

INDICATORS

One Fish, Two Fish

Using The Fisher Transform

Looking for better trading results? This indicator shows you how to identify price reversals in a timely manner.

by John F. Ehlers

The common assumption is that prices have a Gaussian, or normal, probability density function (PDF). A Gaussian PDF is the bell-shaped curve you are probably familiar with, where 68% of all samples fall within one standard deviation around the mean. Do prices follow such a curve? No, but many people think so, and that mistaken assumption is why many trading indicators fail.

Suppose prices were to behave like a square wave, which is a wave that shows gaps between two values, but remains consistent. With that in mind, if you tried to use the price crossing a moving average as a trading system, you would fail because the price, behaving like a square wave, would have already switched to the opposite value by the time the movement was detected. Remember, in a square wave, there are only two price values. Therefore, the probability distribution is 50% that the price will be at one value or the other. The probability distribution of the square wave can be seen in Figure 1. Clearly, this probability function is a long way from the normal bell curve with which we are familiar.

There is no great mystery about the meaning of a probability density function or how it is computed; it is simply the likelihood that price will assume a given value in a given range. Think of it this way: Construct a waveform by arranging beads strung on a series of parallel horizontal

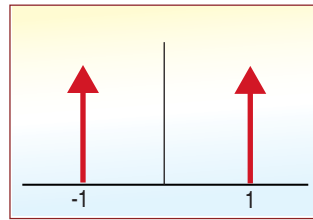


FIGURE 1: THE SQUARE WAVE. The probability distribution of a square wave only has two values.

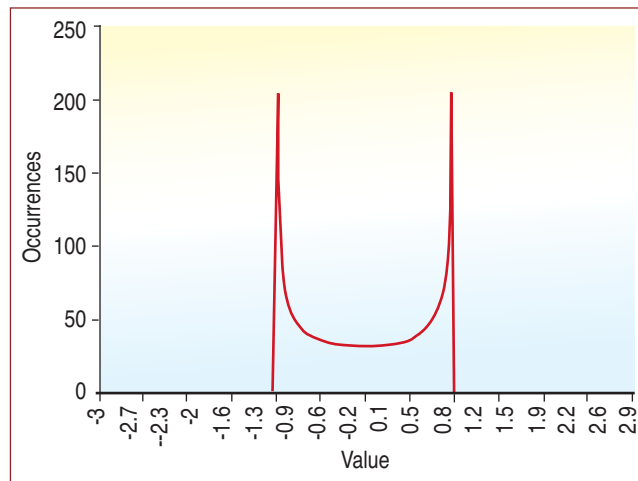


FIGURE 2: SINEWAVE CYCLE PROBABILITY DENSITY FUNCTION. This does not resemble a Gaussian probability density function.

wires. After the waveform is created, turn the frame so the wires are vertical. All the beads will fall to the bottom, and the number of beads on each wire will stack up to demonstrate the probability of the value represented by each wire.

I used that idea, but slightly more sophisticated computer code, to create the probability distribution of a sinewave in Figure 2. In this case, I used a total of 2,000 "beads." This PDF may surprise you, but if you think about it, you will realize that most of the sampled datapoints of a sinewave occur near the maximum and minimum extremes.

The bell curve of a simple sinewave cycle is not at all similar to a Gaussian bell curve. In fact, cycle PDFs are much closer to those of a square wave. The high probability of a cycle being near the extreme values is one of the reasons cycles are difficult to trade. About the only way to trade a cycle successfully is to identify that the market is in a cycle

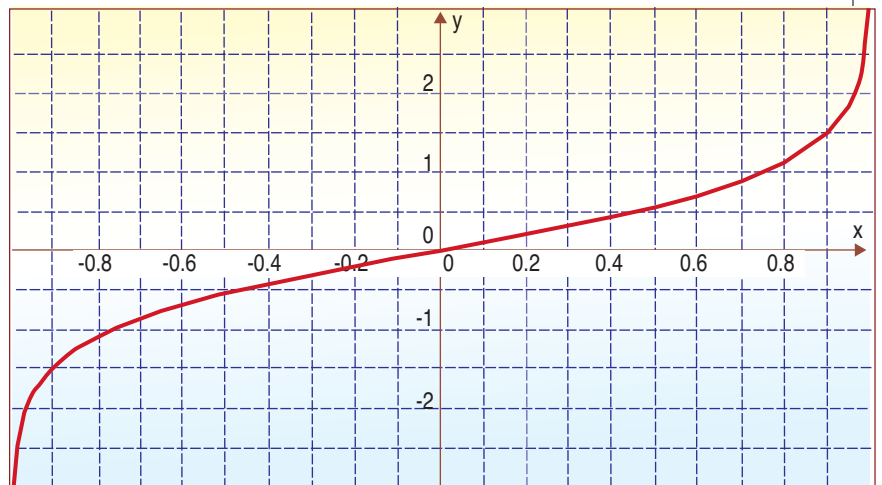


FIGURE 3: NONLINEAR TRANSFER OF THE FISHER TRANSFORM. The inputs (x axis) are converted to outputs (y axis) and have a nearly Gaussian probability distribution function.

mode and, based on the assumption that the cycle will continue into the future, predict the cyclic turning point. This is the technique used in MESA2002 and with the Hilbert sinewave indicator.

CALCULATING IT

The Fisher transform changes the PDF of any waveform so the transformed output has an approximately Gaussian PDF. The Fisher transform equation is:

$$y = .5 * \ln \left[\frac{1 + x}{1 - x} \right]$$

Where: x is the input
 y is the output
 \ln is the natural logarithm

The transfer function of the Fisher transform is displayed in Figure 3.

The input values are constrained to be within the range $-1 < X < 1$. When the input data is near the mean, the gain is approximately unity (the output is approximately equal to the input when $X < 0.5$). By contrast, when the input approaches either limit within the range, the output is greatly amplified. This amplification accentuates the largest deviations from the mean, providing the “tail” of the Gaussian PDF. In Figure 4, the PDF of the Fisher-transformed output is displayed as the red line, compared to the input sinewave PDF. The transformed output probability density function is nearly Gaussian, a radical change.

APPLYING IT

What does this mean for trading? If the prices are normalized to fall within the range from -1 to $+1$ and subjected to the Fisher transform, the extreme price movements are relatively rare events. This means the turning points can be clearly identified. The EasyLanguage code to do this can be seen in Figure 5. Value1 is a function to normalize price within its last 10-day range. The period for the range is adjustable as an input. Value1 is centered on its midpoint and then doubled so that Value1 will swing between the -1 and $+1$ limits. Value1 is also smoothed with an exponentially smoothed moving average (EMA) whose alpha is 0.33. The smoothing may allow Value1 to exceed its 10-day range, so limits are introduced to prevent the Fisher transform from crashing due to having a zero or negative value in the denominator of the argument. The Fisher transform is computed to be the variable “Fish.” Both Fish and Fish-delayed-by-one-bar are plotted to provide a crossover system that identifies the cyclic turning points.

The Fisher transform of the prices within a 10-day channel is plotted in the first subgraph below the price bars in Figure 6. Note that the turning points are not only sharp and distinct, but

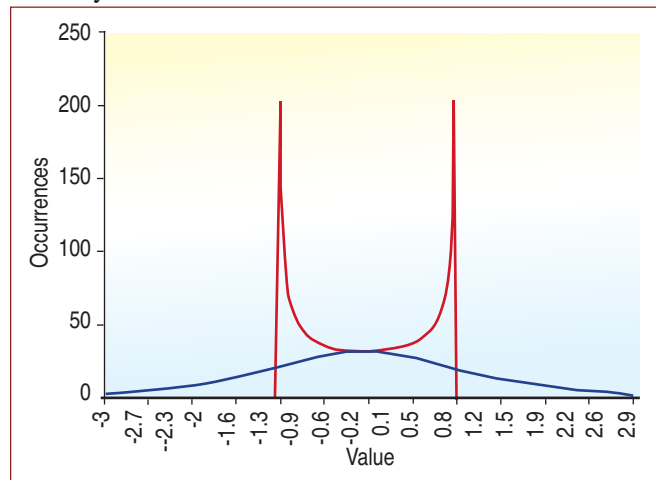


FIGURE 4: FISHER-TRANSFORMED SINEWAVE. The shape nearly resembles a Gaussian probability density function.

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Inputs:      Price((H+L)/2),
             Len(10);

Vars:  MaxH(0),
       MinL(0),
       Fish(0);

MaxH=Highest(Price,Len);
MinL=Lowest(Price,Len);

Value1 = .33*2*((Price-MinL)/(MaxH-MinL)-.5)+.67*Value1[1];
If Value1 > .99 then Value1=.999;
If Value1 < -.99 then Value1=-.999;

Fish = .5*Log((1+Value1)/(1-Value1)) + .5*Fish[1];

Plot1(Fish, "Fisher");
Plot2(Fish[1], "Trigger");

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FIGURE 5: EASLANGUAGE CODE. Here you see how price is normalized to a 10-day channel and its Fisher transform is computed.

they also occur in a timely fashion so profitable trades can be entered. The Fisher transform is also compared to a similarly scaled moving average convergence/divergence (MACD) indicator in subgraph 2 of Figure 6. The MACD is representative of conventional indicators whose turning points are rounded and indistinct in comparison to the Fisher transform. As a result of the rounded turning points, the entry and exit signals are invariably lagging.

The sharp turning points of the Fisher transform mean that these are the positions where the rate of change is the largest. This suggests the use of a momentum function to identify the major turning points. Since a 10-bar channel is used, I multiplied the rate of change of the Fisher transform by 10 and plotted this amplified rate of change over the Fisher transform in the subgraph of Figure 7. The crossing of the amplified rate of change and the Fisher transform clearly identifies each major price turning point.

CONCLUSION

Prices do not have a Gaussian PDF. You can create a nearly Gaussian PDF for prices by normalizing them or creating a normalized indicator such as the relative strength index and applying the Fisher transform. Such a transformed output creates the peak swings as relatively rare events. The sharp turning points of these peak swings clearly and unambiguously identify price reversals in a timely manner. Superior discretionary trading, and mechanical trading systems that perform better can be the result.

John Ehlers is an electrical engineer working in electronic research and development and has been a private trader since 1978. He is a pioneer in introducing maximum entropy spectrum analysis to technical traders through his MESA software.

RELATED READING

- Darley, Roger [2000]. "Updating A New Classic: Optimizing With Hilbert Indicators," *Technical Analysis of STOCKS & COMMODITIES*, Volume 18: November.
- Ehlers, John F. [2000]. "On Lag, Signal Processing, And The Hilbert Transform: Hilbert Indicators Tell You When To Trade," *Technical Analysis of STOCKS & COMMODITIES*, Volume 18: March.

†See Traders' Glossary for definition **S&C**

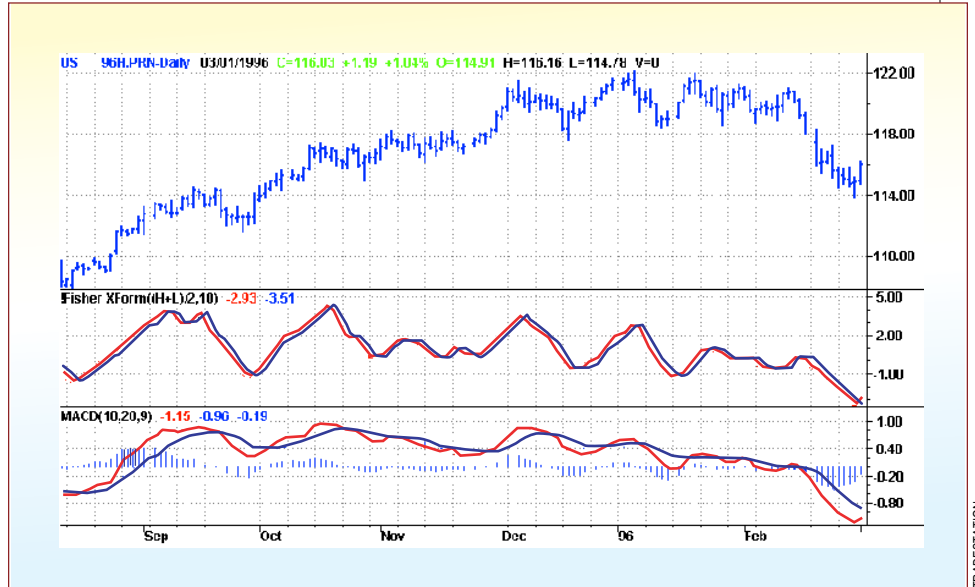


FIGURE 6: FISHER TRANSFORM OF NORMALIZED PRICES. Notice the sharp turning points when compared to conventional indicators such as the moving average convergence/divergence (MACD).

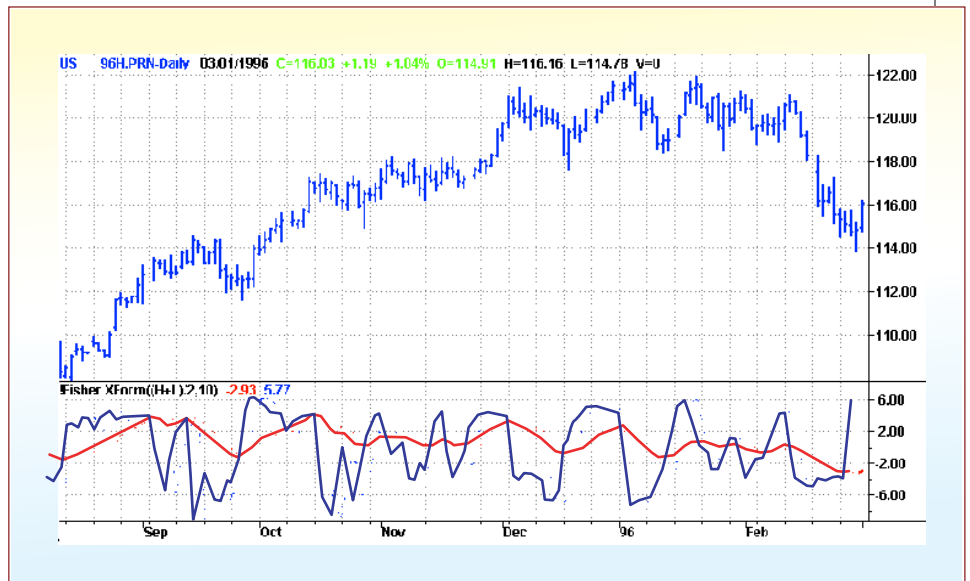


FIGURE 7: MAJOR TURNING POINTS. Crossing of the Fisher transform of normalized prices and 10 times its rate of change clearly identifies major turning points.