ for the price process of figure 8.

The up-bricks are white and the down-bricks are black.

To avoid that, the thus constructed chart of bricks changes direction of the bricks too frequently, we may ask that for a change of direction, a minimum number, say $\$\text{RevAmount} \in \mathbb{N}$, of bricks need to occur before one starts to draw bricks in the new direction. Thus, in order to draw one or more up-bricks,

$\text{Close}[0]$ has to fulfill

$$\text{Close}[0] \geq \begin{cases} \text{High(up-brick)} + \text{Boxsize}, \\ \quad \text{in case of a recent up-brick} \\ \text{High(down-brick)} \\ \quad + \$\text{RevAmount} \cdot \text{Boxsize}, \\ \quad \text{in case of a recent down-brick.} \end{cases} \quad (11)$$

Similarly, in order to draw one or more bricks in down-direction (down-bricks) $\text{Close}[0]$ has to fulfill

$$\text{Close}[0] \leq \begin{cases} \text{Low(down-brick)} - \text{Boxsize}, \\ \quad \text{in case of a recent down-brick} \\ \text{Low(up-brick)} \\ \quad - \$\text{RevAmount} \cdot \text{Boxsize}, \\ \quad \text{in case of a recent up-brick.} \end{cases} \quad (12)$$

Note that up-brick or down-brick in (11) and (12) are not numbers of periods, but as bricks they have upper (High(brick)) and lower (Low(brick)) boundaries. We call this direction series BodyRenkoDir, which depends on Boxsize and $\$\text{RevAmount}$:

Definition 2.4 (**Body Renko Direction**)

BodyRenkoDir[0]

$$:= \begin{cases} 1, & \text{in case of a recent up-brick according to (11)} \\ & \text{and (12)} \\ -1, & \text{in case of a recent down-brick according to (11)} \\ & \text{and (12).} \end{cases}$$

(13)

For $\$RevAmount = 1$, the series RenkoDirection and BodyRenkoDir are equal.

Since new bricks depend only on Close[0], clearly intraperiod highs or lows are ignored. For our application to construct minmax processes, this is problematic. We therefore introduce some kind of continuous Renko chart with

MinimalReversal = Boxsize · $RevAmount

as before, but we refrain from drawing discrete bricks and use instead only one brick with a variable size for each direction.

In case of a recent up-brick (associated with direction equal 1) from the forerunner period, we update the high of the up-brick to High[0], if High[0] is larger than the high of this recent up-brick. If this is not the case, we may get a reversal of the direction process from $+1$ to -1, in case Low[0] is at least
Boxsize + MinimalReversal away from the high of the recent up-brick, i.e.
$\text{High(up-brick)} - \text{Low[0]} \geq \text{Boxsize} \cdot (1 + \$RevAmount)$.
In this case we start a new down-brick. In a similar, but mirrored way, we update lows of recent down-bricks or may get reversals of the direction from -1 to $+1$.

(14)

The resulting direction process will be denoted as RangeRenkoDir series:

Definition 2.5 (**Range Renko Direction**)

RangeRenkoDir[0]

$$:= \begin{cases} 1, & \text{in case of a recent up-brick according} \\ & \text{to (14)} \\ -1, & \text{in case of a recent down-brick according} \\ & \text{to (14).} \end{cases}$$

(15)

In figure 8, we see a price process with current highs of the up-bricks and current lows of the low-bricks calculated according to (11) and (12) (red; Boxsize $= 50$, $\$RevAmount = 2$) and according to (14) (blue; Boxsize $= 100$, $\$RevAmount(\text{down-brick}) = 2$). The associated direction processes based on these bricks are given in figure 9.

2.3. MinMax process

Now that we have constructed several SAR processes, let us outline, how such a process may be used to find relevant minima and maxima. The object that we look for is a minmax process. In the following definition, we use *absolute period numbers* starting at 0 for the first period loaded up to CurrentBarIndex().

Definition 2.6 (**MinMax process**) A *minmax* process is a sequence of alternating minima and maxima with absolute period numbers:

$$\begin{array}{lll} \text{maxima} & a_1 < a_2 < a_3 < \ldots \\ \text{minima} & (b_0 <) \, b_1 < b_2 < b_3 < \ldots \\ \text{with} & (b_0 \leq) \, a_1 \leq b_1 \leq a_2 \leq b_2 \leq a_3 \leq b_3 \leq \ldots \end{array}$$

$$\begin{array}{ll} \text{and} & a_i = b_i \implies b_{i-1} < a_i = b_i < a_{i+1} \\ \text{as well as} & b_i = a_{i+1} \implies a_i < b_i = a_{i+1} < b_{i+1}. \end{array}$$

(16)

The periods numbered b_i should be local relevant minima in the following sense

$$a_i \leq b_i \leq a_{i+1} \quad \text{and} \quad \text{Low}(b_i) \leq \min_{a_i < j \leq a_{i+1}} \text{Low}(j),$$

(17)

and the periods numbered a_i should be local relevant maxima satisfying

$$b_{i-1} \leq a_i \leq b_i \quad \text{and} \quad \text{High}(a_i) \geq \max_{b_{i-1} < j \leq b_i} \text{High}(j).$$

(18)

Furthermore b_i is maximal with the property (17), i.e. any other $\tilde{b}_i \in [a_i, a_{i+1}]$ with $\text{Low}(\tilde{b}_i) \leq \min_{a_i < j \leq a_{i+1}} \text{Low}(j)$ satisfies $\tilde{b}_i \leq b_i$. Similarly, a_i is maximal with the property (18).

In the above definition we use the convention that e.g. High(j) refers to the absolute period number j. For instance, High(0) is the high of the first period loaded, High(1) is one of the second period and so on, or more precisely

$$\text{High(bar)} = \text{High[CurrentBarIndex()} - \text{bar]}.$$

This is necessary, because the period of a relevant maximum does not change as new periods are loaded into the chart.

Remark 2.7 Note that the conditions for relevant extrema in (17) and (18) are asymmetric.

At first glance, it would be desirable to have e.g.

$$\text{Low}(b_i) = \min_{a_i \leq j \leq a_{i+1}} \text{Low}(j).$$

(19)

However, this cannot be archived as the example in figure 10 shows (minimum and maximum occur in the same period; i.e. $b_{i-1} = a_i$).

We have indicated in figure 10 how b_{i-1} and b_i have to be chosen to satisfy (17) and (18). If we wanted to satisfy (19), we would have to choose $b_{i-1} = b_i = a_i$. But this is forbidden according to (16).

Let us start with the construction of such a minmax process.

No matter what *SAR process* we use, for simplicity in the following we will only speak of a Direction series. We begin noting that the change of the direction occurs delayed, since all

Figure 8. Body Renko bricks (red) and Range Renko bricks (blue).

Figure 9. `Direction` processes for figure 8.
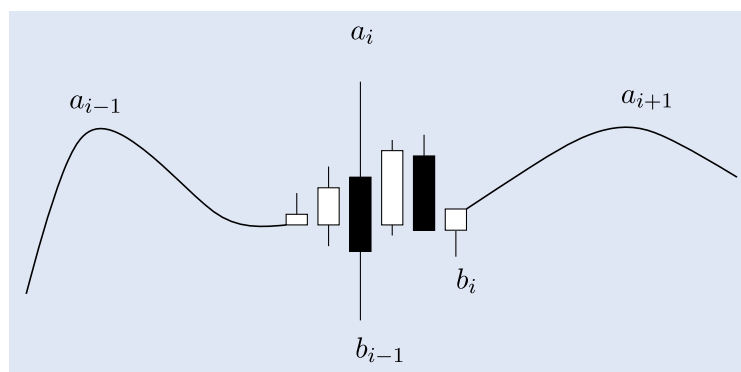
Figure 10. Relevant extrema in (17) and (18).

our stop and reverse indicators are trend following. That means, typically, once the direction changes from $+1$ to -1, the market has already formed out a relevant maximum beforehand. Similarly, a relevant minimum is typically established before the direction changes back. This guides us to the following strategy:

$$\text{Whenever } \texttt{Direction}[0] = 1,$$
we search for a relevant maximum and
$$\text{whenever } \texttt{Direction}[0] = -1, \quad (20)$$
we search for a relevant minimum.

The search, e.g. for a new relevant maximum, should enable us to move the level of the recent high further up as prices proceed growing. I.e. as long as we search for the new maximum,

the so-far found level of that maximum is only temporary. After we switch to searching for a new relevant minimum, however, we have to fix that former temporary maximum level once and forever. This feature is absolutely necessary. Otherwise the resulting minmax process is useless for generating trend signals, because with changing the relevant minimum and maximum levels a posteriori also the trend signals would have to be altered later on.

Before we go further into the construction of the maxima and minima, we need to point out some exceptional situations. Assume the situation of figure 11.

Here the direction during the last few periods is $+1$, however, `Low[0]` lies below the last relevant minimum found at
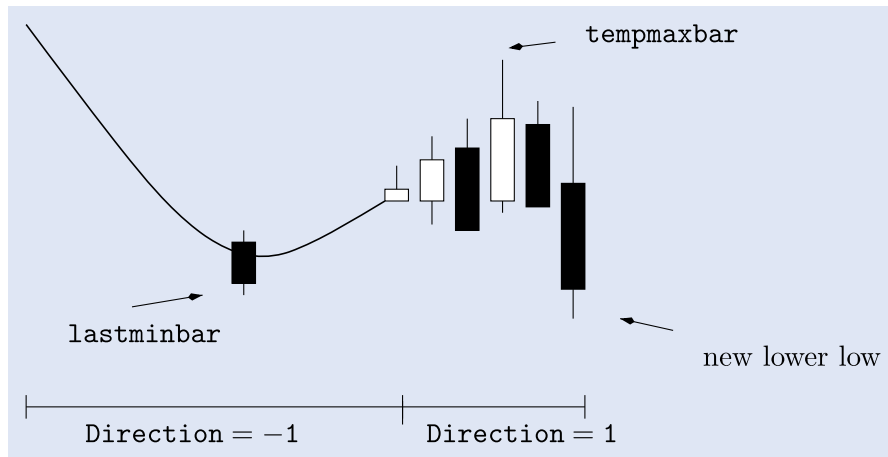
Figure 11.   Exceptional Situation.
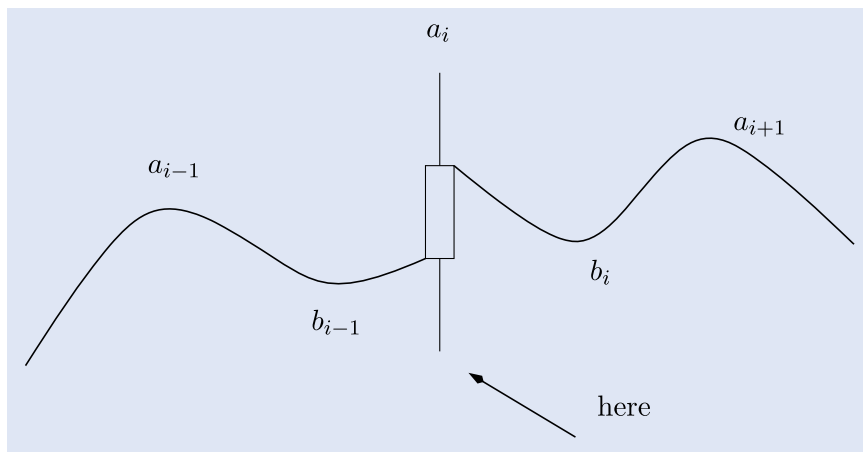
Figure 12.   Relevant extrema: minimum at a_i not detected ($\$\mathtt{Choice} = 1$).

Figure 13. $\mathtt{Minmaxvalue}$ series indicating minima and maxima.

$\mathtt{lastminbar}$, which was already fixed. If we would proceed looking for a new relevant maximum according to (20), we might miss that last low as possible relevant low.

The only way to get around that dilemma, once such an exceptional situation has appeared, is to switch to searching opposite to the direction and fix the so far found temporary maximum. We therefore introduce a new series

$\mathtt{Excep}[0]$
$$:= \begin{cases} -1, & \text{in case an exceptional situation has appeared} \\ 1, & \text{otherwise} \end{cases}$$

and

$$\mathtt{Status}[0] := \mathtt{Excep}[0] \cdot \mathtt{Direction}[0] \qquad (21)$$

yielding a new strategy:
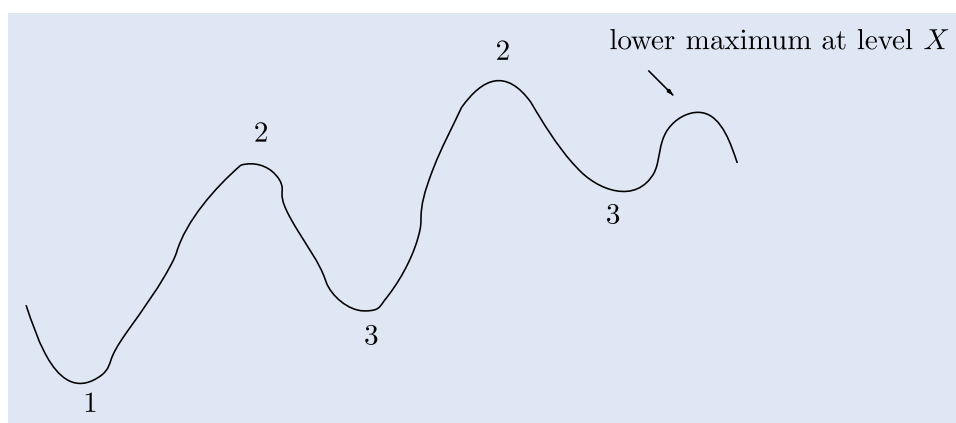
Figure 14.   Up-trend in question.

Basic MinMax Strategy 2.8.

> *Whenever* Status[0] = +1,
> we search for a relevant maximum, and
> *whenever* Status[0] = −1,
> we search for a relevant minimum.

At this point we still have to discuss how the exceptional situations may end. Assuming again the situation of figure 11, there are two possibilities:

(i) The prices continue to fall and eventually the direction changes to −1.
(ii) The prices start rising again and exceed eventually above the last relevant maximum that had to be fixed on entry of the exceptional situation.

In both cases, we switch back to normal, i.e. in case **(i)** we just change Excep[0] back to 1 (yielding Status[0] = −1) and in case **(ii)** we change Excep[0] back to 1 (yielding Status[0] = 1) and additionally fix the recently found minimum, i.e. we continue to search for a new maximum.

For definiteness, we give the update of the Excep series once the last relevant maximum (at absolute period number lastmaxbar) and the last relevant minimum (at absolute period number lastminbar) have been found:

Definition 2.9 **(Update Excep series)**
If $\big(\text{Excep[1]} = -1\big)$ **then**
// *exceptional process was already active*

In the following we use the convention that in case

Status[0] = 1, the last fixed relevant minimum occurred in the period lastminbar and the temporary maximum afterwards is in the period tempmaxbar.

Similarly, in case

Status[0] = −1, the last fixed relevant maximum occurred in the period lastmaxbar and the temporary minimum afterwards is in the period tempminbar.

Hence

> lastminbar ≤ tempmaxbar and lastmaxbar
> ≤ tempminbar (if \$Choice = 0)
> or lastminbar < tempmaxbar and lastmaxbar
> < tempminbar (if \$Choice = 1)

are all absolute period numbers. Depending on whether we allow minima and maxima to occur in the same period, we get different processes of *minima* and *maxima*. This will be modelled with the parameter \$Choice. We will see later that only \$Choice = 0 yields a *minmax* process satisfying all criteria of Definition 2.6.

In order that Definition 2.9 is well defined, we need to know what lastmaxbar and lastminbar is. These two bars can only be defined after the SAR process has finished initializing (say Direction = 0 while initializing).

$$
\text{Excep[0]} := \begin{cases} 1, & \text{in case } \big(\text{Direction[1]} \cdot \text{Direction[0]} = -1\big) \\ & \text{or } \big(\text{Direction[1]} = 1 \text{ and } \text{High(lastmaxbar)} \leq \text{High[0]}\big) \\ & \text{or } \big(\text{Direction[1]} = -1 \text{ and } \text{Low(lastminbar)} \geq \text{Low[0]}\big), \\ -1, & \text{otherwise} \end{cases}
$$

else if $\big(\text{Direction[1]} = \text{Direction[0]}\big)$ **then**
// *check for exceptional situation*

$$
\text{Excep[0]} := \begin{cases} -1, & \text{in case } \big(\text{Direction[0]} = 1\big) \text{ and } \big(\text{Low(lastminbar)} \geq \text{Low[0]}\big) \\ & \text{or } \big(\text{Direction[0]} = -1\big) \text{ and } \big(\text{High(lastmaxbar)} \leq \text{High[0]}\big) \\ 1, & \text{otherwise} \end{cases}
$$
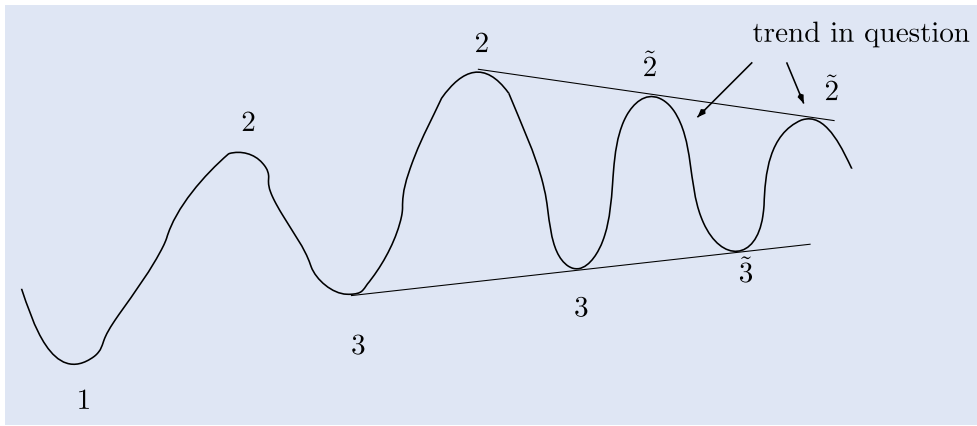
Figure 15.   Often trends in question form a symmetric triangle.
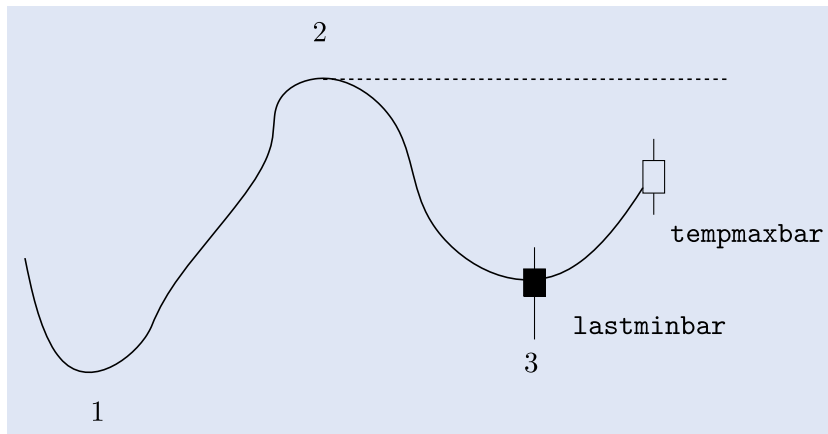
Figure 16.   Possible up-trend (trend in prephase).
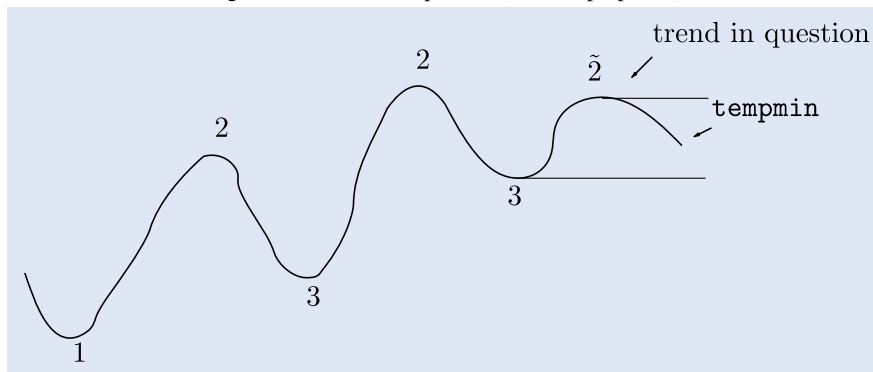
Figure 17.   Simultaneous `UpTrigger` (trend in question) and `DownTrigger` (prephase trend).

Definition 2.10 (**Initial MinMax process**)

If $\Big($`Direction`$[1] = 0$ and `Direction`$[0] = 1\Big)$ **then**

  `lastminbar` := period of lowest low of all
    periods loaded so far.

If $\Big($`Direction`$[1] = 0$ and `Direction`$[0] = -1\Big)$**then**

  `lastmaxbar` := period of highest high of all
    periods loaded so far.

In both cases, we start with `Status`$[0] =$ `Direction`$[0]$, i.e. `Excep`$[0] = 1$.

Having fixed, e.g. the last minimum at `lastminbar`, `tempmaxbar` is the period of the highest high after

`lastminbar` as long as `Status`$[0] = 1$. We therefore can now state how we update the *minmax* process.

Definition 2.11 (**Update MinMax process**)

If$\big($`Status`$[1] = 1\big)$ **then**
 begin

   if$\Big($`High(tempmaxbar)` $\leq$ `High`$[0]\Big)$ **then**

     //update `tempmaxbar`
     `tempmaxbar` := `CurrentBarIndex()`

   if$\big($`Status`$[0] = -1\big)$ **then**

     //`Status` has changed; fix the last maximum

begin
  `lastmaxbar := tempmaxbar`
  `tempminbar := bar of lowest low with`
    `lastmaxbar` $+ \alpha$
      $\leq$ `tempminbar` $\leq$ `CurrentBarIndex()`,
                (22)

where

$$\alpha := \begin{cases} 1, & \text{if } (\$\texttt{Choice} = 1) \text{ or} \\ & \quad (\texttt{lastmaxbar} = \texttt{lastminbar}) \\ 0, & \text{otherwise} \end{cases} \quad (23)$$

  end
  end
If $\quad$ $(\texttt{Status[1]} = -1)$ **then**
  begin
    if $\quad$ $\Big(\texttt{Low(tempminbar)} \geq \texttt{Low[0]}\Big)$ **then**
      `//update tempminbar`
      `tempminbar := CurrentBarIndex()`
    if $\quad$ $(\texttt{Status[0]} = 1)$ **then**
      `//Status has changed; fix the last minimum`
      begin
        `lastminbar := tempminbar`
        `tempmaxbar := bar of highest high with`

        `lastminbar` $+ \alpha \leq$ `tempmaxbar`
          $\leq$ `CurrentBarIndex()` $\quad$ (24)

            `//where` α `is defined as in (23)`
      end
  end

Remark 2.12 $\quad$ If the seldom situation `lastmaxbar` $+ \alpha >$ `CurrentBarIndex()` in (22) occurs, we choose `tempminbar` as the following period, i.e. `CurrentBarIndex()` $+ 1$. Similarly, we proceed with `tempmaxbar` in (24).

This finishes the construction of the minmax process. The sequence of `lastminbars` and `lastmaxbars` will be denoted b_i and a_i, respectively. For $\$\texttt{Choice} = 1$, in (23), we obtain always $\alpha = 1$ yielding

$$a_{i-1} < b_{i-1} < a_i < b_i,$$

because e.g. in (22)

$$\texttt{tempminbar} > \texttt{lastmaxbar}$$

and after a `Status` change `tempminbar` becomes `lastminbar`. Therefore, in case $\$\texttt{Choice} = 1$, we obtain by construction

$$\texttt{Low}(b_i) = \min_{a_i < j < a_{i+1}} \texttt{Low}(j)$$

$$\text{and} \quad \texttt{High}(a_i) = \max_{b_{i-1} < j < b_i} \texttt{High}(j),$$

which is **not** enough to be a minmax process according to Definition 2.6 (cf. (17) and (18)). In fact, this process may fall short to detect all relevant extrema, as is shown in figure 12.

Let us now have a closer look at the case $\$\texttt{Choice} = 0$. In this case α in (23) is only 1 if the relevant minima and maxima occurred at the same bar. Consequently,

$$\begin{aligned} a_i &= b_i &\implies& \quad b_{i-1} < a_i = b_i < a_{i+1} \\ \text{and} \quad b_i &= a_{i+1} &\implies& \quad a_i < b_i = a_{i+1} < b_{i+1} \end{aligned}$$

is satisfied, as is necessary for a minmax process according to Definition 2.6 (cf. (16)).

Again by construction for the case $\$\texttt{Choice} = 0$, we obtain

$$\texttt{Low}(b_i) = \min_{a_i < j \leq a_{i+1}} \texttt{Low}(j), \quad \text{if } b_{i-1} = a_i$$

$$\text{and} \quad \texttt{Low}(b_i) = \min_{a_i \leq j \leq a_{i+1}} \texttt{Low}(j), \quad \text{if } b_{i-1} < a_i.$$

In any case we get

$$\texttt{Low}(b_i) \leq \min_{a_i < j \leq a_{i+1}} \texttt{Low}(j)$$

giving (17). (18) follows analogously. Therefore the process of alternating minima and maxima constructed in Definitions 2.9, 2.10 and 2.11 for $\$\texttt{Choice} = 0$ has all the properties for a minmax process, yielding:

THEOREM 2.13 **(Existence MinMax process)** *For every SAR process it is possible to give (in realtime) a minmax process according to Definition 2.6 that has at least as many extrema as the SAR process has sign changes.*

Proof *Above, we already showed that the process we constructed with* $\$\texttt{Choice} = 0$ *has all the properties of Definition 2.6. Every time a change of the* `Status` *series in Definition 2.11 occurred, we obtained a new extremum. By construction in (21), the number of sign changes of the* `Status` *series is at least as large as the number of sign changes of the* `Direction` *series.*

Remark 2.14 With 'in realtime' we mean that we do decide on all extrema at the current period. Each extremum, once fixed (as `lastminbar` or `lastmaxbar`), will not be changed in retrospective. Only the temporary extrema are updated when new periods are loaded.

Definition 2.15 **(minmaxvalue series)** $\quad$ In order to be able to plot the minmax process, we define a series `minmaxvalue` that contains all relevant minima and maxima. Let $0 \leq p \leq$ `CurrentBarIndex()` be an absolute period number, then

$$\texttt{minmaxvalue}(p) := \begin{cases} \texttt{High}(a_i), & \text{for } b_{i-1} < p \leq a_i \\ \texttt{Low}(b_i), & \text{for } a_i < p \leq b_i. \end{cases} \quad (25)$$

For $p > \max\{\texttt{lastmaxbar}, \texttt{lastminbar}\}$ this series is not yet defined. To indicate the value of the current temporary extremum, we add

$$\texttt{minmaxvalue}(p)$$
$$:= \begin{cases} \texttt{High(tempmaxbar)}, & \text{if } p > \texttt{lastminbar} \\ & \text{and } \texttt{Status[0]} = 1 \\ \texttt{Low(tempminbar)}, & \text{if } p > \texttt{lastmaxbar} \\ & \text{and } \texttt{Status[0]} = -1. \end{cases} \quad (26)$$

Exception In case a bar contains minimum and maximum simultaneously, we want to see both of these levels. Therefore we redefine

$$\texttt{minmaxvalue}(b_i + 1) := \texttt{Low}(b_i), \quad \text{if } a_i = b_i,$$

$$\texttt{minmaxvalue}(a_i + 1) := \texttt{High}(a_i), \quad \text{if } b_{i-1} = a_i. \quad (27)$$

This defines a `minmaxvalue` series, which is able to represent all relevant minima and maxima (see figure 13).
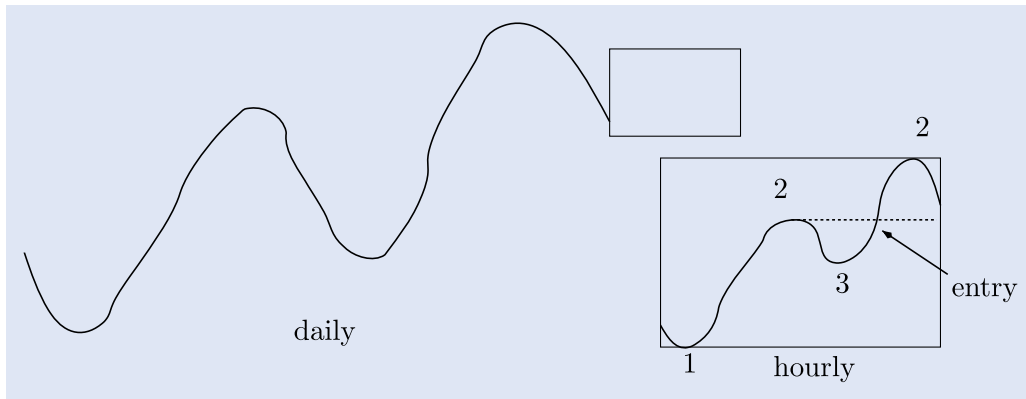
Figure 18.   Multi-scale 1-2-3.

The example in figure 13 corresponds to the MACD based direction process of figures 5 and 6. Note that here two additional extrema occur due to an exceptional situation.

3. Automatic market-technical trends

3.1. *Construction of automatic one two three*

In the last section we have constructed a minmax process according to Definition 2.6. In order to generate the market-technical trend from this process, we introduce the

series
`TempExtremum, LastExtremum, BeforeLastExtremum,`
`ThirdLastExtremum, ForthLastExtremum,`

$$(28)$$

where `TempExtremum` is the value of the extremum associated with `tempminbar` or `tempmaxbar`, i.e.

`TempExtremum[0]`

$$:= \begin{cases} \text{High(tempmaxbar),} & \text{if Status}[0] = 1 \\ \text{Low(tempminbar),} & \text{if Status}[0] = -1, \end{cases}$$

and `LastExtremum` is the value of the extremum associated with `lastminbar` or `lastmaxbar`, i.e.

`LastExtremum[0]`

$$:= \begin{cases} \text{High(lastmaxbar),} & \text{if Status}[0] = -1 \\ \text{Low(lastminbar),} & \text{if Status}[0] = 1. \end{cases}$$

The other three series in (28) are similarly the values of the preceding minima and maxima.

Having these series at hand, it is easy to define a series `Currenttrend`

`Currenttrend[0]`

$$:= \begin{cases} 1, & \text{in case of an up-trend} \\ 0.5, & \text{in case of an up-trend in question} \\ 0, & \text{in case of no trend} \\ -0.5, & \text{in case of a down-trend in question} \\ -1, & \text{in case of a down-trend.} \end{cases} \quad (29)$$

The definition of up-trend and down-trend was already given in Definition 1.1. A trend in question is a situation, where the formal trend definition no longer holds, but the stop level of a former trend (point 3; cf. Definition 1.4) is not yet broken, see figure 14.

Although here a maximum with lower (or equal) level X than the last point 2 has been found, the up-trend could quickly be continued when the prices exceed the value of X (forming a new 1-2-3). We therefore want to track these situations giving:

Definition 3.1 (**Trends in question**) An up-trend (down-trend) is **in question**, if the former `high` (`low`) of the last maximum (minimum) cannot be exceeded, but the stop level of the trend is not yet broken.

Once a trend is in question, it stays in question as long as the relevant highs are monotonically decreasing and the relevant lows are monotonically increasing (cf. figure 15).

The trend numbering can be extended to trends in question as indicated in figure 15.

Remark 3.2 Note that the end of a trend in question can only be a continuation of the old trend at point $\tilde{2}$, or, after breaking the last relevant stop level, the initialization of a new trend in the opposite direction.

The precise knowledge of the relevant extrema (28) gives us the possibility to recognize situations, where trends may occur in one of the following bars. E.g. in figure 16 no up-trend has been established yet, however, it is already clear that an up-trend will form, once the prices cross the indicated level of point 2. We therefore introduce series

`UpPrePhase[0]`

$$:= \begin{cases} 0.5, & \text{if an up-trend is possible (but not yet existent)} \\ 0, & \text{else} \end{cases}$$

`DownPrePhase[0]`

$$:= \begin{cases} -0.5, & \text{if a down-trend is possible (but not yet existent)} \\ 0, & \text{else.} \end{cases}$$

Whenever trends are active or in prephase, we have knowledge of the relevant points 2 and 3 (or $\tilde{2}$ and $\tilde{3}$ for trends in question) giving series

`UpTrigger[0]`

:= point 2 level in an up-trend (active or in prephase)

`DownTrigger[0]`

:= point 2 level in a down-trend (active or in prephase)

`StopTrigger[0]`

:= point 3 of an active trend.

Note that there are situations, where `UpTrigger` and `DownTrigger` are simultaneously meaningful (e.g. in an up-trend in question there might occur a prephase for a down-trend, see figure 17).

Sometimes trends initiate without prior notice through our prephase series. More specifically, this may happen, e.g. in exceptional situations from Subsection 2.3 (see figure 11), because here the change of the `Direction` comes too late to foresee the breakout in the opposite direction.

Another very helpful series that now can be constructed easily is

`Movementtrigger[0]`

$$:= \begin{cases} 1, & \text{in this period a point 2 has been reached in an up-trend} \\ -1, & \text{in this period a point 2 has been reached in a down-trend} \\ 0, & \text{else.} \end{cases}$$

$$(30)$$

Adding up the `Movementtrigger` each time a trend is continued, we obtain the amount of the movements for each trend (series `Movementnumber`).

3.2. Applications of automatic one-two-three

Having the series `Currenttrend` (see (29)) at hand, it is easy to implement all kind of trend related trading ideas.

3.2.1. Trading Trends. A trend trader establishes a long position in an up-trend and a short position in a down-trend.

The position can usually be entered at exactly the point 2 by a stop order, because the series `UpPrePhase` and `DownPrePhase` announce possible trends and `UpTrigger` and `DownTrigger` contain the relevant point 2 level. Alternatively, one may just wait for the end of a period that triggers a new trend and open the position at the close when `Currenttrend` switched from 0 to 1 (long position) or -1 (short position).

The trend trade is open as long as the trend is active. Usually this is handled with a stop at the point 3 (available through the series `StopTrigger`) or shortly behind the point 3. Alternatively, a trend trade may also be closed once a trend is in question (cf. Definition 3.1). Here in addition to the stop at point 3, the trade is closed when `Currenttrend` changes from 1 to 0.5 or from -1 to -0.5.

3.2.2. Trading Movements at Point 2. The idea of a movement trade is to use a part of the movement (cf. Definition 1.4) between the point 3 and (a new) point 2 for the trend. This movement often takes a shorter period of time as the correction (from the former point 2 to the point 3) while crossing a longer range. The typical entry into a movement trade is a stop order at the point 2 as for trend trades. Using the series `Movementtrigger` (see (30)) all the points 2 in a trend can be used for that trading ansatz.

In contrast to the trend trade, the movement trade needs a tighter stop that finishes the trade once the movement is (or seems) over. The stops used for this are all some kind of trailing stops (e.g. `InOutBarsSAR` of Maier-Paape (2010) is a stop based on in- and outbars going back to Voigt (2008)). Another suitable stop for movement trades is the `ParabolicSAR` (cf. Müller and Lindner 2007).

Elaborate statistical data on movement trades based on our method can be found in Wache (2010). Hafizogullari *et al.* (2012a,b,c) investigated breakout systems with entries from our trend indicator and small a priori fixed stops and targets.

3.2.3. Trading Movements out of the Correction. The idea here is to start the trade of the movement earlier than at point 2. One possibility is to start the trade of the movement once a new point 3 has formed in an already active trend that is currently in a correction. With our methods, this is clear when a change of `Status` (cf. (21)) has occurred. If we use a SAR process based on Range Renko Direction (see (15)), this can even be decided intraperiod due to the decision rules in (14). The initial stop lies at the new point 3. After the point 2 has been exceeded, one has to switch to a trailing stop as before (e.g. `InOutBarsSAR` or similar). There are much more entry rules that make sense to enter a trade in the correction (e.g. reversal candles, breaking of a trend line, forming of a trend in a finer time scale, Fibonacci levels), some of which have been successfully backtested with our 1-2-3-trend indicator (see Tronnier 2010). Wache (2010) also used multi-scale 1-2-3 for trading movements (see figure 18).

3.2.4. Other applications. Another application for trading counter-trends can be found in Hafizogullari and Platen (2011), which is based on our 1-2-3-trend indicator and `InOutBarsSAR` as well.

4. Conclusion

The discussion following figure 2 made clear that any definition of market-technical trends is always subjective, because the relevant minima and maxima are subjective. We nevertheless gave a clear and precise derivation of a minmax process (containing all relevant minima and maxima) only based on the prior knowledge of a SAR process.

We furthermore indicated how the minmax process may be used to implement all kind of trend following trading styles automatically. Further work and work in progress concerns empirical statistics on the 1-2-3-trend indicator, which may help to characterize markets and do comparisons between different markets (see Hafizogullari *et al.* 2013).

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